Sensor and actuator optimal location for robust control of a galvanizing process.

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Abstract: The problem of reducing vibrations during continuous hot-dip galvanizing process is addressed. Using the finite difference method the concerned part of the steel production line is modeled by a state space version of the axially moving strip equation which takes into account disturbances that may affect its efficient functioning. The synthesis of an appropriate control law for this process aims to reduce the impact of these disturbances and its implementation requires a definition of the position and the number of the sensor/actuator allowing an optimal reduction. Some numerical simulation of the steel strip behavior are presented and discussed for different sources of vibrations with and without a control system.

Keywords: Optimal sensor location, optimal actuator location, observer-based control, perturbation effect attenuation, galvanizing process.

1. INTRODUCTION

In order to achieve a low carbon emission steel plant, digital transformation and industry 4.0 can be the solution by implementing the digital technologies as making steel production fully automated to allow a good cummunication between the plant and its envoronment (the control unit, sensors, actuators). This will contribute to improve the product quality, the maintenance practise and reduced energy consumption. Some analysis report of digital transformation in steel sectors are detailed in Branca et al. (2020)

In the present article, sensor and actuator placement is applied to a galvanizing process in the steel industry represented on the figure 1. In the galvanizing lines, after being heated and cooled in an annealing furnace, the steel strip is immersed in a bath of liquid zinc and then dried by means of nozzles projecting air in order to form a thin and regular layer of zinc on the surface of the strip. The properties of the steel depend on the thickness of the coating layer, so its control is of great interest. However, a set of disturbances creates vibrations on the strip which breaks uniformity of distance between the strip surface and the gas wiping dies and this can considerably degrade the quality of the coating layer. To limit the impact of these disturbances, in the literature some approaches are presented, a modelbased control method using a feedforward control of the transervse strip profil as detailed in Saxinger et al. (2020), a neural network-based coating weight controller is presented in (Pan et al., 2018), the simultaneous placement of sensors and actuators strategy is investigated in this paper in order to minimize or eliminate disturbances at

wiping zone under constraints of number and location of components (due to physical or economic limitations) with the preservation of the controllability and the observability of the system.

The problem of rejection of disturbances is addressed in many works, such as Daraji et al. (2018), I. Bruant (2016), Li et al. (2012) and Potami (2008). In these cited works, the results have been obtained with the help of methodologies based on applied mathematics and control engineering concepts like optimal component placement



Fig. 1. Schematic diagram of the galvanizing process

strategy Westermayer et al. (2009), Pfister. (2012), using several approaches as gramian-based method Marx et al. (2004), H_2 and H_∞ optimization approach Ambrosio et al. (2012), Munz et al. (2014) also through dynamic output feedback Argha et al. (2016) on different type of systems, including large scale systems as treated by Sakha and Shaker (2017) or Borairi and Soufian (2017).

The present paper is organized as follows. The first step, detailed in the second section, is to develop a model of the steel strip in the galvanizing line taking into account the presence and propagation of vibrations. In the third section, based on the obtained model, the key problem is addressed: the search for the optimal placement of the sensors and actuators to respectively efficiently measure the strip vibrations and to minimize the vibrations magnitude by an appropriate control law. Finally, before concluding, the obtained numerical results are exposed and discussed in the fourth section.

2. STRIP MODELING

The studied case is an axially moving strip supported with touch and stabilizing rolls representing the boundary conditions of the partial differential equation. The strip under tension N_x , traveling at a velocity of v of length L, is excited by a force $F(\zeta, w)$ from the wiping system and cooling boxes. The latter are considered as sources of vibration of the steel strip in addition to the rolls. Applying the Hamilton principle, the governing equation of the axially moving strip is derived as:

$$D\nabla^{(4)}z + \rho h \left(\frac{\partial^2 z}{\partial t^2} + 2v \frac{\partial^2 z}{\partial \zeta \partial t} + v^2 \frac{\partial^2 z}{\partial \zeta^2}\right) - N_x \frac{\partial^2 z}{\partial \zeta^2} + c \left(\frac{\partial z}{\partial t} + v \frac{\partial z}{\partial \zeta}\right) = F(\zeta, t)$$
(1)

where $D, z(\zeta, t), \zeta, h, \rho, c$ are flexural rigidity, transversal displacement, vertical position thickness of the strip, volumic mass and damping coefficient.

