Operating mode recognition. Application to a grinding mill process.

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Abstract— **Process monitoring needs the development of data analysis tools aiming at recognizing, at each time instant, system operating mode using the measurement collected on the system. This communication aims at presenting a method relying on measurement analysis, able to identify operating modes without the knowledge of the mathematical models describing these modes. The proposed method relies on the writing of a *global* model combining, in a multiplicative way, the models describing the different modes. The parameters of this global model are then numerically identified from the available set of measurements. The sensitivity analysis of the global model with regard the input/output variables then provides an indicator to identify, at each instant, the current operating mode. The proposed method is applied on a simplified model of a grinding mill.**

I. Introduction

The complexity of technological as well environmental processes renders their management more and more difficult. This complexity comes from the involved phenomena and their numerous interactions, the dimension of the concerned processes but also because it is desirable to optimize their functioning. Process supervision methods therefore become more and more sophisticated and, during the last two decades, process diagnosis has become a discipline in its own.

A. Motivations

The difficulty of implementing monitoring of a process is highly related to the nature of the changes it undergoes over time. We distinguish on the one hand, the modifications imposed by the operator depending on the production requirements (for example modification of the process set points) and, on the other hand, unwanted changes usually due to the environment of process (not or hardly predictable disturbances). The first changes being perfectly mastered, the second type of change is the need to detect very early, in order to propose actions that can eliminate or minimize the adverse effects of these disturbances.

B. Definitions

Disturbances that affect the behavior of a system can also affect its actuators, its sensors or the components constituting the system itself. Understandably, when the diagnostic operation is not only to detect a change in behavior, but also to determine or locate the affected elements, this step being known under the term fault isolation. This step is usually completed, if possible, by a fault characterization that is to say by an estimate of the amplitude. Based on this magnitude, reflecting the severity of the fault, the control law to counteract the influence of this failure will be defined.

Monitoring can also be done from the knowledge of the different operating modes of the system. Generally, a system is characterized by a nominal operating mode corresponding to normal operation mode. When knowledge of the system is sufficient and when the quality of historical data permits, it is common to have other information characterizing normal and abnormal operating modes. In this case, monitoring, so-called supervised mode, is to detect as quickly as possible the eventual transition from one mode to another, then consider compensatory actions to be taken to restore functioning in the nominal mode.

There are however more difficult situations where different modes of operation have not yet been characterized. In this case, monitoring will be carried out in unsupervised mode. The only information available is the measurements of the system during operation; the latter must contain sufficient information to discern the operating modes even if they were not a priori characterized. In the remainder of this communication, it is this situation that will be exposed.

C. Historic elements

Detecting change of operation modes has been the subject of numerous studies in the field of signal processing. The first of these studies have focused on determining average jumps in signals [11], these jumps are themselves images of changes in a system. These techniques were then generalized to the phase jump detection, variance and frequency [15] and also in estimating regime change time instants [20] [21] [9].

This detection is directly applied to the signals from the sensors, but often detected jumps are not attributable to sensors but changes are the result of system behavior modifications. For this reason, these skip detection techniques must be applied to signals reflecting system behavior changes, such as the signal generated by the innovation sequence of the Kalman filter [16] which is of particular structure. This gave rise to many developments on the construction of indicators suitable for burnout detection. In particular, these indicators have been structured so as to locate and isolate faults and operating modes. Note that most of these techniques rely on the use of models that characterize the normal operation of the systems and more rarely the malfunction situations. In

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addition, some techniques have been developed also in the absence of a model describing the operation of the systems, especially using the principal component analysis techniques [13] to the linear case [2], [3], and their extensions known of kernel methods in the nonlinear case [14], [19].

