Unknown input observer for LPV systems with parameter varying output equation

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Abstract: This paper addresses a discussion about Unknown Input Observers (UIO) for Linear Parameter Varying (LPV) systems designed classically by using the polytopic representation. It is shown that even if the rank conditions ensuring the existence of an UIO are satisfied, the design may fail, due to the polytopic representation of the LPV system. In this paper a new design approach is proposed via a modified UIO for a class of LPV systems with parameter dependent output equation. It is based on parameter algebraic matrix equations and LMI conditions. Examples are provided in order to illustrate the performances of the proposed approach.

Keywords: Linear Parameter Varying systems (LPV), Unknown Input Observers (UIO), Unknown Input (UI) decoupling conditions, LMIs

1. INTRODUCTION

Unknown input observers are of great importance in automatic control and diagnosis. The idea behind UIO design is to decouple the effect of the unknown inputs (UI) from the state estimation error dynamics and to estimate asymptotically the states of the system even in the presence of unknown inputs. These UI can represent faults, disturbances or neglected dynamics in the system. This type of observers can be used in decoupling control, fault estimation, fault tolerant control, etc.

One of the most popular models are the well-known Linear Parameter Varying (LPV) ones, due to their ability to represent a more general class of systems compared to the linear models, and even a class of nonlinear systems in the case where the parameters depend on the state of the system (known as quasi-LPV models). In the context of UIO for LPV systems, there are several works aiming to extend the linear UIO to LPV systems. The commonly used approach is to transform the LPV system in a polytopic form with parameter dependent weighting functions satisfying the convex sum property. The stability analysis is generally done with Lyapunov theory and the established conditions are expressed as Linear Matrix Inequalities for design purposes. For example, in Marx et al. [2007], UIO is designed for both continuous-time and discrete-time descriptor Takagi-Sugeno systems which are similar to polytopic LPV systems. The work has been then extended in many papers, see for example Hamdi et al. [2012]. In Chadli and Karimi [2013], the authors proposed rank conditions ensuring the existence of UIO in polytopic form for a class of Takagi-Sugeno systems. In Briat et al. [2011], an interesting approach is proposed for the design of UIO for time delay LPV systems by exploiting algebraic matrix equalities computation.

In the other hand, these last years the problematic of time derivative estimation of signals has been largely studied and many approaches have been proposed which are robust to noises affecting the signals. In Levant [2003], a sliding mode differentiator has been proposed which provides robust high order time derivatives of a noisy signal. The main interesting property of such a differentiator is the finite time convergence and the exactness of time derivatives estimation for noise-free signals. In Ibrir [2003], another approach has been proposed based on Linear Time Varying differentiator which provides asymptotic time derivatives estimation. More recently, in Fliess et al. [2008], a new non asymptotic differentiator has been introduced using the operational calculus. The interest of such an approach is the transformation of the problem of time derivatives computation as a integrals (numerical low-pass filters) computation in a sliding time window which attenuates considerably the effect of noises affecting the signal. Moreover, no prior knowledge of the statistical properties of the noise is needed.

In this paper, new UIO is proposed for LPV systems where the output of the system is parameter varying. Up to our knowledge, this case is not investigated in the literature. In addition, the proposed observer is more general than the existing ones in the sense that, for the classical approaches using polytopic form, even if the UI decoupling condition is satisfied in the domain of variation of the parameters, classical UIO may not exist, while, the proposed approach provides a solution for the considered systems, and more generally for systems satisfying the decoupling rank condition in the variation range of the parameters.

The paper is organized as follows: In section 2, motivation and problem statement is provided, in particular, for systems in polytopic form and having nonlinear output equation. The section 3 presents an observer with a design procedure for state and unknown input estimation. In section 4, an extension to perturbed output is considered. Finally, in section 5, illustrative examples are provided to compare the proposed approach with respect to existing work and to illustrate the generality of the proposed approach.

