Fault Tolerant Control design for uncertain Takagi-Sugeno systems by trajectory tracking: A descriptor approach

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Abstract

This paper considers the problem of fault tolerant control (FTC) by trajectory tracking for uncertain nonlinear system described by Takagi-Sugeno models. The considered faults are constant, exponential or polynomial. The provided results are easily formulated in terms of Linear Matrix Inequalities by employing the descriptor redundancy property. This latter introduces "virtual" dynamics both in the active FTC control law scheme and in the output error allowing to decouple the gains of the active FTC controller, the observer gain matrices and the system ones. Numerical examples are given to illustrate the efficiency of the proposed approach.

Index Terms

Takagi-Sugeno models, active fault tolerant control, state and fault estimation, diagnosis, descriptor systems.

I. INTRODUCTION

If an unknown input (e.g. a fault) affects a system, it is known that classical control schemes lead to degraded performances or even to the instability of the system. To overcome this drawback, new control strategies have been introduced, especially active and passive fault tolerant control (FTC). The passive FTC has shown its interests [1] [2], since the closed-loop system dynamics is stable and the fault effect is attenuated. In this case, the faults (generally norm bounded) are viewed as external disturbances and the controller structure remains unchanged. Nevertheless, in practice some faults are critical and passive FTC cannot guarantee the system stability. Thus, the active FTC, where the control law is reconfigured according to the occurring fault in order to compensate its effect, is needed to ensure the system stability with satisfactory performances. This can be done by estimating simultaneously the fault and the system state.

Since the introduction of active FTC technique, many works have been developed. In the linear framework, the pseudo-inverse technique leads to determine the controller gains by minimizing the Frobenius norm of the transfer from the fault to the output as proposed by [3] [4] and improved by [5] [6]. In [7] [8] [9], the active FTC gains have been determined, based on eigenstructure assignment, such that the eigenvalues of the controlled faulty system and those of a reference model are identical. Moreover, FTC for linear descriptor systems described by algebraic-differential equations has been addressed in [10].

Some active FTC designs have been proposed for nonlinear systems, considering that the system evolves around operating points [11] [12]. However, these approaches are only valid around these operating points. In [13], active FTC for a class of nonlinear systems is envisaged with the absolute stability theory when the faults only affect the actuators. Although some results in active FTC for nonlinear systems are applied to motor control [14] or in attitude tracking of flexible spacecraft [15], it is still an open problem, as pointed in [16].

An efficient way to represent the behavior of nonlinear systems is to use the multimodel approach introduced by Takagi and Sugeno [17] [18]. Takagi-Sugeno (T-S) models are very interesting since any nonlinear model can be exactly written as a T-S model, on a compact set of the state space. It thus allows extending the linear control theory to nonlinear systems. Many works devoted to T-S systems have been developed, dealing with stability and stabilization [18] [19] [20] [21], observer design [22] [23] [24] [25] [26] [27] and diagnosis [28] [29] [30].

The active FTC design for T-S systems has been poorly studied in literature. In [31], an output active FTC algorithm for vehicle lateral dynamics represented by an uncertain T-S model subject to constant sensor fault has been proposed, but no uncertainties were considered in the output equation. Furthermore, trajectory tracking active FTC design for T-S models subject to constant actuator fault has been investigated by [32] [33] [34] for T-S models without uncertainties and affected only by constant faults.

However, in practice, the faults acting on a system are time varying. Hence, the present work aims to extend the trajectory tracking active FTC to uncertain T-S systems with time varying faults. The faults being here envisaged are modeled by exponential or polynomial functions. The obtained results are formulated in LMI terms by employing the descriptor redundancy property [35] [36] [37] [38] [39]. The main idea is to introduce a ”virtual” dynamics in both active FTC law and output error expressions in order to avoid crossed terms in the LMI and then decrease the number of LMI conditions and consequently relax the conservatism. Indeed, the descriptor formulation is only a mean to obtain relaxed LMI conditions, but singular systems are not the target of the present work. Based on the descriptor approach, active FTC tracking controller design for T-S models without uncertainties subject to time varying faults has been proposed by [40]. In this paper, the FTC design is developed to take into account simultaneously the model uncertainties and the time varying faults occurring in the system. The control law is determined to ensure the state tracking between the healthy system and the faulty one.

This paper is organized as follows. In the next section, the active FTC scheme and the system under study are presented. In section 3, some FTC designs for uncertain T-S systems are established in the case of constant faults, exponential faults and polynomial faults. In the last section, numerical examples are considered to illustrate the applicability and the effectiveness of the proposed approaches.

Notations: In a block matrix, a star $*$ denotes the terms induced by symmetry. The term Π_μ denotes a polytopic matrix defined by $\Pi_\mu = \sum_{i=1}^{r} \mu_i\left(\xi\left(t\right)\right)\Pi_i$, where $\mu_i\left(\xi\left(t\right)\right)$ are scalar weighting functions and Π_i are matrices with appropriate dimensions. $\Phi = diag(\Phi_1, \cdots, \Phi_r)$ is a block diagonal matrix which diagonal entries are defined by Φ_1,..., Φ_r.

The following lemma is needed to provide LMI conditions.

Lemma 1: [41] For any matrices X, $\Sigma(t)$, Y with appropriate dimensions and $\Sigma^T(t)\Sigma(t) \leq I$ and for any positive real number τ, it follows:

$$X^T\Sigma^T(t)Y + Y^T\Sigma(t)X \leq \tau X^TX + \tau^{-1}Y^TY \tag{1}$$

II. Problem statement

T-S models consist in a set of Linear Time Invariant (LTI) models interconnected with nonlinear weighting functions $\mu_i(\xi(t))$. These latter define the contribution of each linear submodel to the overall dynamics of the multiple model. The nonlinear T-S model is given by:

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum\limits_{i=1}^{r} \mu_i(\xi(t))(C_i x(t) + D_i u(t)) \end{cases} \tag{2}$$

where r is the number of submodels defined by the matrices $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{p\times n}$ and $D_i \in \mathbb{R}^{p\times m}$. The nonlinear functions $\mu_i(\xi(t))$ depend on the premise variable $\xi(t)$ which can be measurable ($u(t)$ or $y(t)$) or unmeasurable ($x(t)$). In the present study, the premise variables are assumed to be real time accessible. The functions $\mu_i(\xi(t))$ are known and satisfy the convex sum property:

$$\begin{cases} 0 \leq \mu_i(\xi(t)) \leq 1, \quad i = 1, ..., r, \quad \forall\ t \\ \sum\limits_{i=1}^{r} \mu_i(\xi(t)) = 1, \quad \forall\ t \end{cases} \tag{3}$$

Every nonlinear model can be written as a T-S model on a compact set of the state space [18]. The sector nonlinearity approach [18] [42] allows an exact rewriting of a nonlinear model into a T-S form, without loss of informations.