2.1 Numerical solution

The numerical solution of this type of equation can be obtained by transforming a partial difference equation into an ordinary difference equation by discretizing the spatial variables ζ by the finite difference method. The derivatives of the equation are approximated by differential quotients based on the Taylor series expansions. The equation (1)becomes of the following form:

$$\ddot{z}_{i} = a_{1}z_{i+2} + a_{2}z_{i+1} + a_{3}z_{i} + a_{4}z_{i-1} + a_{5}z_{i-2} a_{6}\dot{z}_{i+1} + a_{7}\dot{z}_{i} + a_{8}\dot{z}_{i-1} + a_{9}f_{act} + a_{10}f_{dist}$$
(2)

where the index i denotes the number of the discretization point on the vertical axis. The equations (2) of the system can be written as:

$$\begin{cases} \dot{x}(t) = Ax(t) + B_u u(t) + B_w w(t) \\ y(t) = Cx(t) \end{cases}$$
(3)

with $x = \begin{pmatrix} z \\ \dot{z} \end{pmatrix}$, $z \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, $w = [w_1, w_2, ...] \in \mathbb{R}^{n_w}$, A, B_u, B_w, C are the state, the input, the disturbance and the output matrix

the output matrices.

For the boundary conditions, the top and the bottom edges are determined by $z(\zeta, t)$, where the followings Dirichlet's conditions are used:

- the bottom of the strip is: $z(0,t) = w_1(t)$,
- the top of the strip is: $z(L,t) = w_2(t)$.

3. SENSOR/ACTUATOR POSITIONING STRATEGY FOR PERTURBATION EFFECT REDUCTION

As stated in the preamble, the objective assigned to this study is the reduction of the disturbance influence on the galvanizing process. The disturbances here result from the steel strip vibrations. Knowing that the system has already been designed to mechanically reduce the impact of these vibrations on its functioning, it remains to design a control law also capable of contributing to this reduction. The problem involves several aspects to be taken into account simultaneously related on the one hand to the sensors and actuators to be used and on the other hand to the control law itself. Concerning the sensors and actuators, two aspects come into consideration: their hardware design and their positioning on the system, this second point being the one considered here.

It is thus a question of finding the location of the sensor(s) allowing an optimal estimation of the disturbance affecting the system, the optimality remaining to be defined. For the actuator(s), it is a question of determining their location in order to optimally reduce the effect of the disturbance, the optimality also remaining to be defined. The choice of the directions of influence of the sensors and actuators are defined in a first synthesis sequence, the structure and parameters of the control law are then to be defined.

Hardware and software constraints could be added to this design. Indeed, for obvious cost reasons, the number of sensors and actuators may be limited. In addition, technical constraints could prohibit the positioning of sensors and actuators in particular locations. As far as the software aspect is concerned, for reasons of computing capacity and time response, one can also imagine that the structure of the control law is constrained. In what follows, for the sake of clarity, these constraints will not be taken into account.

With regard to the possible positioning of sensors and actuators, two sets of positions are defined :

$$E_{a} = \{\zeta_{a,1}, \zeta_{a,2}, \dots, \zeta_{a,P}\} \\ E_{s} = \{\zeta_{s,1}, \zeta_{s,2}, \dots, \zeta_{s,Q}\}$$
(4)

The set $\mathcal{P}(E_a)$, of dimension 2^P , of the parts of E_a allows to consider all the actuator positioning situations and consequently allows to define the set of control matrices B_{ii} associated to it. The same comment can be made about the set of positions E_s and the related matrices C. The result is 2^{P+Q} actuator and sensor placement possibilities. From a practical point of view, the physical constraints prohibiting some sensor/actuator positions may limit the cardinal of these sets. Moreover, in order to limit the computational burden especially in the case of large scale systems, integer optimization tools such as branch and bounds methods may be used. In the following, and this does not prejudice the essence of the proposed method, we limit ourselves to

choosing a sensor among the P and an actuator among the Q, which restricts to $P \times Q$ placement possibilities.

The problem to be solved is therefore the following: which is (are) the best r-subset(s) of a given set of q sensors and p actuators, where $r \leq n$? Beforehand, it is necessary to define what is meant by best and this in relation to the quantification of the perturbation influence reduction.