2. PROBLEM STATEMENT AND MOTIVATION

2.1 Motivating example

Consider the LPV system with unknown input

$$\begin{cases} \dot{x}(t) = A(\rho(t))x(t) + D(\rho(t))d(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where

$$A(\rho(t)) = \begin{pmatrix} 0 & \rho(t) \\ -2 & -3 \end{pmatrix}, D(\rho(t)) = \begin{pmatrix} \rho(t) \\ \rho(t)+1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

where $\rho(t)$ is a time varying parameter. Assume that $\rho(t) \in [1, 4]$. Classically, the system is transformed into polytopic form

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(\rho(t))(A_i x(t) + D_i d(t)) \tag{2}$$

A classical observer for such a system is

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{2} h_i(\rho)(N_i z(t) + L_i y(t)) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases} \tag{3}$$

The UI decoupling condition is then given by

$$(I_2 + EC) D_i = 0, \quad i = 1, 2 \tag{4}$$

Note that the rank condition $rank(CD_i) = rank(D_i)$, $i = 1, 2$ are satisfied. In addition, $rank(CD(\rho)) = rank(D(\rho))$, $\forall\rho(t) \in [1, 4]$. Even if each pair (C, D_i) satisfies the UI decoupling rank condition for $i = 1, 2$, it may not exist a common constant matrix E satisfying simultaneously the conditions (4).

2.2 Problem statement

Consider the LPV system

$$\begin{cases} \dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) + D(\rho(t))d(t) \\ y(t) = C(\rho(t))x(t) \end{cases} \tag{5}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^{n_u}$, $d(t) \in \mathbb{R}^{n_d}$ and $y(t) \in \mathbb{R}^{n_y}$ are, respectively, the state, the control input, the unknown input and the output of the system. Furthermore, $n_y \geq n_d$. The parameter $\rho(t) = [\rho_1, ..., \rho_{n_\rho}]^T$ is a time varying vector belonging to a set Θ (the parameters $\rho_i, i = 1, ..., n_\rho$ are bounded). Assume also that the parameters ρ_i are differentiable.

The matrices of the system (5) are affine with respect to the parameters and given by

$$X(\rho(t)) = X_\rho = X_0 + \rho_1(t)X_1 + ... + \rho_{n_\rho}(t)X_{n_\rho} \tag{6}$$

where

$$X_\rho = X(\rho(t)) \in \{A_\rho, B_\rho, D_\rho, C_\rho\} \tag{7}$$

Let us define the time derivative of X_ρ by the notation

$$\dot{X}_\rho = \frac{dX_\rho}{dt} = \dot{\rho}_1(t)X_1 + ... + \dot{\rho}_{n_\rho}(t)X_{n_\rho} \tag{8}$$

where $\dot{\rho}_i(t)$, $i = 1, ..., n_\rho$ denotes the time derivative of $\rho_i(t)$ which belongs to a compact set Θ_d.

Hypothesis 1. It is assumed that the condition (9) holds.

$$rank\,(C_\rho D_\rho) = rank\,(D_\rho)\,, \quad \forall\rho(t) \in \Theta \tag{9}$$

Note that in some published work about UIO for LPV systems Chadli and Karimi [2013], the systems are transformed in polytopic form from the beginning of the design. Proceeding so, the structure of the UIO is fixed *a priori* which may lead to infeasible problem. In addition, the output of the system is commonly assumed to be linear with respect to the state and the unknown input vector affects also this output (i.e. $y(t) = Cx(t) + Rd(t)$). If the unknown input $d(t)$ is an actuator fault affecting only the state equation, there is no reason to have $y(t) = Cx(t) + Rd(t)$ and the matrix R is zero. In this case the decoupling condition commonly used ($PD_i = 0$, where $P = I_n + EC$) becomes conservative because the rank condition ensuring the existence of the common matrix E is satisfied for a limited class of systems. This case is dealt with in Chadli and Karimi [2013] and rank conditions are proposed for the existence of a constant matrix E such that $PD_i = 0$ is satisfied. Consequently, the aim of this paper is to propose a new algorithm to design UIO for LPV systems by relaxing the conservatism related to the polytopic transformation of the system at the beginning. Secondly, the output matrix C_ρ is parameter varying and the UI affects only the state equation. A discussion will be provided later to illustrate that the proposed approach is more general compared to classical UIO design approaches.