The problem becomes more difficult when the event responsible for the mode change is not known. Indeed, for this unsupervised classification problem, if the different models are unknown, it is necessary to estimate simultaneously model parameters and data partitioning in order to associate to each model data that will allow its identification. As regards the application domain, the detection of regime change is the subject of many studies and this in a variety of fields, such as economics and finance, traffic and epidemics, image analysis, to name a few. The motivation for this interest probably lies in the issues related to the ability to detect as early as possible the change of mode of operation, so as to provide appropriate control strategies. However, little work on production systems or more generally technological systems have been published. Nevertheless include [10] for the detection of regime change operation of aircraft engines (due to the onset of mechanical vibrations), [12] for monitoring flight trajectories using a hybrid representation of their behavior, [25] in mining engineering, [4] in metallurgical engineering and [7] for a chemical process supervision.

The field of environmental monitoring is also the subject of many applications. In [8], the authors compare two probabilistic strategies to detect changes in the regime of a river. In [18] and [23], the application relates to the detection of regime change in marine ecosystems.

In all these applications, it is noted that it is necessary to know the models characterizing different operating system. This is to be compared with the proposed method in which these models are not required. Conversely, a single model, resulting in a certain multiplicative form all operating regimes, is constructed without knowing the parameters of each operating mode. The main contribution of the proposed method is to detect mode changes without knowing the model parameters characterizing each mode. The number of operating modes (described by so-called local models) as well as the model structures describing each of these modes are known a priori. The method relies on the estimation of the parameters of a "global" model of the system, resulting from a multiplicative combination of local models. The sensitivity analysis of the global model with regard the input/output variables then provides an indicator to detect changes in the operating mode.

D. Hypotheses

As previously mentioned, the proposed mode recognition method doesn't need *a priori* the knowledge of the models describing these modes.

In the sequel, the following assumptions are assumed:

- The number of operating modes is *a priori* known and limited to 2;
- The input/output models describing each operating modes are linear in their variables;
- A set of measurements collected on the system assumed to operate according to all its potential operating modes (here the two modes) is available;
- The inputs of the system are sufficiently persistent to allow the identification of its behaviour.

II. Main goal of the proposed method

The main goal of the proposed method is to detect the change of operating mode of a system based on the analysis of the measurements of its different input/output variables. The proposed method relies on the writing of a *global* model combining in a multiplicative way the models describing the different modes. The parameters of this global model are then numerically identified from the available set of measurements. The sensitivity analysis of the global model with regard the input/output variables then provides an indicator to identify, at each instant, the current operating mode. Subsection A uses a very simple model allowing to give the principle of the method.

A. System with two input variables

1) Local and global models: Let us denote y the output variable and x_1, x_2 the two input variables. The models describing, at the discrete time instant k, the two considered operating modes are written as:

$$\begin{cases} \text{Mode } M_1 & : \quad y_k + b_1\, x_{1,k} + a_1\, x_{2,k} = 0 \\ \text{Mode } M_2 & : \quad y_k + b_2\, x_{1,k} + a_2\, x_{2,k} = 0 \end{cases} \tag{1}$$

Depending on the operating conditions, the system behaviour is described at a particular time instant k by one of the two models M_1 or M_2. From the knowledge, at instant k, of the measurement triple $y_k, x_{1,k}, x_{2,k}$, it is desirable to identify the operating mode of the system. As the parameters a_i and b_i of these models are not know, a matching test of the measurement triple to M_1 or M_2 is not possible. Contrarily, this triple necessary verifies the global model defined by the following multiplicative form:

$$(y_k + b_1\, x_{1,k} + a_1\, x_{2,k})(y_k + b_2\, x_{1,k} + a_2\, x_{2,k}) = 0 \tag{2}$$

that can be also written as:

$$\begin{aligned} & p_0\, y_k^2 + p_1\, y_k\, x_{1,k} + p_2\, x_{1,k}^2 + p_3\, y_k\, x_{2,k} + \\ & p_4\, x_{1,k}\, x_{2,k} + p_5\, x_{2,k}^2 = 0 \end{aligned} \tag{3}$$

where the parameter p_0 can be arbitrarily chose equal to 1.