The system (2) is considered as a reference model corresponding to a fault-free T-S model to be tracked by the uncertain T-S model affected by faults defined by:

$$\begin{cases} \dot{x}_f(t) = \sum\limits_{i=1}^{r} \mu_i(\xi(t)) \left(\bar{A}_i x_f(t) + \bar{B}_i u_f(t) + G_i f(t)\right) \\ y_f(t) = \sum\limits_{i=1}^{r} \mu_i(\xi(t)) \left(\bar{C}_i x_f(t) + \bar{D}_i u_f(t) + W_i f(t)\right) \end{cases} \tag{4}$$

where $x_f(t) \in \mathbb{R}^n$, $y_f(t) \in \mathbb{R}^p$, $f(t) \in \mathbb{R}^q$ and $u_f(t) \in \mathbb{R}^m$ are respectively the faulty state, the faulty output, the fault affecting the system behavior and the fault tolerant control. The matrices $G_i \in \mathbb{R}^{n\times q}$ and $W_i \in \mathbb{R}^{p\times q}$ are the known distribution matrices of the faults on the system. The uncertain matrices of the system (4) are defined by:

$$\bar{X}_i = X_i + \Delta X_i(t),\ X \in \{A, B, C, D\} \tag{5}$$

where $\Delta A_i(t)$, $\Delta B_i(t)$, $\Delta C_i(t)$ and $\Delta D_i(t)$ are time varying unknown matrices describing the bounded model uncertainties, defined by:

$$\Delta X_i(t) = M_i^x \delta_i^x(t) N_i^x,\ (X, x) \in \{(A, a), (B, b), (C, c), (D, d)\} \tag{6}$$

where M_i^a, M_i^b, M_i^c, M_i^d, N_i^a, N_i^b, N_i^c and N_i^d are known matrices with appropriate dimensions and the matrix functions $\delta_i^a\,(t)$, $\delta_i^b\,(t)$, $\delta_i^c\,(t)$ and $\delta_i^d\,(t)$ are bounded by:

$$\delta_i^x(t)\,(\delta_i^x(t))^T \leq I,\ x \in \{a,b,c,d\} \tag{7}$$

The aim of this work is to design a fault tolerant controller ensuring the tracking between the faulty system state (4) and the healthy one (2). In other words, the objective is to find the control law $u_f(t)$ minimizing the difference between the reference state $x(t)$ of (2) and the faulty state $x_f(t)$ of (4). The proposed active FTC control law is:

$$\begin{cases} u_f\,(t) = u\,(t) + u_c\,(t) \\ u_c\,(t) = K_\mu\,(x\,(t) - \hat{x}_f\,(t)) - K_\mu^f \hat{f}\,(t) \end{cases} \tag{8}$$

where $K_i \in \mathbb{R}^{m\times n}$ and $K_i^f \in \mathbb{R}^{m\times q}$ are the state feedback gains to be determined, $\hat{x}_f(t)$ and $\hat{f}(t)$ are the estimated state and fault.

The controller design scheme is depicted by figure 1. The FTC law (8) requires the estimation of both the faulty

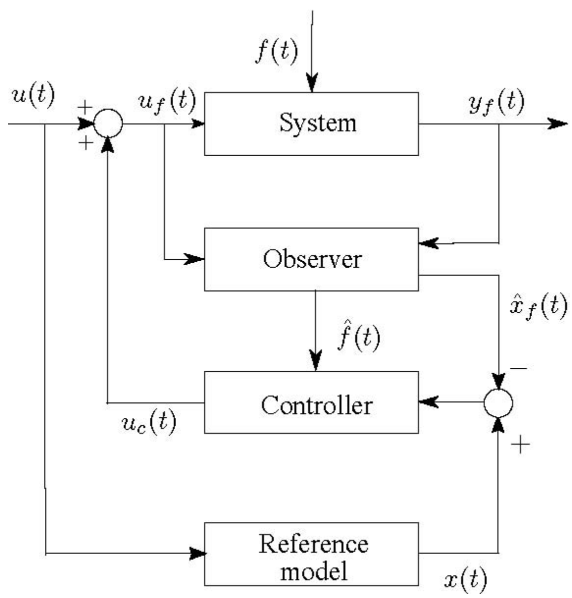

Fig. 1. Tracking fault tolerant controller design scheme

state and the fault. These informations are provided by a proportional integral observer (PIO) or a proportional multiple integral observer (PMIO) depending on the fault model.

III. LMI conditions for fault tolerant controller design

In this section, it is considered that some a priori knowledge about the fault characteristics is available. Indeed, the fault may be a polynomial function, a constant one or described by an exponential function.

A. *Polynomial fault case*

It is here assumed that the fault affecting the system behavior can be approximated by a k^{th} order polynomial, on a finite time window. Let us denote the $(k+1)$ first time derivatives of the fault by:

$$\phi_\ell(t) = \frac{d^\ell f(t)}{dt^\ell}, \ \ell = 1, \dots, k+1 \tag{9}$$

Obviously, $f(t)$ being a k^{th} order polynomial, it follows that $\phi_{k+1}(t) = 0$. The faulty state and the fault estimations, needed for the control law (8), are provided by a proportional multiple integral observer (PMIO), since this kind of observer is known to estimate polynomial unknown input. The proposed observer is defined by:

$$\begin{cases} \dot{\hat{x}}_f(t) = A_\mu \hat{x}_f(t) + B_\mu u_f(t) + G_\mu \hat{f}(t) + L_\mu^1 \left(y_f(t) - \hat{y}_f(t)\right) \\ \dot{\hat{f}}(t) = L_\mu^2 (y_f(t) - \hat{y}_f(t)) - \hat{f}(t) \\ \dot{\hat{\phi}}_\ell(t) = L_\mu^{\ell+2} \left(y_f(t) - \hat{y}_f(t)\right) - \hat{\phi}_\ell(t), \ \ell = 1, \dots, k \\ \hat{y}_f(t) = C_\mu \hat{x}_f(t) + D_\mu u_f(t) + W_\mu \hat{f}(t) \end{cases} \tag{10}$$

where the matrices $L_i^1 \in \mathbb{R}^{n \times p}$ and $L_i^\ell \in \mathbb{R}^{q \times p}$ ($\ell = 2, \dots, k+2$), respectively defining the polytopic matrices L_μ^1 and L_μ^ℓ, are the observer gains to be determined with the controller gains K_μ and K_μ^f in (8).

In the remaining of the paper, for space convenience, the time variable t will be omitted when there is no ambiguity.