3. MAIN RESULT

Let us consider the system (5) and the following proposed observer

$$\begin{cases} \dot{z}(t) = N_\rho z(t) + G_\rho u(t) + L_\rho y(t) \\ \hat{x}(t) = z(t) - E_\rho y(t) \end{cases} \tag{10}$$

Note that the matrices N_ρ, G_ρ, L_ρ and E_ρ are parameter varying, their structure will be defined later, they are not a priori in a polytopic form. For simplicity, the notation X_ρ is used but the matrices of the observer may depend also of the time derivative of the parameter $\rho(t)$. Notice also that the matrix E_ρ is parameter varying while that of the classical observer (3) is constant. The estimated state is denoted $\hat{x}(t)$. Let us define the state estimation error $e(t) = x(t) - \hat{x}(t)$, by replacing the expression of $\hat{x}(t)$ from (10), it follows

$$e(t) = x(t) - z(t) + E_\rho C_\rho x(t) \tag{11}$$

$$= \underbrace{(I_n + E_\rho C_\rho)}_{P_\rho} x(t) - z(t) \tag{12}$$

The time derivative of $e(t)$ is given by

$$\dot{e}(t) = P_\rho \dot{x}(t) + \dot{P}_\rho x(t) - \dot{z}(t) \tag{13}$$
$$= \left(P_\rho A_\rho - L_\rho C_\rho - N_\rho P_\rho + \dot{P}_\rho\right) x(t)$$
$$+ (P_\rho B_\rho - G_\rho) u(t) + P_\rho D_\rho d(t) + N_\rho e(t) \tag{14}$$

Under the conditions

$$P_\rho A_\rho - L_\rho C_\rho - N_\rho P_\rho + \dot{P}_\rho = 0 \tag{15}$$
$$G_\rho = P_\rho B_\rho \tag{16}$$
$$P_\rho D_\rho = 0 \tag{17}$$

The state estimation error dynamics becomes

$$\dot{e}(t) = N_\rho e(t) \tag{18}$$

Proposition 2. There exists an unknown input observer (10) for the system (5) if and only if the following conditions (C1-C4) hold

(C1) The system $\dot{e}(t) = N_\rho e(t)$ is asymptotically stable
(C2) $P_\rho A_\rho - L_\rho C_\rho - N_\rho P_\rho + \dot{P}_\rho = 0$
(C3) $G_\rho - P_\rho B_\rho = 0$
(C4) $P_\rho D_\rho = 0$

Note that under the conditions (C2), (C3) and (C4) of the proposition 2, the state estimation error becomes

$$\dot{e}(t) = N_\rho e(t) \tag{19}$$

And if N_ρ is chosen as a stable matrix $\forall \rho(t) \in \Theta$, then the state estimation error tends to zero when t tends to infinity. Hence, the state of the system is estimated asymptotically and the unknown input is completely decoupled.

Now, let us analyse under what conditions there exists a solution satisfying the algebraic matrix equalities (C2), (C3) and (C4) in proposition 2. First, consider the condition $P_\rho D_\rho = 0$ which can be made in the form

$$E_\rho C_\rho D_\rho = -D_\rho \tag{20}$$

The solution E_ρ is then obtained by

$$E_\rho = -D_\rho \underbrace{(C_\rho D_\rho)^T \left(C_\rho D_\rho (C_\rho D_\rho)^T\right)^{-1}}_{(C_\rho D_\rho)^-} \tag{21}$$

Note that a solution E_ρ exists if the matrix $(C_\rho D_\rho)^-$ exists $\forall \rho(t) \in \Theta$ and this matrix exists if and only if the condition in hypothesis 1 is satisfied. Secondly, after computing the matrix E_ρ it is easy to compute the matrix G_ρ such that

$$G_\rho = P_\rho B_\rho = (I_n + E_\rho C_\rho) B_\rho \tag{22}$$

Finally, one obtains for the condition (2) in the proposition 2

$$N_\rho = P_\rho A_\rho - K_\rho C_\rho + \dot{P}_\rho \tag{23}$$

with $K_\rho = L_\rho + N_\rho E_\rho$, so the state estimation error dynamics is reduced to

$$\dot{e}(t) = \left(P_\rho A_\rho - K_\rho C_\rho + \dot{P}_\rho\right) e(t) \tag{24}$$