Remark 1: The equations (2) and (3) can be compared in order to establish the relations between the local model parameters a_i, b_i and the global ones p_i. In certain cases, depending on some rank conditions, local model parameters can be expressed from global ones. However, in that communication, the persued objective is restricted to the identification of the current operating mode without providing the model describing each mode (at least without searching to estimate explicitly the local model parameters).
With the following definitions:

$$\begin{aligned} z_k &= \begin{bmatrix} y_k & x_{1,k} & x_{2,k} \end{bmatrix}^T \\ R_2 &= \frac{1}{2} \begin{bmatrix} 2p_0 & p_1 & p_3 \\ p_1 & 2p_2 & p_4 \\ p_3 & p_4 & 2p_5 \end{bmatrix} \end{aligned} \tag{4}$$

the global model (3) can be written as:

$$z_k^T R_2 z_k = 0 \tag{5}$$

2) Identification of the global model parameters: We assume now that we have a set of measurements collected on the system during a period where it operates according the two modes M_1 or M_2. As the global model (3) is linear in p_i, a classical least squares method can be used for the parameter identification. More generally, when the considered system have more than one output variables, the parameters can be easily obtained using a Principal Component Analysis (see section III)

3) Mode change indicator: The global model is now identified. The problem now is to recognize, from the knowledge of a new triple of measurements, the mode M_1 or M_2 according which the system operates. This can be done by analysing the direction of the gradient vector of the global model with regard the different variables as it is shown below. Let us define:

$$\begin{aligned} r_k \;\; = \;\; & p_0\, y_k^2 + p_1\, y_k x_{1,k} + p_2\, x_{1,k}^2 + p_3 y_k x_{2,k} + \\ & p_4\, x_{1,k} x_{2,k} + p_5\, x_{2,k}^2 \end{aligned} \tag{6}$$

The gradient σ_k of r_k with regard the variables y_k, $x_{1,k}$ et $x_{2,k}$ is:

$$\sigma_k = \begin{bmatrix} 2p_0 y_k + p_1 x_{1,k} + p_3 x_{2,k} \\ p_1 y_k + 2p_2 x_{1,k} + p_4 x_{2,k} \\ p_3 y_k + p_4 y_{1,k} + 2p_5 x_{2,k} \end{bmatrix} \tag{7}$$

Equation (7) provides an explicit form of the model gradient combining the two operating modes. It depends on numerical values of the system variables and on the known parameters p_i of the global model but do not involve the unknown local model parameters a_i, b_i.

This gradient vector can be used as a mode indicator. At instant k, the measurement triple is $y_k, x_{1,k}, x_{2,k}$. If, at that instant, the system operates according to model M_1, we have $y_k = -b_1 x_{1,k} - a_1 x_{2,k}$ and if it operates according M_2: $y_k = -b_2 x_{1,k} - a_2 x_{2,k}$. Substituting these two expressions in (7) leads to:

$$\begin{aligned} \sigma_{k,1} &= (b_2 - b_1) x_{1,k} + (a_2 - a_1) x_{2,k} \begin{bmatrix} 1 \\ b_1 \\ a_1 \end{bmatrix} \\ \sigma_{k,2} &= (b_2 - b_1) x_{1,k} + (a_2 - a_1) x_{2,k} \begin{bmatrix} 1 \\ b_2 \\ a_2 \end{bmatrix} \end{aligned} \tag{8}$$

The magnitude of $\sigma_{k,1}$ and $\sigma_{k,2}$ are time varying, but each retain a constant direction. Therefore, at instant k, the gradient of r is oriented according one of the two following directions:

$$\bar{\sigma}_1 = \begin{bmatrix} 1 \\ b_1 \\ a_1 \end{bmatrix}, \qquad \bar{\sigma}_2 = \begin{bmatrix} 1 \\ b_2 \\ a_2 \end{bmatrix} \tag{9}$$

More generally, for an available set of measurements at different time instants, the gradient vector orients according the two directions $\bar{\sigma}_1$ or $\bar{\sigma}_2$ only which respectively characterize the modes M_1 et M_2. It is therefore very simple to identify, at each time instant, according which mode the system operates and thus to detect a change of mode.

