A method to avoid the unmeasurable premise variables in observer design for discrete time TS systems

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Outline of the talk

Problem statement

- The T-S approach of nonlinear estimation
- Background on TS observer design
- Background on nonlinear observers
- The proposed approach: immersion and TS observer design
- A preliminary example

Main result

- Immersion-based Observer design algorithm
- Some extensions (PIO, continuous time)
- Illustrative Example

Concluding remarks

- Given a discrete-time nonlinear system:

$$(NLS)\ \begin{cases} x_{k+1} = f(x_k, u_k) \\ y_k = g(x_k, u_k) \end{cases}$$

the goal is to **estimate** x_k from the knowledge of u_k and y_k

- Using the Nonlinear Sector Appraoch:

$$(NLS)\ \begin{cases} x_{k+1} = f(x_k, u_k) \\ y_k = g(x_k, u_k) \end{cases} \Rightarrow \ (TS)\ \begin{cases} x_{k+1} = A_h x_k + B_h u_k \\ y_k = C_h x_k + D_h u_k \end{cases}$$

where $0 \leq h_i(\xi_k) \leq 1$ and $\sum_{i=1}^{r} h_i(\xi_k) = 1$ and $\sum_{i=1}^{r} h_i(\xi_k) X_i = X_h$.

- **Observer design performed on the T-S system**

- Depending on the non-linearities of (NLS) $\rightarrow \xi(u_k), \xi(y_k), \xi(x_k), \ldots$
$\rightarrow \xi$ may be measurable (MPV) or unmeasurable (UMPV)

If the premise variable ξ_k are measurable (i.e. $\xi(u, y)$): MPV

- T-S observer:

$$(TSO)\ \begin{cases} \hat{x}_{k+1} = A_h\hat{x}_k + B_h u_k + L_h(y_k - \hat{y}_k) \\ \hat{y}_k = C_h\hat{x}_k + D_h u_k \end{cases}$$

  the observer gains L_i are determined such that: $\hat{x}_k \to x_k$

- Observer design:
  - state estimation error $e_k = x_k - \hat{x}_k$ obeys to:
    $$e_{k+1} = (A_h - L_h C_h)e_k$$
  - easily put as an LMI problem and solved:
    $$(A_h - L_h C_h)^T P (A_h - L_h C_h) - P < 0$$
  - classical relaxation schemes:
    (Tuan et al, IEEE TFS, 2001), Polya (Sala & Ariño, FSS, 2007), descriptor approach (Tanaka et al, IEEE TFS, 2007), Sum-Of-Squares (Tanaka et al., IEEE TFS, 2009), ...

If the premise variable ξ_k are unmeasurable (i.e. $\xi(x)$): UMPV

- T-S observer depending on the estimated premise variable:

$$(TSO)\ \begin{cases} \hat{x}_{k+1} = A_{\hat{h}}\hat{x}_k + B_{\hat{h}}u_k + L_{\hat{h}}(y_k - \hat{y}_k)) \\ \hat{y}_k = C_{\hat{h}}\hat{x}_k + D_{\hat{h}}u_k \end{cases} \qquad \sum_{i=1}^{r} h_i(\hat{\xi}_k)X_i = X_{\hat{h}}$$

- TS system and estimation error are rewritten:
  - Lipschitz approach and majoration:
    $$x_{k+1} = A_{\hat{h}}x_k + B_{\hat{h}}u_k + \delta_k, \text{ with } \delta_k \text{ Lipschitz in } x_k$$
    (Bergsten & Palm, FUZZ'IEEE, 2000), (Ichalal et. al., IET CTA, 2010), ...
  - Pseudo-perturbation approach and $\mathcal{L}_2$-attenuation or ISS:
    $$x_{k+1} = A_{\hat{h}}x_k + B_{\hat{h}}u_k + \delta_k$$
    (Ichalal et. al., IEEE MSC 2012), (Ichalal et. al., IEEE MED, 2012), ...
  - Pseudo-uncertainty approaches and matrix majorations:
    $$x_{k+1} = (A_{\hat{h}} + \Delta A_k)x_k + (B_{\hat{h}} + \Delta B_k)u_k$$
    (Ichalal et. al., IEEE CDC 2009), (Ichalal et. al., IEEE MED, 2009), ...