If the pair $(P_\rho A_\rho + \dot{P}_\rho, C_\rho)$ is detectable $\forall \rho(t) \in \Theta$ and $\forall \dot{\rho}(t) \in \Theta_d$, then the free matrix K_ρ can be computed in order to stabilize the state estimation error dynamics. In order to derive LMI conditions that ensure the asymptotic convergence of the state estimation error $e(t)$, it is easy to transform the matrices $P_\rho A_\rho + \dot{P}_\rho$ and C_ρ in a polytopic form. Indeed, since $\rho(t)$ and its time derivative are bounded, using the sector nonlinearity transformation Tanaka and Wang [2001], parameter dependent matrices can be written as polytopic matrices and the LPV system can be equivalently rewritten as a polytopic LPV one where the weighting functions depend of $\rho(t)$ and its time derivative. Then, it follows

$$\begin{cases} P_\rho A_\rho + \dot{P}_\rho = \displaystyle\sum_{i=1}^{r} \mu_i(\rho, \dot{\rho}) \mathcal{A}_i \\ C_\rho = \displaystyle\sum_{i=1}^{r} \mu_i(\rho, \dot{\rho}) \mathcal{C}_i \end{cases} \tag{25}$$

Consequently, the gain K_ρ takes the form

$$K_\rho = \sum_{i=1}^{r} \mu_i(\rho, \dot{\rho}) \mathcal{K}_i \tag{26}$$

With the polytopic notations (25) and (26), the state estimation error dynamics (24) becomes

$$\dot{e}(t) = \sum_{i=1}^{r} \sum_{i=1}^{r} \mu_i(\rho, \dot{\rho}) \mu_j(\rho, \dot{\rho}) (\mathcal{A}_i - \mathcal{K}_i \mathcal{C}_j) e(t) \tag{27}$$

The stability of systems of the form (27) are largely studied using the Lyapunov theory and different Lyapunov functions to deal with the conservatism of the LMI conditions. For instance, let us give the solution obtained by a quadratic Lyapunov function $V(e(t)) = e^T(t) X e(t)$ where $X = X^T > 0$ and the Tuan's lemma Tuan et al. [2001].

The system (27) is asymptotically stable if there exists a common matrix $X = X^T > 0$ and gain matrices $\bar{K}_i$, $i = 1..., r$ solution to the following LMI conditions

$$\begin{cases} \Xi_{ii} < 0 & i = 1, ..., r \\ \dfrac{1}{r-2} \Xi_{ii} + \Xi_{ij} + \Xi_{ji} < 0 & i \neq j \end{cases} \tag{28}$$

where

$$\Xi_{ij} = \mathcal{A}_i^T X + X \mathcal{A}_i - \mathcal{C}_j^T \bar{K}_i^T - \bar{K}_i \mathcal{C}_j \tag{29}$$

After solving the LMIs (28), the gains $\mathcal{K}_i$ in (27) are obtained from

$$\mathcal{K}_i = \bar{K}_i X^{-1}, \quad i = 1, ..., r \tag{30}$$

The matrix K_ρ is then given by the matrix (26) which leads to compute the matrices L_ρ and N_ρ as follows

$$L_\rho = K_\rho - N_\rho E_\rho \tag{31}$$
$$N_\rho = P_\rho A_\rho + \dot{P}_\rho - K_\rho C_\rho \tag{32}$$

Unknown input estimation From the output equation of (5), the time derivative of $y(t)$ is

$$\dot{y}(t) = \left(C_\rho A_\rho + \dot{C}_\rho\right) x(t) + C_\rho B_\rho u(t) + C_\rho D_\rho d(t) \tag{33}$$

Since the assumption 1 is satisfied, the unknown input can be expressed, by model inversion, as follows

$$d(t) = (C_\rho D_\rho)^- \left(\dot{y}(t) - \left(C_\rho A_\rho + \dot{C}_\rho\right) x(t) - C_\rho B_\rho u(t)\right) \tag{34}$$

When the state estimation error $e(t)$ converges to zero, we have $\hat{x}(t) \longrightarrow x(t)$, then the following UI estimation $\hat{d}(t)$ is obtained

$$\hat{d}(t) = (C_\rho D_\rho)^- \left(\dot{y}(t) - \left(C_\rho A_\rho + \dot{C}_\rho\right) \hat{x}(t) - C_\rho B_\rho u(t)\right) \tag{35}$$

The convergence of $\hat{d}(t)$ towards $d(t)$ can be analysed by defining the unknown input estimation error $e_d(t) = d(t) - \hat{d}(t)$, which leads to

$$\hat{d}(t) = (C_\rho D_\rho)^- \left(\left(C_\rho A_\rho + \dot{C}_\rho\right) e(t) + C_\rho D_\rho d(t)\right) \quad (36)$$

it follows

$$e_d(t) = -(C_\rho D_\rho)^- \left(C_\rho A_\rho + \dot{C}_\rho\right) e(t) \quad (37)$$

Knowing that $e(t)$ converges asymptotically to zero, then $e_d(t)$ also converges asymptotically to zero.