Let us remark that equation (8) expresses the gradient on the basis of the measurements $x_{1,k}, x_{2,k}$ which are known and the local model parameters a_1, b_1, a_2, b_2 which are unknown. Then, this expression is not useful for the numerical evaluation of the gradient but provides a theoretical explanation about the direction taken by this vector.

Remark 2: The proposed procedure, established for two operating modes, can be easily extended to any number of modes.

4) Implementation of the proposed method: On a practical point of view, the gradient calculus is done using its definition (7) based on the knowledge of the global model parameters. Indeed, (9) cannot be used as it depends on the unknown local model parameters. Therefore, the procedure for determining, at each time, the operating mode of the system can be sum up as:

- from previously acquired data on a system that covered all operating modes, estimate the global model parameters p_i with a least squares method,
- at each time k, using the global model parameters, evaluate, from the inputs and outputs of the system, the gradient vector σ_k
- Compare σ_k with $\bar{\sigma}_1$ and $\bar{\sigma}_2$ and recognize the operating mode.

B. Generalization to linear models of any order

The generalization to a linear system described by n input variables x_i is immediate. This generalization is particularly useful when the exogeneous variables are introduced progressively into the model with the objective to determine its structure. The two modes are then described by:

$$\left\{ \begin{array}{lclcl} \text{Mode 1} & : & y_k - \theta_1^T v_k & = & 0 \\ \text{Mode 2} & : & y_k - \theta_2^T v_k & = & 0 \end{array} \right. \tag{10}$$

y_k and $v_k = \begin{bmatrix} x_{1,k} & \dots & x_{n,k} \end{bmatrix}$ denoting respectively the exogeneous (output) variable and the exogeneous (input) variable vector; θ_1 et θ_2 are the parameter vectors of the models describing the two operating modes. The global model:

$$r_k = (y_k - \theta_1^T v_k)\,(y_k - \theta_2^T v_k) \tag{11}$$

has the following gradient with regard y_k and v_k :

$$\left\{ \begin{aligned} \frac{\partial r_k}{\partial y_k} &= y_k - \theta_2^T v_k + y_k - \theta_1^T v_k \\ \frac{\partial r_k}{\partial v_k} &= -\theta_1\,(y_k - \theta_2^T v_k) - \theta_2\,(y_k - \theta_1^T v_k) \end{aligned} \right. \tag{12}$$

Consequently, if y_k and v_k are the measurements issued from the operating mode M_1, then $y_k = \theta_1^T v_k$, that leads to the following expression of the gradient:

$$\left\{ \begin{aligned} \frac{\partial r_k}{\partial y_k} &= (\theta_1 - \;\theta_2)^T v_k \\ \frac{\partial r_k}{\partial v_k} &= -\theta_1 (\theta_1 - \;\theta_2)^T v_k \end{aligned} \right. \tag{13}$$

So, for data collected on the system that operates according to M_1, the gradient vector orients according the specific fixed direction defined by $\begin{bmatrix}1 & -\theta_1\end{bmatrix}^T$. Identically, when measurements come from the system operating according the M_2 mode, the gradient vector orients according another given direction defined by $\begin{bmatrix}1 & -\theta_2\end{bmatrix}^T$.