The dynamic system under consideration is equipped with a u control and subjected to a w perturbation which act respectively on the system through two matrices B_u and B_w :

$$\dot{x}(t) = Ax(t) + \underbrace{[b_1 \ b_2 \ \dots \ b_P]}_{B_u} u(t) + B_w w(t)$$
$$y(t) = \underbrace{[c_1 \ c_2 \ \dots \ c_Q \ 0 \ \dots \ 0]^T}_{C} x(t), \qquad y \in \mathbb{R}^m$$
(5)

The expression (5) involves two parameters (p,q) which define where the command u and the sensors y respectively intervene. This formulation thus takes into account the possibility for the command to act on the dynamics of the system in different positions $B_u(p) = b_p, p = 1, ..., P$. In the same way the parameter q indicates the possible choices of sensors defining the measured outputs of the system $C(q) = c_q, q = 1, ..., Q$.

The two parameters p and q are directly related to actuator and sensor positioning. In the following these two parameters must be optimally chosen.

Remark 1. A more general situation could take into account the possibility that the disturbance could impact the system dynamics in different ways through different directions E(r), with r playing a role analogous to p and q. This is particularly the case in the rapid cooling section before the zinc bath, where the strip is cooled and excited by high-speed gas jets. This cooling is necessary, but the position in the space of the cooling box is not unique and therefore the impact of its position could be taken into account (to see more Renard, M. and Beaujard, K. (2009)).

We are now interested in active vibration control. The aim is to minimize the effect of the w disturbance by optimally positioning the sensor (parameter p) and the actuator of the system (parameter q).

Remark 2. In Potami (2008), the author proposes to simplify the problem because of the computational cost by adopting an assumption of collocation of the sensor/actuator couple, by imposing p = q. Here, for the presentation of our method, we propose to keep the general case and thus to relax this hypothesis.

3.1 Observed state feedback control law

In the present paper the system is controlled by an observed state feedback control law. The applied control law is thus defined by:

$$u(t) = -K\hat{x}(t) \tag{6}$$

where the observed state denoted \hat{x} is provided by a Luenberger observer described by:

$$\hat{x}(t) = (A - LC(q))\hat{x}(t) + B_u(p)u(t) + Ly(t)$$
(7)

where $\tilde{x}(t) = x(t) - \hat{x}(t)$ and p denotes the location of the actuator providing the control. The two gains K and L are to be determined.

In its augmented form, the system (7) can also be written:

$$\begin{cases} \dot{x}_a(t) = A_a(p,q)x_a(t) + B_a w(t) \\ y(t) = C_a x_a(t) + \nu(t) \end{cases}$$
(8)

with $x_a(t) = \begin{pmatrix} x(t) \\ \tilde{x}(t) \end{pmatrix}$, $B_a = \begin{bmatrix} B_w \\ B_w \end{bmatrix}$, $C_a = \begin{bmatrix} C & 0 \end{bmatrix}$, C select all displacements states $z, \nu(t)$ measurement noise and

$$A_a(p,q) = \begin{pmatrix} A - B_u(p)K(p) & B_u(p)K(p) \\ 0 & A - L(q)C(q) \end{pmatrix}$$

The Kalman filter gain L is given by:

$$L(q) = X_l C^T(q) V^{-1} (9)$$

where X_l is the solution of the following Riccati equation:

 $AX_l + X_l A^T - X_l C^T(q) R^{-1} C(q) X_l + Q_w = 0$ (10) where $Q_w = B_w W B_w^T$ is related to the disturbance distribution and where W, V are the spectral densities of w(t), v(t) respectively. The feedback gain K is given by:

$$K(p) = M^{-1} B_u^T(p) X_k$$
(11)

where X_k is the solution of the following Riccati equation:

 $A^{T}X_{k} + X_{k}A - X_{k}B_{u}(p)M^{-1}B_{u}^{T}(p)X_{k} + S = 0$ (12) with a quadratic cost function defined as:

$$\Phi = \int_0^\infty \left(\| x(t) \|_S^2 + \| u(t) \|_M^2 \right) dt$$
 (13)

where $||x(t)||_{S}^{2}$ is defined by: $||x(t)||_{S}^{2} = x(t)^{T}Sx(t)$ and the same for $||u(t)||_{M}^{2}$.

3.2 Observer and controller synthesis

The expression (7) describes the dynamics of the augmented state. Since the objective of the control is to reduce the influence of the w disturbance on the state of the system, its state matrix $A_a(p,q)$ must be adequately structured. If the choice of the positions p and q is made, then the gains K and L of the controller and the observer should be adjusted. For each value of p and q, the control efficiency must be quantified. Several performance indexes may be used as the \mathcal{H}_2 or \mathcal{H}_∞ -norms of the closed-loop system. The chosen optimality criterion here is the \mathcal{H}_2 gain as discussed bellow.