Remark 3. In the proposed observer, the first time derivative of the output $y(t)$ and / or that of the parameters are needed. It can be computed by different differentiators such as sliding mode differentiators Levant [2003] or algebraic differentiators Fliess et al. [2008].

3.1 Illustrative example and discussions

Let us consider the LPV system (5) with the matrices

$$A_\rho = \begin{pmatrix} 0 & \rho(t) \\ 1-\rho(t) & -3 \end{pmatrix}, B_\rho = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, D_\rho = \begin{pmatrix} \rho(t) \\ 1+\rho(t) \end{pmatrix}$$

$$C_\rho = (\,\rho(t)\ \ 0\,)$$

and $\rho(t) \in [2,4]$, $\forall t$. Note that, the system is observable and the rank condition in assumption 1 is satisfied $\forall \rho(t) \in [2,4]$ and $\forall \dot{\rho}(t)$. This can be seen from the observability matrix

$$O_\rho = \begin{pmatrix} C_\rho \\ C_\rho A_\rho + \dot{C}_\rho \end{pmatrix} = \begin{pmatrix} \rho(t) & 0 \\ \dot{\rho}(t) & \rho^2(t) \end{pmatrix} \quad (38)$$

where $rank(O_\rho) = 2$, since $\rho(t)$ and $\rho(t)^2$ are strictly positive definite. The condition in assumption 1 is

$$rank\left(\rho^2(t)\right) = rank\left(\rho(t)\right) = 1, \forall \rho(t) \in [\,2\ \ 4\,] \quad (39)$$

Discussion on the classical UIO for LPV systems: For this system, the classical UIO of the form

$$\begin{cases} \dot{z}(t) = N_\rho x(t) + G_\rho u(t) + L_\rho y(t) \\ \hat{x}(t) = z - Ey(t) \end{cases} \quad (40)$$

does not exist because the matrix C_ρ is time varying and the matrix E is constant. The decoupling condition for the observer (40) is given by

$$E = -D_\rho \left(C_\rho D_\rho\right)^{-1} = \begin{pmatrix} -\dfrac{1}{\rho(t)} \\ -\dfrac{1+\rho(t)}{\rho^2(t)} \end{pmatrix} \quad (41)$$

but the matrix E as defined classically is constant so this solution is not acceptable.

In a second situation, let us assume that C_ρ is not parameter varying but given by $C = (1 \quad 0)$. In this case, prior to the UIO design, the system can be put in a potytopic form according to

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{2} \mu_i(\rho)\left(A_i x(t) + Bu(t) + D_i d(t)\right) \\ y(t) = Cx(t) \end{cases} \quad (42)$$

where

$$A_1 = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 4 \\ -3 & -3 \end{pmatrix} \quad (43)$$

$$D_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, D_2 = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \quad (44)$$

In this situation, the decoupling condition is

$$(I + EC)D_i = 0 \quad (45)$$

Since the condition $rank(CD_i) = rank(D_i)$ holds, the solution E is then given by

$$E = -D_i\left(CD_i\right)^{-1} \quad (46)$$

The solution E satisfying the two equations $i = 1, 2$ exists if (Chadli and Karimi [2013])

$$rank\left(C\,[\,D_1\ \ D_2\,]\right) = rank\left([\,D_1\ \ D_2\,]\right) \quad (47)$$

This condition is not satisfied because $rank(C[D_1 \ \ D_2]) = 1$ and $rank([D_1 \ \ D_2]) = 2$, then, the common matrix E satisfying the two decoupling conditions does not exist, which implies that the classical polytopic UIO does not exist.

As a conclusion, the classical UIO does not exist because the output matrix C_ρ is parameter varying. Moreover, even when the matrix C is constant and satisfies the decoupling conditions, the classical UIO design may fail to provide a solution for UIO design. Whereas, the proposed UIO design for LPV systems (5) with parameter dependent output matrices C_ρ replaces the constant matrix E by the matrix E_ρ which is parameter varying and the system is not transformed into polytopic form at the beginning of the design. Then the decoupling condition is

$$E_\rho = -D_\rho \underbrace{\left(C_\rho D_\rho\right)^T \left(C_\rho D_\rho \left(C_\rho D_\rho\right)^T\right)^{-1}}_{(C_\rho D_\rho)^{-1}} = \begin{pmatrix} -\dfrac{1}{\rho(t)} \\ -\dfrac{\rho(t)+1}{\rho^2(t)} \end{pmatrix} \quad (48)$$

which is defined $\forall \rho \in [2,4]$. And the matrix G_ρ is given by

$$G_\rho = \begin{pmatrix} 0 \\ -\dfrac{\rho(t)+1}{\rho(t)} \end{pmatrix} \quad (49)$$