In what concern the method implementation, as the θ_1 and θ_2 parameter vectors are unknown, the gradient calculus must be done using the global model (13) written in a linear form with regard the parameters using the so-called Véronèse's transformation[1] :

$$\begin{aligned} r_k = {} & p_0\, y_k^2 + p_1\, y_k\, x_{1,k} + p_2\, x_{1,k}^2 + p_3\, y_k\, x_{2,k} + \\ & p_4\, x_{1,k}\, x_{2,k} + p_5\, x_{2,k}^2 + \cdots + p_m\, x_{n,k}^2, \quad m = \tfrac{(n+1)(n+2)}{2} \end{aligned} \tag{14}$$

With $z_k = \begin{bmatrix}x_k & y_k^T\end{bmatrix}^T$ and:

$$r_k = z_k^T\, R_n\, z_k \tag{15}$$

the gradient with regard the vector z_k is defined by:

$$\frac{\partial r_k}{\partial z_k} = 2\, R_n\, z_k \tag{16}$$

where the matrix R_n only depends on global model parameters. Let us remark the construction of this matrix can be done systematically. As an example, the partition of R_3, for a model with three exogeneous variables is easily established from matrices R_2 and R_1 related to systems with respectively 2 and 1 exogeneous variables. Indeed:

$$R_3 = \left[\begin{array}{ccc|c|c} 2p_0 & p_1 & p_3 & p_6 & p_{10} \\ p_1 & 2p_2 & p_4 & p_7 & p_{11} \\ p_3 & p_4 & 2p_5 & p_8 & p_{12} \\ \hline p_6 & p_7 & p_8 & 2p_9 & p_{13} \\ \hline p_{10} & p_{11} & p_{12} & p_{13} & 2p_{14} \end{array}\right]$$

Remark 3: The writings (10) or (11) can be extended to dynamic (linear) models. This can be done by including delayed measurements in the vector v.

III. Example: grinding mill process

A. Simple model of a grinding mill process

Classically [17], the granularity $g_i(t)$ of the output products of a grinding mill is related to that $g_{e,i}(t)$ of the input products by a mass balance taking into account the selection function S and the breakage one B whose elements are s_i and b_i. For an constant input flowrate Q and a constant load W in the ball mill, a model taking into account two granulometric fractions only can be written as:

$$\left\{ \begin{aligned} \dot{g}_2(t) &= \frac{1}{\tau}(g_{e,2}(t) - g_2(t)) - g_2(t)s_2 + g_1(t)s_1b_1 \\ \dot{g}_1(t) &= \frac{1}{\tau}(g_{e,1}(t) - g_1(t)) - g_1(t)s_1 \end{aligned} \right. \tag{17}$$

[1] Véronèse's transformation of order 2 is the application $\nu_2 : R^n \to R^d$, with $d = \binom{n+1}{2}$, defined by:

$$\nu_2([x_1, \ldots, x_n]^T) = [x_1^2, x_1x_2, x_1x_3, \ldots, x_1x_n, \ldots, x_n^2]^T$$

As a direct consequence, any polynom of order 2 can be written as a linear combination of the monomials $x^\ell = x_1^{n_1} x_2^{n_2} \ldots x_n^{n_n}$, with $0 \le n_i \le 2$ and $\sum_{i=1}^n n_i = 2$.

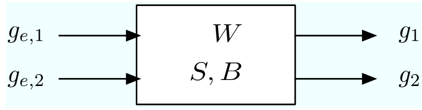

Fig. 1. Input and output granularity distributions

whith $\tau = W/Q$ and where the index $\bullet_1$ denotes the most coarse granular fraction. At steady state, the expression of the output granularity can be deduced:

$$\left\{ \begin{array}{lcl} g_1 &=& \gamma\, g_{e,1} \\ g_2 &=& \alpha\, g_{e,1} + \beta b\, g_{e,2} \end{array} \right. \tag{18}$$

where the t variable was omitted and:

$$\left\{ \begin{array}{l} \alpha = \dfrac{\tau\, s_1\, b_1}{(1+\tau\, s_1)(1+\tau\, s_2)} \\ \beta = \dfrac{1}{1+\tau\, s_2}, \qquad \gamma = \dfrac{1}{1+\tau\, s_1} \end{array} \right.$$