Let us define the state tracking error, the state and the fault estimation errors, the fault derivative estimation errors, the difference between the nominal and the FTC laws, denoted in the next, by the control error and the output estimation error, given respectively by:

$$\begin{cases} e_p = x - x_f \\ e_s = x_f - \hat{x}_f \\ e_f = f - \hat{f} \\ \tilde{\phi}_\ell = \phi_\ell - \hat{\phi}_\ell \\ e_u = u - u_f \\ e_y = y_f - \hat{y}_f \end{cases} \tag{11}$$

Using equations (2), (4) and (10), the tracking error dynamics, the state and fault estimation error dynamics are expressed as follows:

$$\dot{e}_p = \bar{A}_\mu e_p + \bar{B}_\mu e_u - G_\mu f - \Delta A_\mu x - \Delta B_\mu u \tag{12}$$

$$\dot{e}_s = A_\mu e_s - \Delta B_\mu e_u + \Delta B_\mu u + G_\mu e_f - L_\mu^1 e_y + \Delta A_\mu \left(x - e_p\right) \tag{13}$$

$$\dot{e}_f = \phi_1 - L_\mu^2 e_y - e_f + f \tag{14}$$

The estimation error of the k first derivatives of the fault are given by

$$\dot{\tilde{\phi}}_\ell = \phi_{\ell+1} + \phi_\ell - \tilde{\phi}_\ell - L_\mu^{\ell+2} e_y, \ \ell = 1, \dots, k-1 \tag{15}$$

$$\dot{\tilde{\phi}}_k = \phi_k - \tilde{\phi}_k - L_\mu^k e_y \tag{16}$$

The output estimation error expression is given by:

$$e_y = C_\mu e_s - \Delta C_\mu e_p + W_\mu e_f - \Delta D_\mu e_u + \Delta D_\mu u + \Delta C_\mu x \tag{17}$$

The substitution of (8) and (17) in the above error dynamic equations (12), (13) and (14) implies to multiply the matrices of the faulty system and the observer ones with those of the controller and the observer to be determined. This coupling introduces cross terms leading to double summations and then to conservative results. A way to avoid the coupling terms is to formulate the error dynamics in the descriptor form. The idea is to rewrite the output error equation (17) and the active FTC law (8) as static equations encompassed in a descriptor system:

$$0\dot{e}_y = C_\mu e_s + \Delta C_\mu (x - e_p) + W_\mu e_f + \Delta D_\mu (u - e_u) - e_y \tag{18}$$

$$0\dot{e}_u = e_u + K_\mu e_p + K_\mu e_s + K_\mu^f e_f - K_\mu^f f \tag{19}$$

The combination of (12), (13), (14), (18) and (19) leads to the following descriptor form:

$$\tilde{E}\dot{e} = \tilde{A}_\mu e + \tilde{B}_\mu \omega \tag{20}$$

where $\tilde{E} = diag(I_{2n+q(k+1)}, 0_{m+p})$,

$$\tilde{A}_\mu = \begin{pmatrix} \bar{A}_\mu & 0 & 0 & 0 & \cdots & 0 & \bar{B}_\mu & 0 \\ -\Delta A_\mu & A_\mu & G_\mu & 0 & \cdots & 0 & -\Delta B_\mu & -L_\mu^1 \\ 0 & 0 & -I & 0 & \cdots & 0 & 0 & -L_\mu^2 \\ 0 & 0 & 0 & -I & \cdots & 0 & 0 & -L_\mu^3 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -I & 0 & -L_\mu^{(k+2)} \\ -K_\mu & -K_\mu & -K_\mu^f & 0 & \cdots & 0 & -I & 0 \\ -\Delta C_\mu & C_\mu & W_\mu & 0 & \cdots & 0 & -\Delta D_\mu & -I \end{pmatrix} \tag{21}$$

$$\tilde{B}_\mu = \begin{pmatrix} -\Delta A_\mu & -G_\mu & 0 & \cdots & \cdots & \cdots & 0 & -\Delta B_\mu \\ \Delta A_\mu & 0 & 0 & \cdots & \cdots & \cdots & 0 & \Delta B_\mu \\ 0 & I & I & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & I & I & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & 0 & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & I & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 0 & I & 0 \\ 0 & K_\mu^f & 0 & \cdots & \cdots & \cdots & 0 & 0 \\ \Delta C_\mu & 0 & 0 & \cdots & \cdots & \cdots & 0 & \Delta D_\mu \end{pmatrix} \tag{22}$$

$$e^T = \begin{pmatrix} e_p^T & e_s^T & e_f^T & \tilde{\phi}_1^T & \cdots & \tilde{\phi}_k^T & e_u^T & e_y^T \end{pmatrix} \tag{23}$$

$$\omega^T = \begin{pmatrix} x^T & f^T & \phi_1^T & \phi_2^T & \ldots & \phi_{(k-1)}^T & \phi_k^T & u^T \end{pmatrix} \tag{24}$$

The conditions ensuring the stability of the descriptor system (20) and the attenuation level from the perturbation-like term ω to the error dynamics e are provided in the following theorem.

Theorem 1: The system (20) describing the different estimation errors is stable and the $\mathcal{L}_2$-gain from the fault and its derivatives to the state tracking error, the state and fault estimation error and the estimation error of the fault derivatives is bounded by $\sqrt{\bar{\gamma}}$, if there exists matrices $\bar{X}_1$ invertible, $\bar{X}_2$, $\bar{K}_i$, $\bar{K}_i{}^f$, $P_j = P_j^T > 0$, for $j = 1, 2, ..., k+3$, $\bar{L}_{\ell,i}$, for $\ell = 1, 2, ..., k+2$ and positive scalars $\bar{\gamma}$, τ_1, τ_2, τ_3 and τ_4 such that the following LMIs are verified, for $i \in \{1, ..., r\}$:

$$\begin{pmatrix} \tilde{\Pi}_i & * & * \\ \tilde{\Theta}_i & -\tilde{\phi}_i & 0 \\ \tilde{\Sigma}_i & 0 & -\bar{\xi} \end{pmatrix} < 0 \tag{25}$$

where

$$\bar{\xi} = diag(\tau_1 I, \tau_2 I, \tau_3 I, \tau_4 I)$$

$$\tilde{\phi}_i = diag(\bar{\gamma} I - \tau_1 (N_i^a)^T N_i^a - \tau_3 (N_i^c)^T N_i^c, \bar{\gamma} I, \bar{\gamma} I, \cdots, \bar{\gamma} I, \bar{\gamma} I - \tau_2 (N_i^b)^T N_i^b - \tau_4 (N_i^d)^T N_i^d)$$

and $\tilde{\Pi}_i$, $\tilde{\Theta}_i$ and $\tilde{\Sigma}_i$ are defined by:

$$\begin{aligned}
\tilde{\Pi}_{1,i} =& P_1 A_i + A_i^T P_1 + Q_1 + \tau_1 \left(N_i^a\right)^T N_i^a + \tau_3 \left(N_i^c\right)^T N_i^c \\
\tilde{\Pi}_{2,i} =& P_2 A_i + A_i^T P_2 + Q_2 \\
\tilde{\Pi}_{\ell,i} =& -2P_\ell + Q_\ell, \text{ for } \ell = 3, \ldots, k+3 \\
\tilde{\Pi}_{82,i} =& \bar{X}_2^T C_i - \bar{L}_{1,i} \\
\tilde{\Pi}_{83,i} =& \bar{X}_2^T W_i - \bar{L}_{2,i} \\
\tilde{\Pi}_{71,i} =& B_i^T P_1 - \bar{K}_i \\
\tilde{\Pi}_{7,i} =& -\bar{X}_1 - \bar{X}_1^T + \tau_2 \left(N_i^b\right)^T N_i^b + \tau_4 \left(N_i^d\right)^T N_i^d \\
\tilde{\Pi}_{8,i} =& -\bar{X}_2 - \bar{X}_2^T
\end{aligned}$$