Finally, we have

$$P_\rho A_\rho + \dot{P}_\rho = \begin{pmatrix} 0 & 0 \\ \dfrac{\dot{\rho}(t)}{\rho^2(t)} - \rho(t) + 1 & -\rho(t) - 4 \end{pmatrix} \quad (50)$$

At this stage, the matrices $P_\rho A_\rho + \dot{P}_\rho$ and C_ρ can be transformed into a polytopic form and the parameter varying gain L_ρ can be designed by solving the LMI conditions (28).

4. BOUNDED STATE ESTIMATION ERROR: PERTURBED OUTPUT

The proposed design approach for UIO is extended in this section to LPV systems with perturbed measurements expressed by

$$\begin{cases} \dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) + D(\rho(t))d(t) \\ y(t) = C(\rho(t))x(t) + \eta(t) \end{cases} \quad (51)$$

where $\eta(t)$ represents a vector of bounded perturbation. Assume also that the first time derivative of $\eta(t)$ is bounded. The UIO observer (10) is then designed in order to decouple the unknown input $d(t)$ and minimize the effect of $\eta(t)$ on the state estimation error. Computing the state estimation error $e(t) = x(t) - \hat{x}(t)$ leads to

$$e(t) = \underbrace{(I + E_\rho C_\rho)}_{P_\rho} x(t) - z(t) + E_\rho \eta(t) \quad (52)$$

its dynamics obeys to the following differential equation

$$\dot{e}(t) = \left(P_\rho A_\rho - L_\rho C_\rho - N_\rho P_\rho + \dot{P}_\rho\right) x(t) + (P_\rho B_\rho - G_\rho)\, u(t) + P_\rho D_\rho d(t) + N_\rho e(t) + \dot{E}_\rho \eta(t) + E_\rho \dot{\eta}(t) \quad (53)$$

Under the assumption 1, there exists a matrix E_ρ allowing to decouple the unknown input $d(t)$ from the dynamics of the state estimation error. In addition, the conditions (C2-C4) of proposition 2 leads to

$$\dot{e}(t) = N_\rho e(t) + \tilde{E}_\rho \tilde{\eta}(t) \quad (54)$$

where

$$\tilde{\eta}(t) = \begin{pmatrix} \eta(t) \\ \dot{\eta}(t) \end{pmatrix}, \tilde{E}_\rho = \begin{pmatrix} E_\rho & \dot{E}_\rho \end{pmatrix} \quad (55)$$

which can be made in the form

$$\dot{e}(t) = \left(P_\rho A_\rho - K_\rho C_\rho + \dot{P}_\rho\right) e(t) + \tilde{E}_\rho \tilde{\eta}(t) \quad (56)$$

In the perturbation-free case, the gain K_ρ is selected in order to ensure asymptotic convergence toward zero of the state estimation error. In the present case with outputs affected by the perturbation $\eta(t)$, the gain is selected in order to both stabilizes the state estimation error and minimizes the effect of $\tilde{\eta}(t)$ on the state estimation error. To this purpose, the polytopic transformations of the known matrices leads to

$$\dot{e}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\rho, \dot{\rho}) \mu_j(\rho, \dot{\rho}) \left((\mathcal{A}_i - \mathcal{K}_i \mathcal{C}_j)\, e(t) + \tilde{\mathcal{E}}_i \tilde{\eta}(t) \right) \quad (57)$$

The Lemma 4 gives sufficient LMI conditions that ensure the following specifications

- $e(t) \to 0$ asymptotically if $\tilde{\eta}(t) = 0$
- Bounded error $e(t)$ if $\tilde{\eta}(t) \neq 0$

Lemma 4. Given a scalar $\alpha > 0$, if there exist a symmetric and positive definite matrix Q, gain matrices $\bar{\mathcal{K}}_i$ and positive scalars c and γ solution to the following optimization problem

$$\min_{Q, \mathcal{K}_i, c, \gamma} \gamma$$

s.t.