In that example, the system has two inputs and two outputs; then it is characterized by two models. However, the previous described method (section II) can be applied on each model. Clearly this enriches the identification of the operating mode of the sytem. Besides, it's possible to consider an interrelated output model eliminating the $g_{e,1}$ variable between the two equations (18):

$$g_2 = \delta\, g_1 + \beta\, g_{e,2} \tag{19}$$

with:

$$\delta = \frac{\alpha}{\gamma} = \frac{\tau\, s_1\, b_1}{(1+\tau\, s_2)}$$

Although redundant with the two equations (18), certain parameters don't intervene in this equation (19). Therefore, it can be used to confirm or disconfirm the presence of a mode change.

Consider the three model equations (18, 19) and two sets of grinding parameter values $(\alpha_i, \beta_i, \gamma_i, \delta_i,\ i = 1, 2)$ corresponding to two operating modes. The three global models can then be written as:

$$\begin{aligned} r_1 &= (g_1 - \gamma_1\, g_{e,1})(g_1 - \gamma_2\, g_{e,1}) \\ r_2 &= (g_2 - \alpha_1\, g_{e,1} - \beta_1 g_{e,2})(g_2 - \alpha_2\, g_{e,1} - \beta_2 g_{e,2}) \\ r_3 &= (g_2 - \delta_1\, g_1 - \beta_1\, g_{e,2})(g_2 - \delta_2\, g_1 - \beta_2\, g_{e,2}) \end{aligned} \tag{20}$$

The local model parameters $(\alpha_i, \beta_i, \gamma_i, \delta_i)$ being unknown, let us recall that the proposed method only relies on the global model obtained by multiplicative combination of the local models. Using formulation (15), model (19) is written:

$$\begin{aligned} r_1 &= \begin{bmatrix} g_1 & g_{e,1} \end{bmatrix} R_1 \begin{bmatrix} g_1 \\ g_{e,1} \end{bmatrix} \\ r_2 &= \begin{bmatrix} g_2 & g_{e,1} & g_{e,2} \end{bmatrix} R_2 \begin{bmatrix} g_2 \\ g_{e,1} \\ g_{e,2} \end{bmatrix} \\ r_3 &= \begin{bmatrix} g_1 & g_2 & g_{e,2} \end{bmatrix} R_3 \begin{bmatrix} g_1 \\ g_2 \\ g_{e,2} \end{bmatrix} \end{aligned} \tag{21}$$

where the matrices R_i defined as in (4) are defined using global model parameters:

$$R_1 = \begin{bmatrix} 2p_{1,0} & p_{1,1} \\ p_{1,1} & 2p_{1,2} \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 2p_{2,0} & p_{2,1} & p_{2,3} \\ p_{2,1} & 2p_{2,2} & p_{2,4} \\ p_{2,3} & p_{2,4} & 2p_{2,5} \end{bmatrix} \tag{22}$$

$$R_3 = \begin{bmatrix} 2p_{3,0} & p_{3,1} & p_{3,3} \\ p_{3,1} & 2p_{3,2} & p_{3,4} \\ p_{3,3} & p_{3,4} & 2p_{3,5} \end{bmatrix}$$

As explained in section II, the parameters $p_{i,j}$ of the three global models are easily identified from the measurements $\{g_1, g_{e,1}\}$, $\{g_2, g_{e,1}, g_{e,2}\}$ et $\{g_1, g_2, g_{e,2}\}$. A most elegant approach consists in expressing the three global models as functions of all the input/output variable vector z:

$$z = \begin{bmatrix} g_1 & g_2 & g_{e,1} & g_{e,2} \end{bmatrix} \tag{23}$$

under the form:

$$r_i = z^T R_i z \quad i = 1, 2, 3 \tag{24}$$

with:

$$R_1 = \frac{1}{2} \begin{bmatrix} 2p_{1,0} & 0 & p_{1,1} & 0 \\ 0 & 0 & 0 & 0 \\ p_{1,1} & 0 & 2p_{1,2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_2 = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2p_{2,0} & p_{2,1} & p_{2,3} \\ 0 & p_{2,1} & 2p_{2,2} & p_{2,4} \\ 0 & p_{2,3} & p_{2,4} & 2p_{2,5} \end{bmatrix} \tag{25}$$

$$R_3 = \frac{1}{2} \begin{bmatrix} 2p_{3,0} & p_{3,1} & 0 & p_{3,3} \\ p_{3,1} & 2p_{3,2} & 0 & p_{3,4} \\ 0 & 0 & 0 & 0 \\ p_{3,3} & p_{3,4} & 0 & 2p_{3,5} \end{bmatrix}$$

This formulation is particularly useful for the estimation of the three global model parameters $p_{i,j}$ that share the same data measurement set z. The Principal Component Analysis (PCA) is well suited for that estimation. The vector v of variables intervening in the data matrix on which the PCA is applied comes from equation (25) and the usage of Véronèse's application:

$$\begin{aligned} v \;=\; [\, & g_1^2 \;\; g_1 g_2 \;\; g_1 g_{e,1} \;\; g_1 g_{e_2} \;\; g_2^2 \;\; g_2 g_{e,1} \\ & g_2 g_{e,2} \;\; g_{e,1}^2 \;\; g_{e,1} g_{e,2} \;\; g_{e,2}^2 \,] \end{aligned} \tag{26}$$

In v, the variables that appear come from developing products defining r_i (24). The variable measurements $(g_1, g_2, g_{e,1}, g_{e,2})$ being known at each time instant k, the values v_k of v are also known. That's allows to build the observation matrix:

$$Z = \begin{bmatrix} v_1 & v_2 & \dots & v_N \end{bmatrix}^T \tag{27}$$

on which the PCA is applied in order to extract all the redundancy equations, i.e. the global model of the system. The parameters of the three global models are then generated by the eigenvectors of the matrix $Z^T Z$ that correspond to the three null eigenvalues (or, due to the presence of noise, to the three least eigenvalues).

B. Mode change indicators

The mode change indicators are provided by the gradients of the expressions (25) with regard the variables z: $g_i = \frac{\partial r_i}{\partial z}$. Explicitly, the eight indicators are obtained:

$$I_1 = \begin{bmatrix} 2p_{1,0} g_1 \;+\; p_{1,1} g_{e,1} \\ p_{1,1} g_1 + 2p_{1,2} g_{e,1} \end{bmatrix}$$

$$I_2 = \begin{bmatrix} 2p_{2,0}\, g_2 + p_{2,1}\, g_{e,1} + p_{2,3}\, g_{e,2} \\ p_{2,1}\, g_2 + 2p_{2,2}\, g_{e,1} + p_{2,4}\, g_{e,2} \\ p_{2,3}\, g_2 + p_{2,4}\, g_{e,1} + 2p_{2,5}\, g_{e,2} \end{bmatrix} \tag{28}$$

$$I_3 = \begin{bmatrix} 2p_{3,0} g_1 + p_{3,1}\, g_{e,1} + p_{3,3}\, g_{e,2} \\ p_{3,1} g_1 + 2p_{3,2}\, g_{e,1} + p_{3,4}\, g_{e,2} \\ p_{3,3} g_1 + p_{3,4}\, g_{e,1} + 2p_{3,5}\, g_{e,2} \end{bmatrix}$$

Let us recall that, for the measurement set $(g_1, g_2, g_{e,1}, g_{e,2})$ each gradient vector orients in only two distinct directions, each of them being the image of a mode. To get rid of their magnitude variations, each gradient vector could be normed which eases their interpretation.