3.3 Optimal positioning of the sensor/actuator pair

Remember that the purpose of the control associated with the placement of the sensor/actuator pair is to reduce the influence of the disturbance on the state (or part of the state) of the system. With (7), the transfer function reflecting this influence is :

$$T_{x_aw}(p,q,s) = (sI - A_a(p,q))^{-1} B_a$$
(14)

and the values of p and q (related to the choice of the sensor/actuator pair) jointly influence the dynamics and the gain of this transfer function. An adequate setting of p and q should therefore satisfy two constraints: (i) a "fast" dynamic and (ii) a "low" static gain ; in the following, the second constraint will be preferred. The parameterization of the gain $T_{x_aw}(p,q,s)$ as a function of p and q thus explains the role of the sensor/actuator

pair on the reduction of the impact of the disturbance. In view of (14), an analytical expression of the gain $T_{x_aw}(p,q)$ is complex to explain as a function of p and q. For this reason, the search for the sensor/actuator pair is based on a numerical procedure. For that, one can initially calculate the H_2 -norm of the transfer $T_{x_aw}(p,q,s)$:

$$\|T_{x_aw}(p,q)\|_2^2 = trace\left(B_a^T \tilde{X}(p,q)B_a\right)$$
(15)

where $\tilde{X}(p,q)$ is the solution of the following Lyapunov equation:

$$A_{a}(p,q)^{T}\tilde{X}(p,q) + \tilde{X}(p,q)A_{a}(p,q) + C_{a}^{T}C_{a} = 0$$
 (16)

The use of the expression (7) then proceeds by a numerical evaluation of the norm $|| T_{x_aw}(p,q) ||_2^2$ as a function of p and q. As previously indicated, if the analysis is limited to a sensor among P and an actuator among Q, there are thus $P \times Q$ configurations to be tested, the one qualified as optimal corresponds to the minimum of the norm with respect to p and q:

$$\{\hat{p}, \hat{q}\} = argmin_{p,q} \parallel T_{x_a w}(p,q) \parallel_2^2 \tag{17}$$

Algorithm (1) summarizes the different steps of the proposed approach for the synthesis of the optimal control allowing an attenuation of the effect of disturbances. The optimal control synthesis encompasses the optimal placement of a sensor/actuator couple.

Algorithm 1 Sensor/actuator positioning
for $q = 1$ to Q do
for $p = 1$ to P do
Compute gains K and L from (9) - (13)
Update a system (8)
Solve (16) in respect to $\tilde{X}(p,q)$
Compute the norm $ T_{x_a w}(p,q) _2^2$ (15)
end for
end for

Using $|| T_{x_aw}(p,q) ||_2^2$ solve (17) to obtain the optimal positionning \hat{p}, \hat{q}

Remark 3. Partial attenuation of the effects of the disturbance.

With the previous formulation (8), the influence of the perturbation is taken into account on all the components of the augmented state vector. The user may prefer to focus this influence on certain components of this state z_a . It is then sufficient to define the vector $z_a(t) = C_{z_a} x_a(t)$ as the output of the system (8) where C_{z_a} selects the components z_a of the state x_a on which one wishes to pay attention in terms of influence of the perturbation w. In this case, the matrix $T_{z_aw}(p,q)$ is then substituted to the matrix $T_{x_aw}(p,q)$ and its norm can then be analyzed as previously according to the values of p and q.

Remark 4. Reduction of the calculation load: collocation constraint.

Despite the proposed limitation of placing only one sensor among P and only one actuator among Q, the number of candidate positions is $P \times Q$. A collocation technique can be considered as envisaged in the remark (2). It seems more relevant to adopt a matching or preference technique based on the a priori definition of sensor/actuator pairs that are eligible for technical or financial reasons for example. The table 1 gives an example of matching between 5 sensors $c_i, i = 1, \ldots, 5$ and 3 actuators $a_i, i = 1, \ldots, 3$. As examples, the actuator a_1 can only be used with the sensors c_1 or c_4 , no actuator can be used with the sensor c_2 . Given these a priori pairings, the number of sensor/actuator pairs to be tested is limited.

Table 1. Sensor/actuator pairings

	c_1	c_2	c_3	c_4	c_5
a_1	×	•		×	
a_2			×	×	
a_3	×	•		×	×

4. NUMERICAL RESULTS AND DISCUSSION

In this section, some results are presented. The presented case study is split in three steps. In order to be maximally robust to the disturbances, the first step consists in determining the worst disturbance source on the strip. The second step consists in applying the proposed algorithm for sensor and actuator location. The third step is the comparison of the system responses with and without the considered observed state feedback control.