$$\tilde{\Pi}_i = \begin{pmatrix} \tilde{\Pi}_{1,i} & 0 & 0 & 0 & \cdots & 0 & * & 0 \\ 0 & \tilde{\Pi}_{2,i} & * & 0 & \cdots & 0 & * & * \\ 0 & G_i^T P_2 & \tilde{\Pi}_{3,i} & 0 & \cdots & 0 & * & * \\ 0 & 0 & 0 & \tilde{\Pi}_{4,i} & \cdots & 0 & 0 & * \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \tilde{\Pi}_{k+3,i} & 0 & * \\ \tilde{\Pi}_{71,i} & -\bar{K}_i & -\bar{K}_i^f & 0 & \cdots & 0 & \tilde{\Pi}_{7,i} & 0 \\ 0 & \tilde{\Pi}_{82,i} & \tilde{\Pi}_{83,i} & -\bar{L}_{3,i} & \cdots & -\bar{L}_{k+2,i} & 0 & \tilde{\Pi}_{8,i} \end{pmatrix} \tag{26}$$

$$\tilde{\Theta}_i = \begin{pmatrix} -\tau_1 \left(N_i^a\right)^T N_i^a - \tau_3 \left(N_i^c\right)^T N_i^c & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -G_i^T P_1 & 0 & P_3 & 0 & \cdots & 0 & (\bar{K}_i^f)^T & 0 \\ 0 & 0 & P_3 & P_4 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & P_{k+3} & 0 & * \\ 0 & 0 & 0 & 0 & \cdots & 0 & -\tau_2 \left(N_i^b\right)^T N_i^b - \tau_4 \left(N_i^d\right)^T N_i^d & 0 \end{pmatrix} \tag{27}$$

$$\tilde{\Sigma}_i = \begin{pmatrix} \left(M_i^a\right)^T P_1 & -\left(M_i^a\right)^T P_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \left(M_i^b\right)^T P_1 & -\left(M_i^b\right)^T P_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \left(M_i^c\right)^T \bar{X}_2 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \left(M_i^d\right)^T \bar{X}_2 \end{pmatrix} \tag{28}$$

The observer and controller gains are then computed by:

$$K_i = \left(\bar{X}_1^T\right)^{-1} \bar{K}_i, \quad K_i^f = \left(\bar{X}_1^T\right)^{-1} \bar{K}_i^f, \quad L_i^\ell = \left(P_{\ell+1}\right)^{-1} \left(\bar{L}_{\ell,i}\right)^T \tag{29}$$

Proof: Let us consider the weighted $\mathcal{L}_2$ constraint given by:

$$\int_0^t e^T(\sigma) Q e(\sigma) d\sigma < \gamma^2 \int_0^t \omega^T(\sigma)\omega(\sigma) d\sigma \tag{30}$$

where γ is the attenuation level from the perturbation-like term ω to the errors e in (20) and Q is a known symmetric semi positive definite weighting matrix chosen as $Q = diag(Q_1, Q_2, Q_3, Q_4, \cdots, Q_{k+3}, 0, 0)$. It is well known that the constraints (30) is satisfied if there exists a Lyapunov function $V(e)$ such that:

$$\dot{V}(e) + e^T Q e - \gamma^2 \omega^T \omega < 0 \tag{31}$$

Let us consider the following Lyapunov function candidate given by:

$$V(e) = e^T \tilde{E}^T P e, \ \tilde{E}^T P = P^T \tilde{E} \geq 0 \tag{32}$$

where $P = diag(P_1, P_2, P_3, P_4, \ldots, P_{k+3}, \bar{X}_1, \bar{X}_2)$. It can be shown that the LMI (25) implies (31) for $V(e)$ defined by (32). The mathematical details are given in appendix. ■

Remark 1: Thanks to the descriptor redundancy approach, the T-S system (20) is defined by a unique summation whereas interconnecting a T-S system with a T-S controller (8) and a T-S observer (10) often leads to triple summations. Thus, the number of LMI conditions in the theorem 1 is linear in the number of submodels denoted r, contrarily to most of the previously published works on state estimation or diagnosis for T-S systems where the number of LMI conditions is polynomial in r [18], [30], [32], [33], [34].

Remark 2: Since the descriptor redundancy approach is used to write the closed-loop system (20), the matrix gains to be determined (namely K_i, K_i^f and L_i^j) are neither pre-multiplied nor post-multiplied by any other matrix in the entries of $\tilde{A}_i$ and $\tilde{B}_i$. As a consequence, the Lyapunov stability conditions are easily expressed as LMI in theorem 1 with simple variable changes. This is obtained without the need of conservative upper bounding usually introduced to linearize the coupled terms.

B. Constant fault case

Constant fault may occurs in practical situations when a sensor or an actuator is affected by a constant bias. It can also be considered as an approximation of slowly varying fault. The constant fault case is obviously a special case of the polynomial one. In this case, the fault satisfies:

$$\frac{d}{dt}\left(f\left(t\right)\right)=0 \tag{33}$$

As a consequence, the PMIO can be reduced to a PIO defined by

$$\begin{cases} \dot{\hat{x}}_f(t) = A_\mu \hat{x}_f(t) + B_\mu u_f(t) + G_\mu \hat{f}(t) + L_\mu^1\left(y_f\left(t\right) - \hat{y}_f\left(t\right)\right) \\ \dot{\hat{f}}(t) = L_\mu^2 (y_f(t) - \hat{y}_f(t)) - \hat{f}(t) \\ \hat{y}_f(t) = C_\mu \hat{x}_f(t) + D_\mu u_f(t) + W_\mu \hat{f}(t) \end{cases} \tag{34}$$

where the matrices $L_i^1 \in \mathbb{R}^{n\times p}$ and $L_i^2 \in \mathbb{R}^{q\times p}$, respectively defining the polytopic matrices L_μ^1 and L_μ^2, are the observer gains to be determined taking into account specifications about the reconstruction errors.
Such observers are known to be efficient when estimating systems affected by slowly varying unknown inputs [43].