Some variable changes may ease the observer design

- Nonlinear transformation (Krener & Respondek, SIAM JCO, 1985):

$$(NLS)\ \begin{cases} x_{k+1} = f(x_k, u_k) \\ y_k = g(x_k, u_k) \end{cases} \Rightarrow \begin{cases} z_{k+1} = Az_k + \varphi(u_k, y_k) \\ y_k = Cz_k \end{cases}$$

  with $z_k = \Phi(x_k)$ and $dim(x_k) = dim(z_k)$

- Immersion (Besançon & Ţiclea, IEEE TAC, 2007):

$$(NLS)\ \begin{cases} x_{k+1} = f(x_k, u_k) \\ y_k = g(x_k, u_k) \end{cases} \Rightarrow\ (IS)\ \begin{cases} z_{k+1} = Az_k + \varphi(u_k, y_k) \\ y_k = Cz_k \end{cases}$$

  with $z_k = \Phi(x_k)$ and **dim(x) < dim(z)**

- the nonlinear part of (IS) $\varphi(u_k, y_k)$ depends on accessible signals
- the state of (NLS) can be deduced from the state of (IS)
- the state estimate $\hat{x}$ is deduced from the extended state estimate $\hat{z}$

$$\text{Nonlinear system: } \begin{cases} x_{k+1} = f(x_k, u_k) \\ y_k = Cx_k \end{cases}$$

$\Downarrow$ **Immersion**

$$\text{Q-LPV system: } \begin{cases} z_{k+1} = A(y_k, u_k)z_k + \underbrace{B(y_k, u_k)u_k}_{\varphi(u_k, y_k)} \\ \\ y_k = \mathcal{C}z_k \end{cases}$$

$\Downarrow$ **Nonlinear Sector Transformation**

$$\text{TS system: } \begin{cases} z_{k+1} = \mathcal{A}_h z_k + \mathcal{B}_h u_k \\ y_k = \mathcal{C}z_k \end{cases} \quad \textit{with } h_i(y, u)$$

$\Downarrow$ **Observer design for TS MPV system**

$$\text{TS Observer: } \begin{cases} \hat{z}_{k+1} = \mathcal{A}_h \hat{z}_k + \mathcal{B}_h u_k + \mathcal{L}_h(y_k - \hat{y}_k) \\ \hat{y}_k = \mathcal{C}\hat{z}_k \end{cases}$$

- Given a nonlinear system

$$\begin{cases} \begin{pmatrix} x_{1,k+1} \\ x_{2,k+1} \end{pmatrix} = \begin{pmatrix} x_{2,k} \\ -2(x_{1,k})^3 + 2x_{1,k} + 0.3x_{1,k}x_{2,k} + u_k \end{pmatrix} \\ \quad y_k \quad = \begin{pmatrix} 0 & 1 \end{pmatrix} x_k \end{cases}$$

- Initialize the augmented state by $z_k = x_k$ and use $y_k = z_{2,k}$

$$\begin{cases} \begin{pmatrix} z_{1,k+1} \\ z_{2,k+1} \end{pmatrix} = \begin{pmatrix} z_{2,k} \\ 2z_{1,k} + u_k + 0.3z_{1,k}y_k \end{pmatrix} + \begin{pmatrix} 0 \\ -2(z_{1,k})^3 \end{pmatrix} \\ \quad y_k \quad = \begin{pmatrix} 0 & 1 \end{pmatrix} z_k \end{cases}$$

- Augment the state with $z_{3,k} = z_{1,k}^3 \Rightarrow z_{3,k+1} = (z_{2,k})^3 = (y_k)^2 z_{2,k}$

$$\begin{cases} \begin{pmatrix} z_{1,k+1} \\ z_{2,k+1} \\ z_{3,k+1} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 2 + 0.3y_k & 0 & -2 \\ 0 & (y_k)^2 & 0 \end{pmatrix} \begin{pmatrix} z_{1,k} \\ z_{2,k} \\ z_{3,k} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} u_k \\ \quad y_k \quad = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} z_k \end{cases}$$