The first step consists to find the worst distribution of disturbances which maximally influences the propagation of vibrations on a steel strip. For u = 0, equation (3) becomes :

$$\begin{cases} \dot{x}(t) = Ax(t) + B_w w(t) \\ y(t) = Cx(t) \end{cases}$$
(18)

The idea is to find the value of the matrix $B_w(p_w)$ that maximizes the effect of w(t), which leads to find the spatial component that amplifies the effect of vibrations quantified by its 2-norm:

$$\{\hat{\zeta_w}\} = \operatorname{argmax} \parallel T_{xw}(\zeta_w) \parallel_2^2 \tag{19}$$

The 2-norm is evaluated by the following expression :

 $\|T_{xw}(\zeta_w)\|_2^2 = trace\left(CX_w(\zeta_w)C^T\right)$ (20)

where $X_w(\zeta_w)$ is the solution of the Lyapunov equation :

$$AX_w(\zeta_w) + X_w(\zeta_w)A^T + B_w(\zeta_w)B_w^T(\zeta_w) = 0$$
(21)

For a strip of length L = 40m, the two rolls have more impact on the propagation of vibrations (the values are normalized $|| T_{xw}(\zeta)_{bottom} ||_2^2 = 1$, $|| T_{xw}(\zeta)_{top} ||_2^2 = 0.99$).

On the figure 2 are shown the variation of the 2-norm depending on the location ζ on the strip without the boundary conditions.



Fig. 2. Values of the normalized 2-norm depending on the placement of disturbances.

Remark 5. Since the top and bottom roll vibration are the most impacting on the steel strip (see 2), only these cases will be studied. The middle of the band represents the third worst possible location for disturbance.

The second step consists to find the optimal placement of actuators and sensors on a strip to design the optimal observer-based controller.

On the figure 3 are gathered some values of the 2-norm according to the locations of the actuators and sensors. These values are normalized by the maximal obtained value. Many locations are possible since n = 113 but for the sake of clarity the figure 3 only displays some representative curves among the 113 possible ones. Each curve represents, for a given placement of a sensor, the variation of the 2-norm depending on the location of the actuators.

The steel strip is excited by vibrations of the rolls with real disturbances values of a galvanizing line.

- Case A: Only the bottom roll are excited (first column of B_w).
- Case B: The top and the bottom rolls are excited (first and second column of B_w).

The optimal placement obtained for a single sensor and a single actuator in case A and B is at $\{p_{opt}, q_{opt}\} =$ $\{1,1\}$ and for 2 actuators and 2 sensors in case A and B is at $\{p_{opt}, q_{opt}\}_A = \{\{1,2\}, \{1,2\}\}, \{p_{opt}, q_{opt}\}_B =$ $\{\{1,113\}, \{1,113\}\}$ respectively.



Fig. 3. Values of normalized 2-norm of the system according to the placement of the sensors and actuators.

In the third step, the behavior of the strip is compared in three different cases: in the open loop case only affected by disturbances and in two closed loop settings with one or two actuator(s) and sensor(s) in the wiping zone. For the system with 1 actuator and 1 sensor, as previously mentioned the optimal location is $\{p_{opt}, q_{opt}\} = \{1, 1\}$. In the closed loop setting with 2 actuators and 2 sensors, the optimal location is $\{p_{opt}, q_{opt}\}_A$ and $\{p_{opt}, q_{opt}\}_B$.

Remark 6. As it can be seen on the figure 4, the control based on the proposed optimal sensor and actuator location efficiently reduces the steel strip vibrations caused by the bottom (and top) roll(s) in both cases.



Fig. 4. Comparison of the system responses with and without active vibration control (top: without control, middle: controlled with 1 sensor and 1 actuator, bottom: controlled with 2 sensors and 2 actuators.

5. CONCLUSION AND PERSPECTIVE

The aim of this paper is to jointly handle with the placement of sensors and actuators as well as the synthesis of a feedback control for the reduction of disturbances. The proposed approach analyzes the influence of the position of the sensor / actuator on the estimation of the disturbance and to reduce its impact on the state of the system. The first numerical results obtained after simulation of the vibratory state of a galvanizing process give realistic positions of the sensor / actuator couple. The continuation of this work covers different directions with the priority of using a two-dimensional model of the galvanizing process and the establishment of numerical results at the placement of several sensors and actuators. In the short term, tests on a pilot site in the ongoing instrumentation phase will be carried out for the purpose of validating the proposed approach.

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