Similarly to what have been done in the previous section, the state tracking error, the state and fault estimation errors, the difference between the nominal and the FTC laws are defined by (11). The tracking error dynamics and the state estimation error are still defined by (12) and (13) respectively, whereas the fault estimation error is slightly modified and is now defined by:

$$\dot{e}_f = -L_\mu^2 e_y - e_f + f \tag{35}$$

The output estimation error and the active FTC law are also still defined by (18) and (19) respectively. The combination of all these differential and algebraic equations leads to the following descriptor form:

$$\tilde{\tilde{E}}\dot{\tilde{e}}(t) = \tilde{\tilde{A}}_\mu \tilde{e}(t) + \tilde{\tilde{B}}_\mu \tilde{\omega}(t) \tag{36}$$

where $\tilde{\tilde{E}} = diag(I, I, I, 0, 0)$, $\tilde{e}^T = (e_p^T \; e_s^T \; e_f^T \; e_u^T \; e_y^T)$ and

$$\tilde{\omega} = \begin{pmatrix} x \\ f \\ u \end{pmatrix} \quad \tilde{\tilde{A}}_\mu = \begin{pmatrix} \bar{A}_\mu & 0 & 0 & \bar{B}_\mu & 0 \\ -\Delta A_\mu & A_\mu & G_\mu & -\Delta B_\mu & -L_\mu^1 \\ 0 & 0 & -I & 0 & -L_\mu^2 \\ -K_\mu & -K_\mu & -K_\mu^f & -I & 0 \\ -\Delta C_\mu & C_\mu & W_\mu & -\Delta D_\mu & -I \end{pmatrix} \quad \tilde{\tilde{B}}_\mu = \begin{pmatrix} -\Delta A_\mu & -G_\mu & -\Delta B_\mu \\ \Delta A_\mu & 0 & \Delta B_\mu \\ 0 & I & 0 \\ 0 & K_\mu^f & 0 \\ \Delta C_\mu & 0 & \Delta D_\mu \end{pmatrix} \tag{37}$$

The equation (36) explicits the evolution of the error dynamics and, in particular, the influence of the fault on these errors. The gains L_i^1 and L_i^2 of the observer and the gains K_i and K_i^f of the controller clearly appear in the definition of the state matrices $\tilde{\tilde{A}}_\mu$ and $\tilde{\tilde{B}}_\mu$.
It can be mentioned that the descriptor system (36) is the same as (20) where the state variables $\tilde{\phi}_\ell$ and the inputs ϕ_ℓ have been removed. Consequently, the matrices $\tilde{\tilde{A}}_\mu$ and $\tilde{\tilde{B}}_\mu$ in (37) are obtained from $\tilde{A}_\mu$ and $\tilde{B}_\mu$ defined by (21) and (22) respectively by selecting the appropriate rows and columns.
The sufficient LMI stability conditions of the closed-loop system (36) are given in the following theorem.

Theorem 2: The system (36) describing the tracking and estimation errors is stable and the $\mathcal{L}_2$-gain from the faults to the state tracking error, the state and the fault estimation errors is bounded by $\sqrt{\bar{\gamma}}$, if there exists matrices $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $P_3 = P_3^T > 0$, $\bar{X}_1$ invertible, $\bar{X}_2$, $\bar{K}_i$, $\bar{K}_i^f$, $\bar{L}_{1,i}$, $\bar{L}_{2,i}$ and positive scalars $\bar{\gamma}$, τ_1, τ_2, τ_3 and τ_4, such that the following LMIs hold for $i \in \{1, ..., r\}$:

$$\Lambda_i < 0 \tag{38}$$

where Λ_i is defined by:

$$\Lambda_i = \begin{pmatrix} \tilde{\Pi}_{1,i} & 0 & 0 & * & 0 & * & * & 0 & * & * & 0 & 0 \\ 0 & \tilde{\Pi}_{2,i} & * & * & * & 0 & 0 & 0 & * & * & 0 & 0 \\ 0 & G_i^T P_2 & \tilde{\Pi}_{3,i} & * & * & 0 & * & 0 & 0 & 0 & 0 & 0 \\ \tilde{\Pi}_{71,i} & -\bar{K}_i & -\bar{K}_i^f & \tilde{\Pi}_{7,i} & 0 & 0 & * & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{\Pi}_{82,i} & \tilde{\Pi}_{83,i} & 0 & \tilde{\Pi}_{8,i} & 0 & 0 & 0 & 0 & 0 & * & * \\ \Lambda_{61,i} & 0 & 0 & 0 & 0 & \Lambda_{6,i} & 0 & 0 & 0 & 0 & 0 & 0 \\ -G_i^T P_1 & 0 & P_3 & (\bar{K}_i^f)^T & 0 & 0 & -\bar{\gamma} I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Lambda_{84,i} & 0 & 0 & 0 & \Lambda_{8,i} & 0 & 0 & 0 & 0 \\ (M_i^a)^T P_1 & -(M_i^a)^T P_2 & 0 & 0 & 0 & 0 & 0 & 0 & -\tau_1 I & 0 & 0 & 0 \\ (M_i^b)^T P_1 & -(M_i^b)^T P_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\tau_2 I & 0 & 0 \\ 0 & 0 & 0 & 0 & (M_i^c)^T \bar{X}_2 & 0 & 0 & 0 & 0 & 0 & -\tau_3 I & 0 \\ 0 & 0 & 0 & 0 & (M_i^d)^T \bar{X}_2 & 0 & 0 & 0 & 0 & 0 & 0 & -\tau_4 I \end{pmatrix} \tag{39}$$

where $\tilde{\Pi}_{1,i}$, $\tilde{\Pi}_{2,i}$, $\tilde{\Pi}_{3,i}$, $\tilde{\Pi}_{71,i}$, $\tilde{\Pi}_{7,i}$, $\tilde{\Pi}_{82,i}$, $\tilde{\Pi}_{83,i}$, and $\tilde{\Pi}_{8,i}$ are defined in theorem 1 and

$$\begin{aligned} \Lambda_{61,i} &= -\tau_1 \left(N_i^a\right)^T N_i^a - \tau_3 \left(N_i^c\right)^T N_i^c \\ \Lambda_{6,i} &= -\bar{\gamma} I + \tau_1 \left(N_i^a\right)^T N_i^a + \tau_3 \left(N_i^c\right)^T N_i^c \\ \Lambda_{84,i} &= -\tau_2 \left(N_i^b\right)^T N_i^b - \tau_4 \left(N_i^d\right)^T N_i^d \\ \Lambda_{8,i} &= -\bar{\gamma} I + \tau_2 \left(N_i^b\right)^T N_i^b + \tau_4 \left(N_i^d\right)^T N_i^d \end{aligned}$$

The observer and controller gains are then computed by:

$$K_i = \left(\bar{X}_1^T\right)^{-1} \bar{K}_i, \quad K_i^f = \left(\bar{X}_1^T\right)^{-1} \bar{K}_i^f, \quad L_i^1 = (P_2)^{-1} \left(\bar{L}_{1,i}\right)^T, \quad L_i^2 = (P_3)^{-1} \left(\bar{L}_{2,i}\right)^T \tag{40}$$

Proof: The proof is very similar to the one of Theorem 1. The considered weighted $\mathcal{L}_2$ constraint is now given by:

$$\int_0^t \tilde{e}^T(\sigma) Q \tilde{e}(\sigma) d\sigma < \gamma^2 \int_0^t \tilde{\omega}^T(\sigma) \tilde{\omega}(\sigma) d\sigma \tag{41}$$

where γ is the attenuation level from the perturbation-like term $\tilde{\omega}$ to the errors $\tilde{e}$ in (36) and Q is a known symmetric semi positive definite weighting matrix chosen as $Q = diag(Q_1, Q_2, Q_3, 0, 0)$. It is well known that the constraint (41) is satisfied if there exists a Lyapunov function $V(\tilde{e})$ such that:

$$\dot{V}(\tilde{e}) + \tilde{e}^T Q \tilde{e} - \gamma^2 \tilde{\omega}^T \tilde{\omega} < 0 \tag{42}$$