- Use the nonlinear sector transformation, with MPV $\xi_k = [y_k \ (y_k)^2]^T$

$$\begin{cases} z_{k+1} = \mathcal{A}_h z_k + \mathcal{B}_h u_k \\ \quad y_k = \mathcal{C} z_k \end{cases}$$

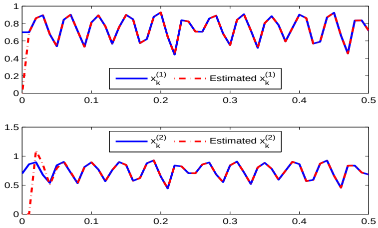
- The TS system and TS Observer:

$$\begin{cases} z_{k+1} = \mathcal{A}_h z_k + \mathcal{B}_h u_k \\ y_k \quad = \mathcal{C} z_k \end{cases} \quad \textit{and} \quad \begin{cases} \hat{z}_{k+1} = \mathcal{A}_h \hat{z}_k + \mathcal{B}_h u_k + \mathcal{L}_h(y_k - \hat{y}_k) \\ \hat{y}_k \quad = \mathcal{C} \hat{z}_k \end{cases}$$

- Quadratic Lyapunov function: $V(e_k) = e_k^T P e_k$
- Solving the LMI in P and K_i for $i = 1, \ldots, 4$, gives the TSO gains:

$$\begin{pmatrix} -P & \mathcal{A}_i^T P - \mathcal{C}^T K_i^T \\ P\mathcal{A}_i - K_i \mathcal{C} & -P \end{pmatrix} < 0 \Rightarrow \mathcal{L}_i = P^{-1} \mathcal{K}_i$$

- State estimates: $\hat{x}_{1,k} = \hat{z}_{1,k}$ and $\hat{x}_{2,k} = \hat{z}_{2,k}$



(Immersion)

[**Step 1**] - Initialize the new variables: $z_{i,k} = x_{i,k}$, for $i = 1, \dots, dim(x)$.

[**Step 2**] - For each new defined variable $z_{l,k}$, compute $z_{l,k+1}$.
- What depends on measured signals is included in the Q-LPV matrices: $A(y_k, u_k)$ and $B(y_k, u_k)$.
- Remaining non-linearities are defined as new variables $z_{l+\dots,k}$.

[**Step 3**] - If new variables are defined, go to **Step 2**. Else, go to **Step 4**

(Observer design)

[**Step 4**] - Apply the nonlinear sector transformation on the Q-LPV system:

$$z_{k+1} = A(y_k, u_k) z_k + B(y_k, u_k) u_k.$$

[**Step 5**] - Apply any observer design for TS system with MPV

PI Observer design

- If the NLS is affected by slow dynamics unknown inputs (UI) d_k
- Include the UI (or the non-linearities depending on it) in the extended state z_k:

$$\begin{cases} z_{k+1} = \mathcal{A}_h z_k + \mathcal{B}_h u_k + \mathcal{B}_h^d d_k \\ (d_{k+1} \simeq d_k \ assumed) \\ y_k = \mathcal{C} z_k \end{cases}$$

- Design a Proportional-Integral T-S Observer:

$$\begin{cases} \hat{z}_{k+1} = \mathcal{A}_h \hat{z}_k + \mathcal{B}_h u_k + \mathcal{B}_h^d \hat{d}_k + \mathcal{L}_h (y_k - \hat{y}_k) \\ \hat{d}_{k+1} = \hat{d}_k + \mathcal{K}_h (y_k - \hat{y}_k) \\ \hat{y}_k = \mathcal{C} \hat{z}_k \end{cases}$$

Continuous time case

- Roughly speaking, the computation of $z_{i,k+1}$ in [**Step 2**] becomes $\frac{dz_i}{dt}$
- For further details, see (Ichalal et. al., IFAC ICONS, 2016)