C. Numerical results

The realized trials, with $\tau = 1.5$, are dedicated to the detection of changes in the grinding parameters s_1, s_2, b_1. Two trials are shown, the first one is related to the modification of the selection parameter s_1 which takes the value 0.25 along the whole simulation horizon except between the time instants 10 to 23 where its value is 0.35. The corresponding granular distributions are shown in figure 2. The figure 4, which presents the time evolution of only one component of the gradient vector, perfectly highlights this change of operating mode.

The second trial concerns a modification of the breakage parameter b_1 which evolves from 0.30 to 0.35 from time instants 10 to 23. Figure 3 shows the resulting granular distributions. The time evolution of the three indicators, shown in figure 5, visualizes the mode change, but only on two components of the gradient vector. This preferential sensitivity can be easily explained by the model dependence with regard to the parameters that induce the mode change. The table I precises the influence ($\times$ mark) of the grinding parameters s_1, s_2, b_1 on the parameters $\alpha, \beta, \gamma, \delta$ of the global models as well as the models r_1, r_2, r_3. The parameters s_2 et b_1 have the same structural influence and modify two indicators only, the parameter s_1 influencing the three indicators.

IV. CONCLUSION

The recognition strategy of active mode of a system was presented in a restrictive context (limited and known number of operating modes, absence of measurement noises, etc.). However it is an original approach for operating mode recognition that takes place in the system supervision framework. The main contribution consists in the ability to discriminate and to recognize operating modes of a system

	s_1	s_2	b_1
α	$\times$	$\times$	$\times$
β	.	$\times$	.
γ	$\times$	.	.
δ	$\times$	$\times$	$\times$
r_1	$\times$	.	.
r_2	$\times$	$\times$	$\times$
r_3	$\times$	$\times$	$\times$

TABLE I
VARIABLE OCCURENCES

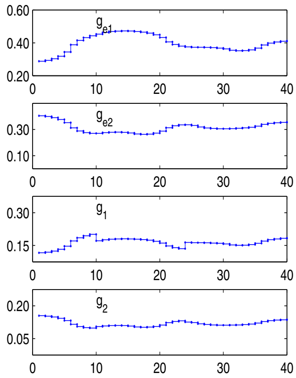

Fig. 2. Input and output granularities with the variation of s_1

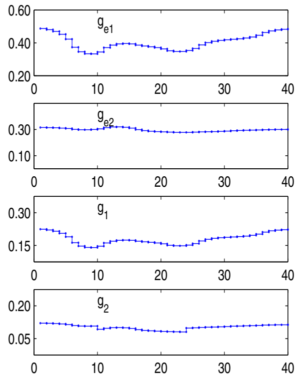

Fig. 3. Input and output granularities with the variation of b_1

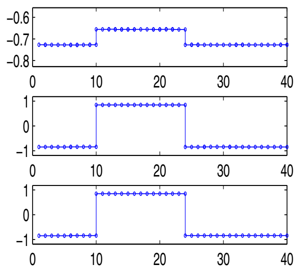

Fig. 4. Gradients of the global models. Variation of s_1

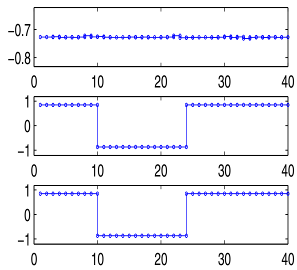

Fig. 5. Gradients of the global models. Variation of b_1

without the precise knowledge (parameter values) of the models describing each mode.

The numerical application, applied on a very simple example, has the advantage to explain with straightforwardness the method implementation. Some stated assumptions can be easily relaxed. It is the case of the number of modes and the order of linear models describing the different modes.

A important topic that requires a deep analysis and necessitates further developments concerns the measurement noise influence. In that context, the analysis must probably relies on the design of mode indicators taking into account simultaneously the distance between two operating mode (which must be defined) and the upper bounds of the measurement noises (in a set membership approach) or the probability density function of the noise (in a stochastic framework).

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