Let us consider the following Lyapunov function candidate given by:

$$V(\tilde{e}) = \tilde{e}^T \tilde{\bar{E}}^T P \tilde{e},\ \ \tilde{\bar{E}}^T P = P^T \tilde{\bar{E}} \geq 0 \tag{43}$$

where $P = diag(P_1, P_2, P_3, \bar{X}_1, \bar{X}_2)$. Using the quadratic Lyapunov function candidate (43) and (36), the inequality (42) leads to:

$$\begin{pmatrix} \tilde{e} \\ \tilde{\omega} \end{pmatrix}^T \Psi_\mu \begin{pmatrix} \tilde{e} \\ \tilde{\omega} \end{pmatrix} < 0 \tag{44}$$

with

$$\Psi_\mu = \begin{pmatrix} \tilde{\bar{A}}_\mu^T P + P^T \tilde{\bar{A}}_\mu + Q & * \\ \tilde{\bar{B}}_\mu^T X & -\gamma^2 I \end{pmatrix} \tag{45}$$

Similarly to the proof of Theorem 1, it can be shown that the LMI (38) implies (44) and thus (42). ■

C. *Exponential fault case*

In this section, the fault is assumed to be defined, on a finite time window $[t_0\ t_f]$, by an exponential function which parameters are uncertain. Thus, each component of the fault is given by:

$$f_i\left(t\right) = e^{\alpha_i t + \beta_i}, \text{for } t_0 \leq t \leq t_f \tag{46}$$

where $\alpha_i, \beta_i \in \mathbb{R}$, for $i = 1, ..., q$.

To model as closely as possible the dynamics of the exponential fault, the parameters α_i in (46) can be expressed as:

$$\begin{cases} \alpha_i = \alpha_{0,i} + \Delta\alpha_i \\ \alpha = diag\begin{pmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_q \end{pmatrix} \end{cases} \tag{47}$$

where $\alpha_{0,i}$ and $\Delta\alpha_i$ respectively represent the nominal and the uncertain parts of the parameters α_i.

$$\begin{cases} \alpha_0 = diag\begin{pmatrix} \alpha_{0,1} & \alpha_{0,2} & \cdots & \alpha_{0,q} \end{pmatrix} \\ \Delta\alpha = diag\begin{pmatrix} \Delta\alpha_1 & \Delta\alpha_2 & \cdots & \Delta\alpha_q \end{pmatrix} \end{cases} \tag{48}$$

It is assumed that there exists a known diagonal positive definite matrix $\nu \in \mathbb{R}^{q \times q}$, such that the uncertain part of the fault model can be bounded as:

$$\Delta\alpha^T \Delta\alpha \leq \nu \tag{49}$$

Using the PI observer (34) when the fault are described by (46), the fault estimation dynamics is given by:

$$\dot{e}_f = \dot{f} - \dot{\hat{f}} \tag{50}$$

$$= \alpha f - L_\mu^2 e_y + \hat{f} \tag{51}$$

By adding and subtracting f in (51), one can obtain:

$$\dot{e}_f = -L_\mu^2 e_y - e_f + (\alpha + I)\, f \tag{52}$$

The concatenation of (12), (13), (52), (18) and (19) leads to:

$$\tilde{\tilde{E}}\dot{\tilde{e}}(t) = \tilde{\tilde{A}}_\mu \tilde{e}(t) + \tilde{\tilde{B}}_\mu \tilde{\omega}(t) \tag{53}$$

where $\tilde{e}$, $\tilde{\omega}$, $\tilde{\tilde{E}}$ and $\tilde{\tilde{A}}_\mu$ have been already defined in (36) and $\tilde{\tilde{B}}_\mu$ is now defined by

$$\tilde{\tilde{B}}_\mu = \begin{pmatrix} -\Delta A_\mu & -G_\mu & -\Delta B_\mu \\ \Delta A_\mu & 0 & \Delta B_\mu \\ 0 & \alpha + I & 0 \\ 0 & K_\mu^f & 0 \\ \Delta C_\mu & 0 & \Delta D_\mu \end{pmatrix} \tag{54}$$

In order to compute the gains L_μ^1, L_μ^2, K_μ and K_μ^f, LMI conditions are given in the following theorem.

Theorem 3: The system (53) describing the different estimation errors is stable and the $\mathcal{L}_2$-gain from the fault to the state tracking error, the state and fault estimation errors is bounded by $\sqrt{\bar{\gamma}}$, if there exists matrices $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $P_3 = P_3^T > 0$, $\bar{X}_1$ invertible, $\bar{X}_2$, $\bar{K}_i$, $\bar{K}_i^f$, $\bar{L}_{1,i}$, $\bar{L}_{2,i}$, positive scalars $\bar{\gamma}$, τ_1, τ_2, τ_3, τ_4 and η such that the following LMIs are verified for $i \in \{1, ..., r\}$:

$$\left(\begin{array}{c|c} \Lambda_i + T^T\psi + \psi^T T + \eta\vartheta^T\vartheta & * \\ \hline \psi & -\eta I \end{array}\right) < 0 \tag{55}$$

where Λ_i is given in theorem 2 and

$$T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{56}$$

$$\psi = \begin{pmatrix} 0 & 0 & P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{57}$$

$$\vartheta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \nu^{1/2} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{58}$$

The observer and controller gains are then computed by:

$$K_i = \left(\bar{X}_1^T\right)^{-1} \bar{K}_i, \quad K_i^f = \left(\bar{X}_1^T\right)^{-1} \bar{K}_i^f, \quad L_i^1 = (P_2)^{-1} \left(\bar{L}_{1,i}\right)^T, \quad L_i^2 = (P_3)^{-1} \left(\bar{L}_{2,i}\right)^T \tag{59}$$

Proof: The proof of this theorem is similar to the previous one. Considering the quadratic Lyapunov function candidate defined by (43) and the $\mathcal{L}_2$ constraint (41), the development of (42) leads to:

$$\begin{pmatrix} \tilde{e} \\ \tilde{\omega} \end{pmatrix}^T \tilde{\Psi}_\mu \begin{pmatrix} \tilde{e} \\ \tilde{\omega} \end{pmatrix} < 0 \tag{60}$$

where $\tilde{\Psi}_\mu$ is defined by

$$\tilde{\Psi}_\mu = \Psi_\mu + Z^T\bar{\psi} + \bar{\psi}^T Z + X^T\bar{\psi} + \bar{\psi}^T X \tag{61}$$

where Ψ_μ is defined in (45), Z, X and $\bar{\psi}$ are defined by:

$$Z = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \alpha_0 & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \Delta\alpha & 0 \end{pmatrix}$$

$$\bar{\psi} = \begin{pmatrix} 0 & 0 & P_3 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Applying Lemma 1 on (61), one can obtain:

$$\Psi_\mu + Z^T\bar{\psi} + \bar{\psi}^T Z + X^T\bar{\psi} + \bar{\psi}^T X \leq \Psi_\mu + Z^T\bar{\psi} + \bar{\psi}^T Z + \eta\bar{\vartheta}^T\bar{\vartheta} + \eta^{-1}\bar{\psi}^T\bar{\psi} \tag{62}$$

where $\bar{\vartheta} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \nu^{1/2} \end{pmatrix}$. Following the same steps of the previous proofs and applying the Schur complement on the term $\eta^{-1}\bar{\psi}^T\bar{\psi}$, the sufficient LMI conditions proposed in theorem 3 follows. ■