$$c - \alpha\gamma \leq 0 \quad (58)$$

$$\begin{cases} \Pi_{ii} < 0 & i = 1, ..., r \\ \dfrac{1}{r-2}\Pi_{ii} + \Pi_{ij} + \Pi_{ji} < 0 & i \neq j \end{cases} \quad (59)$$

where

$$\begin{pmatrix} \mathcal{A}_i^T Q + Q\mathcal{A}_i - \bar{\mathcal{K}}_i \mathcal{C}_j + \mathcal{C}_j^T \bar{\mathcal{K}}_i^T + \alpha Q & Q\tilde{\mathcal{E}}_i \\ \tilde{\mathcal{E}}_i^T Q & -cI \end{pmatrix} < 0 \quad (60)$$

then the state estimation error is bounded and satisfies the inequality

$$\|e(t)\|_2 < \sqrt{\frac{\alpha_2}{\alpha_1}} \|e(0)\|_2 \exp\left(-\frac{\alpha}{2}t\right) + \sqrt{\frac{c}{\alpha\alpha_1}} \|\tilde{\eta}(t)\|_\infty \quad (61)$$

where $\alpha_1 > 0$ and $\alpha_2 > 0$ denote respectively the lower and the upper eigenvalues of the matrix Q. The gains of the observer are obtained from $\mathcal{K}_i = Q^{-1}\bar{\mathcal{K}}_i$.

Proof. The proof is similar to that provided in Ichalal et al. [2012], then it is omitted. The reader can follow the same reasoning with a quadratic Lyapunov function $V(e(t)) = e^T(t)Qe(t)$, $Q = Q^T > 0$.

5. SIMULATION EXAMPLE

Let us consider the system (5) with the matrices

$$A_\rho = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\rho(t) & -3 & -\rho(t) \end{pmatrix}, D_\rho = \begin{pmatrix} 2\rho(t) & 0 \\ 0 & 1 \\ 0 & -\rho(t) \end{pmatrix}$$

with the parameter $\rho(t) = 2 + sin(t)$. It is clear that$1 \leq \rho(t) \leq 3$. Let us assume a linear output $y(t) = Cx(t)$ where

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If the polytopic approach is used the system will be transformed into

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i(\rho(t))\, (A_i x(t) + D_i d(t)) \quad (62)$$

where

$$A_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -1 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -3 & -3 \end{pmatrix}$$

$$D_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}, D_2 = \begin{pmatrix} 6 & 0 \\ 0 & 1 \\ 0 & -3 \end{pmatrix}$$

and

$$\mu_1(\rho(t)) = \frac{3 - \rho(t)}{2}, \mu_2(\rho(t)) = \frac{\rho(t) - 1}{2} \quad (63)$$

The rank condition given in Chadli and Karimi [2013] is not satisfied because $rank\,(C\,[\,D_1 \;\, D_2\,]) \neq rank\,([\,D_1 \;\, D_2\,])$ which implies that the UIO proposed in Chadli and Karimi [2013] does not exist. However, with the proposed approach, it is clear that the rank condition $rank\,(CD_\rho) = rank\,(D_\rho) = 2$ is satisfied $\forall \rho(t) \in [1, 3]$. Therefore, the proposed observer (10) exists and its matrices are given by

$$E_\rho = \begin{pmatrix} -1 & 0 \\ 0 & \dfrac{1}{\rho(t)} \\ 0 & -1 \end{pmatrix}, P_\rho = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & \dfrac{1}{\rho(t)} \\ 0 & 0 & 0 \end{pmatrix}$$

The matrix $\dot{P}_\rho$ is given by

$$\dot{P}_\rho = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\dfrac{\dot{\rho}(t)}{\rho^2(t)} \\ 0 & 0 & 0 \end{pmatrix}$$

Finally, the matrix N_ρ is given by

$$N_\rho = \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ -1 & -\dfrac{3}{\rho(t)} & -\dfrac{\dot{\rho}(t)}{\rho^2(t)} \\ 0 & 0 & 0 \end{pmatrix}}_{P_\rho A_\rho + \dot{P}_\rho} - K_\rho C \quad (64)$$