- Given a nonlinear system with UI d_k:

$$\begin{cases} \begin{pmatrix} x_{1,k+1} \\ x_{2,k+1} \end{pmatrix} = \begin{pmatrix} x_{2,k} \\ -2(x_{1,k})^3 + 2x_{1,k} + 0.3x_{1,k}x_{2,k} + x_{1,k}d_k \end{pmatrix} \\ \quad y_k \quad = \begin{pmatrix} 0 & 1 \end{pmatrix} x_k \end{cases}$$

- Initialize the augmented state by $z_k = x_k$. With $y_k = z_{2,k}$, it follows:

$$\begin{cases} \begin{pmatrix} z_{1,k+1} \\ z_{2,k+1} \end{pmatrix} = \begin{pmatrix} z_{2,k} \\ 2z_{1,k} + 0.3z_{1,k}y_{2,k} \end{pmatrix} + \begin{pmatrix} 0 \\ -2(z_{1,k})^3 + z_{1,k}d_k \end{pmatrix} \\ \quad y_k \quad = \begin{pmatrix} 0 & 1 \end{pmatrix} z_k \end{cases}$$

- Augment the state with $z_{3,k} = z_{1,k}^3$ and $z_{4,k} = x_{1,k}d_k$.
  Then: $z_{3,k+1} = (z_{2,k})^3 = (y_k)^2 z_{2,k}$ and $z_{4,k+1} = y_k d_k$

$$\begin{cases} \begin{pmatrix} z_{1,k+1} \\ z_{2,k+1} \\ z_{3,k+1} \\ z_{4,k+1} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 2 + 0.3y_k & 0 & -2 & 1 \\ 0 & (y_k)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_{1,k} \\ z_{2,k} \\ z_{3,k} \\ z_{4,k} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ y_k \end{pmatrix} d_k \\ \quad y_k \quad = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix} z_k \end{cases}$$

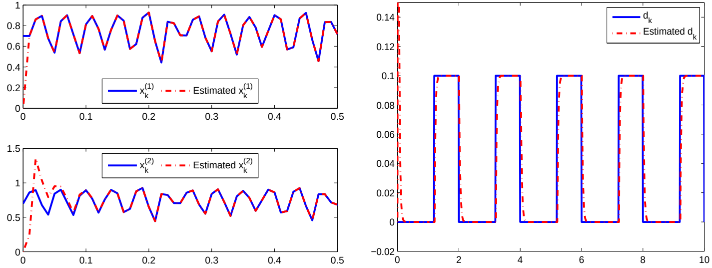
- The nonlinear sector transformation, with $\xi_k = [y_k \; (y_k)^2]^T$, leads to:

$$\begin{cases} z_{k+1} = \mathcal{A}_h z_k + \mathcal{B}_h^d d_k \\ y_k \quad = \mathcal{C} z_k \end{cases}$$

- The PI TS Observer is designed by stabilizing:

$$\begin{pmatrix} z_{k+1} - \hat{z}_{k+1} \\ d_{k+1} - \hat{d}_{k+1} \end{pmatrix} = \begin{pmatrix} \mathcal{A}_h - \mathcal{L}_h \mathcal{C} & \mathcal{B}_h^d \\ \mathcal{K}_h \mathcal{C} & I \end{pmatrix} \begin{pmatrix} z_k - \hat{z}_k \\ d_k - \hat{d}_k \end{pmatrix}$$

- State and UI estimation : $\hat{x}_{1,k} = \hat{z}_{1,k}$, $\hat{x}_{2,k} = \hat{z}_{2,k}$, and $\hat{d}_k = \hat{z}_{4,k} / \hat{z}_{1,k}$



Concluding remarks

- \+ This work is an attempt to bridge observer design in the MPV / UMPV cases
- \+ Any observer design for TS systems with MPV can be used.
- \+ The immersion in a Q-LPV system is less restrictive than the usual immersion in state affine or linear systems.
- \+ The original system state is included in the augmented system state $\rightarrow$ no reverse nonlinear transformation is needed to have $\hat{x}$ from $\hat{z}$.
- \+ The extension to joint state and UI estimation is easy.
- \+ The continuous time case can be dealt with similarly.
- \- The nonlinear systems that can be immersed into Q-LPV systems are not characterized.

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