IV. Simulation example

Let us consider the nonlinear reference and faulty systems (2) and (4), with $r = 2$ defined by:

$$A_1 = \begin{pmatrix} 0 & -1 & -0.5 \\ -1 & -3 & 2.5 \\ 1 & 1 & -2 \end{pmatrix}, B_1 = \begin{pmatrix} 2.5 \\ 1 \\ 2 \end{pmatrix}, C_1^T = \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -2 & 1 & 1 \\ 0 & -3 & 3 \\ 0.5 & 1 & -3 \end{pmatrix}, B_2 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, C_2^T = \begin{pmatrix} 0 \\ -0.5 \\ 0 \end{pmatrix}$$

$$D_1 = 1,\ D_2 = 0.8,\ W_1 = 1.5,\ W_2 = 1,$$

$$G_1^T = \begin{pmatrix} 1 & 0.5 & 0 \end{pmatrix}, G_2^T = \begin{pmatrix} 0.6 & 0.5 & 0.5 \end{pmatrix},$$

$$N_1^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -0.15 & 0.1 \\ 0.15 & 0 & -0.2 \end{pmatrix}, N_2^a = \begin{pmatrix} -0.15 & 0 & 0 \\ 0 & -0.15 & 0 \\ 0 & 0 & -0.1 \end{pmatrix},$$

$$M_1^a = \begin{pmatrix} 0.1 & 0.3 & 0.2 \\ 0 & 0.15 & 0 \\ 0.15 & 0.1 & 0.2 \end{pmatrix}, M_2^a = \begin{pmatrix} 0.1 & 0.2 & 0.2 \\ 0.1 & 0.25 & 0.1 \\ 0.1 & 0.2 & 0.15 \end{pmatrix},$$

$$M_1^{bT} = \begin{pmatrix} 0.15 & 0.25 & 0.2 \end{pmatrix}, M_2^{bT} = \begin{pmatrix} 0.2 & 0.25 & 0.2 \end{pmatrix}$$

$$N_1^b = 0.2, N_2^b = 0.1, N_1^d = 0.25, N_2^d = 0.15,$$

$$N_1^c = \begin{pmatrix} 0.2 & 0.15 & 0 \end{pmatrix}, N_2^c = \begin{pmatrix} 0.1 & 0.05 & 0.15 \end{pmatrix},$$

$$M_1^d = 0.1, M_2^d = 0.15, M_1^c = 0.2, M_2^c = 0.25$$

The uncertainties are defined by:

$$\delta_1^a(t) = \delta_2^a(t) = 0.5\sin^2(t)$$

$$\delta_1^b(t) = \delta_2^b(t) = 0.1\cos(t)\sin(t)$$

$$\delta_1^c(t) = \delta_2^c(t) = 0.5\sin(t)$$

$$\delta_1^d(t) = \delta_2^d(t) = 0.3\cos^2(t)$$

The nominal input signal and the nonlinear weighting functions are respectively given by:
$u(t) = \sin(\cos(0.5t)\,t)$, $\mu_1(u(t)) = \frac{1-\tanh(0.5-u(t))}{2}$ and $\mu_2(u(t)) = 1 - \mu_1(u(t))$.

A. *Case of second order polynomial fault*

The fault occurring when $8\,s \leqslant t \leqslant 18\,s$ is modeled by the following second order polynomial:

$$f(t) = 0.03(t-8)(t-18) \tag{63}$$

Figure 2 (left) represents, on the same graph, the reference model states and the faulty system states with and without FTC. Clearly, when the fault occurs, the faulty system states deviate from the reference one. Implementing the proposed strategy, one observes that, firstly, the fault is correctly estimated (figure 2 right) and, secondly, the faulty system states follow the reference trajectory with a small error. These simulation results show the effectiveness of the synthesized observer and active FTC controller compared to the classical controller, since the fault effect is compensated and the tracking between the faulty system states and the reference model ones is ensured. Figure 3 allows the comparison of the nominal control and active FTC signals.
In the next, other kinds of faults (third order polynomial, sinusoidal and constant faults) are considered, while the FTC law design is still computed for a second order polynomial fault. This illustrates the robustness of the above FTC and observer design face to poorly modeled time varying faults.

B. *Case of third order polynomial fault*

The considered fault affecting the system behavior at $8\,s \leq t \leq 18\,s$ is defined by:

$$f(t) = 0.015(t-8)(t-18)^2 \tag{64}$$

The simulation results are given by the figures 4 and 5. From these results, one can remark that even if the FTC controller and observer are synthesized for a second order polynomial fault, when the fault is a third order polynomial it is still well estimated and compensated.

C. *Case of sinusoidal fault*

The considered fault occurring at $8\,s \leq t \leq 18\,s$ is defined by:

$$f(t) = 0.6\sin(t-8) \tag{65}$$

The simulation results are given by figures 6 and 7. Even if the fault is sinusoidal, the active FTC controller is once again designed to cope with a second order polynomial fault. One can see that, although the model of the fault is false, the controller ensures a very good tracking between the faulty system states and the fault-free system ones.

V. Conclusion

In this paper, the problem of active FTC design for uncertain faulty nonlinear systems represented by Takagi-Sugeno models is treated. The aim of the FTC is to ensure the state trajectory tracking. Three kinds of faults have been considered. The first one consists in faults modeled by polynomial functions. Constant faults are then treated as a special case of the first ones. The last one deals with faults modeled by exponential functions. Based on Lyapunov theory, a new approach dealing with the considered faults is proposed. This approach has been easily formulated in LMI terms by using the descriptor redundancy property, which allowing to express the error dynamics of the closed-loop system in descriptor form. Finally, academic examples have been considered to illustrate the efficiency of the proposed approaches.

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APPENDIX A

PROOF OF THE THEOREM 1

Substituting (20) in (31), one can obtain:

$$\begin{pmatrix} e \\ \omega \end{pmatrix}^T \begin{pmatrix} \tilde{A}_\mu^T P + P^T \tilde{A}_\mu + Q & * \\ \tilde{B}_\mu^T P & -\gamma^2 I \end{pmatrix} \begin{pmatrix} e \\ \omega \end{pmatrix} < 0 \tag{66}$$

The inequality (66) fulfilled if:

$$\begin{pmatrix} \tilde{A}_\mu^T P + P^T \tilde{A}_\mu + Q & * \\ \tilde{B}_\mu^T P & -\gamma^2 I \end{pmatrix} < 0 \tag{67}$$

In order to obtain LMI conditions, matrix P is chosen as follows: $P = diag(P_1, P_2, P_3, P_4, \ldots, P_{k+3}, \bar{X}_1, \bar{X}_2)$. According to (32), we can find that, for $j = 1, 2, ..., (k+3)$ P_j are symmetric positive definite matrices and $\bar{X}_1$, $\bar{X}_2$ are free matrices with $\bar{X}_1$ invertible. Note that, the chosen structure of P is not unique. Indeed, there are several possibilities from the condition (32) to define the matrix P structure.