After transforming the matrix $P_\rho A_\rho + \dot{P}_\rho$ in polytopic form with the new parameters $z_1(t) = -\frac{3}{\rho(t)}$, $z_2(t) = -\frac{\dot{\rho}(t)}{\rho^2(t)}$, the gains $\mathcal{K}_i$ are computed with the LMI conditions given in (28), and are given by

$$K_1 = K_3 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \\ 0 & 0.5 \end{pmatrix}, K_2 = K_4 = \begin{pmatrix} 0.5 & 0 \\ 0 & -2.1 \\ 0 & 0.5 \end{pmatrix}$$

The matrix L_ρ is computed directly from the equation $L_\rho = K_\rho - N_\rho E_\rho$. The figures 1 and 2 depict the state and the unknown input estimation. It can be seen that the provided estimation is asymptotic and the unknown inputs can be estimated without any condition on their time variations. A second simulation is performed by adding random measurement noises bounded by 0.05, which provides the results in figures 3 and 4. One can see that the results are acceptable because the time derivatives of the outputs and the parameters are computed by a sliding mode differentiator which is significantly less sensitive to noises.

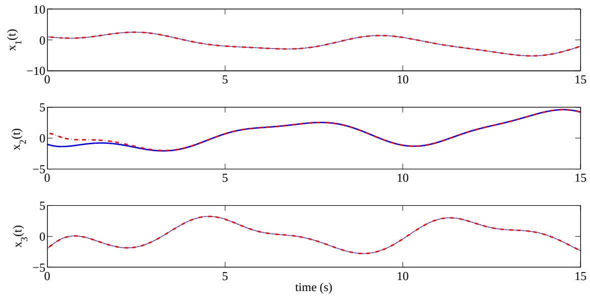

Fig. 1. States (blue) and estimates (red)

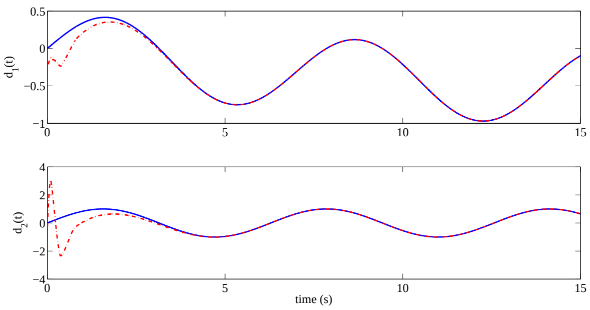

Fig. 2. Unknown inputs (blue) and estimates (red)

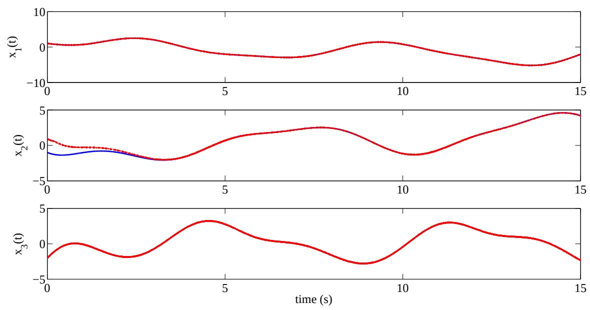

Fig. 3. State estimation with noise measurements

6. CONCLUSION

This paper proposed general UIO designs for LPV systems by using algebraic parameter varying matrix equations and LMIs conditions. The considered LPV systems encompass parameter varying output equations. It is illustrated that transforming a LPV system in polytopic form may restrict the feasibility of UIO design because the structure of the observer is also fixed *a priori*. As a conclusion, the proposed approach is more general than the classical UIOs in the sense that the polytopic transformation is made at the end of the observer matrix characterization in order to establish LMI conditions. In addition, the proposed approach is applicable for systems with parameter varying output equation, i.e. where the observation matrix is parameter varying. An extension is provided for systems with perturbed measurements, aiming to stabilize the state estimation error while the perturbation term is minimized.

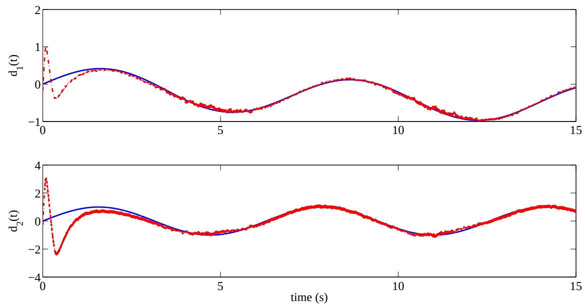

Fig. 4. Uknown input estimation (with noise measurements)

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