Considering the matrices $\tilde{A}_\mu$ and $\tilde{B}_\mu$ in (20) and P, the inequality (67) becomes:

$$\begin{pmatrix} \Pi_\mu & * \\ \tilde{\Upsilon}_\mu & -\gamma^2 I \end{pmatrix} < 0 \tag{68}$$

where Π_μ, $\tilde{\Upsilon}_\mu$ are defined by

$$\Pi_\mu = \begin{pmatrix} \Pi_{1,\mu} & * & 0 & 0 & \cdots & 0 & * & * \\ -P_2 \Delta A_\mu & \tilde{\Pi}_{2,\mu} & 0 & 0 & \cdots & 0 & * & * \\ 0 & G_\mu^T P_2 & \tilde{\Pi}_{3,\mu} & 0 & \cdots & 0 & * & * \\ 0 & 0 & 0 & \tilde{\Pi}_{4,\mu} & \cdots & 0 & * & * \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \tilde{\Pi}_{k+3,\mu} & * & * \\ \Pi_{71,\mu} & \Pi_{72,\mu} & -\bar{X}_1^T K_\mu^f & 0 & \cdots & 0 & -\bar{X}_1 - \bar{X}_1^T & * \\ -\bar{X}_2^T \Delta C_\mu & \Pi_{82,\mu} & \Pi_{83,\mu} & -L_{3,\mu}^T P_4 & \cdots & -L_{k+2,\mu}^T P_{k+3} & -\bar{X}_2^T \Delta D_\mu & -\bar{X}_2 - \bar{X}_2^T \end{pmatrix} \tag{69}$$

$$\Pi_{1,\mu} = P_1 A_i + A_i^T P_1 + Q_1 \qquad \Pi_{71,\mu} = -\bar{X}_1^T K_\mu + \bar{B}_\mu^T P_1$$

$$\Pi_{72,\mu} = -\bar{X}_1^T K_\mu - \Delta B_\mu^T P_2 \qquad \Pi_{82,\mu} = \bar{X}_2^T C_\mu - \left(L_\mu^1\right)^T P_2$$

$$\Pi_{83,\mu} = \bar{X}_2^T W_\mu - \left(L_\mu^2\right)^T P_3$$

$$\tilde{\Upsilon}_\mu = \begin{pmatrix} -\Delta A_\mu^T P_1 & \Delta A_\mu^T P_2 & 0 & 0 & \cdots & 0 & 0 & \Delta C_\mu^T \bar{X}_2 \\ -G_\mu^T P_1 & 0 & P_3 & 0 & \cdots & 0 & (K_\mu^f)^T \bar{X}_1 & 0 \\ 0 & 0 & P_3 & P_4 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & P_{k+3} & 0 & 0 \\ -\Delta B_\mu^T P_1 & \Delta B_\mu^T P_2 & 0 & 0 & \cdots & 0 & 0 & \Delta D_\mu^T \bar{X}_2 \end{pmatrix} \tag{70}$$

With (5-7) and applying lemma 1, (68) yields:

$$\begin{pmatrix} \bar{\Pi}_\mu & * & * \\ \bar{\varphi}_\mu & -\phi_\mu & 0 \\ \Sigma_\mu & 0 & -\bar{\xi} \end{pmatrix} < 0 \tag{71}$$

where ϕ_μ, $\bar{\Pi}_\mu$, $\bar{\varphi}_\mu$ and Σ_μ are defined by

$$\begin{aligned} \phi_\mu =& diag(\gamma^2 I - \tau_1 (N_\mu^a)^T N_\mu^a - \tau_3 \left(N_\mu^c\right)^T N_\mu^c, \gamma^2 I, \\ & \gamma^2 I, \cdots, \gamma^2 I, \gamma^2 I - \tau_2 (N_\mu^b)^T N_\mu^b - \tau_4 \left(N_\mu^d\right)^T N_\mu^d) \end{aligned}$$

$$\bar{\Pi}_\mu = \begin{pmatrix} \tilde{\Pi}_{1,\mu} & 0 & 0 & 0 & \cdots & 0 & * & 0 \\ 0 & \tilde{\Pi}_{2,\mu} & * & 0 & \cdots & 0 & * & * \\ 0 & G_\mu^T P_2 & \tilde{\Pi}_{3,\mu} & 0 & \cdots & 0 & * & * \\ 0 & 0 & 0 & \tilde{\Pi}_{4,\mu} & \cdots & 0 & * & * \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \tilde{\Pi}_{k+3,\mu} & * & * \\ -\bar{X}_1^T K_\mu & -\bar{X}_1^T K_\mu & -\bar{X}_1^T K_\mu^f & 0 & \cdots & 0 & \tilde{\Pi}_{7,\mu} & 0 \\ 0 & \Pi_{82,\mu} & \Pi_{83,\mu} & -L_{3,\mu}^T P_4 & \cdots & -L_{k+2,\mu}^T P_{k+3} & 0 & \tilde{\Pi}_{8,\mu} \end{pmatrix} \tag{72}$$

$$\bar{\varphi}_\mu = \begin{pmatrix} -\tau_1 {N_\mu^a}^T N_\mu^a - \tau_3 {N_\mu^c}^T N_\mu^c & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -G_\mu^T P_1 & 0 & P_3 & 0 & \cdots & 0 & (K_\mu^f)^T \bar{X}_1 & 0 \\ 0 & 0 & P_3 & P_4 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & P_{k+3} & 0 & * \\ 0 & 0 & 0 & 0 & \cdots & 0 & -\tau_2 \left(N_\mu^b\right)^T N_\mu^b - \tau_4 \left(N_\mu^d\right)^T N_\mu^d & 0 \end{pmatrix} \tag{73}$$

$$\Sigma_{\mu} = \begin{pmatrix} \left(M_{\mu}^{a}\right)^{T} P_1 & -\left(M_{\mu}^{a}\right)^{T} P_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \left(M_{\mu}^{b}\right)^{T} P_1 & -\left(M_{\mu}^{b}\right)^{T} P_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \left(M_{\mu}^{c}\right)^{T} \bar{X}_2 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \left(M_{\mu}^{d}\right)^{T} \bar{X}_2 \end{pmatrix} \tag{74}$$

Considering the following bijective variable changes: $\bar{X}_1^T K_{\mu} = \bar{K}_{\mu}$, $\bar{X}_1^T K_{\mu}^f = \bar{K}_{\mu}^f$, $\gamma^2 = \bar{\gamma}$ and $\left(L_{\mu}^{\ell}\right)^T P_{\ell+1} = \bar{L}_{\ell,\mu}$ for $\ell = 1, \ldots, k+2$ and applying the Schur complement on $\bar{\Pi}_{\mu}$, the sufficient LMI conditions in theorem 1 hold.

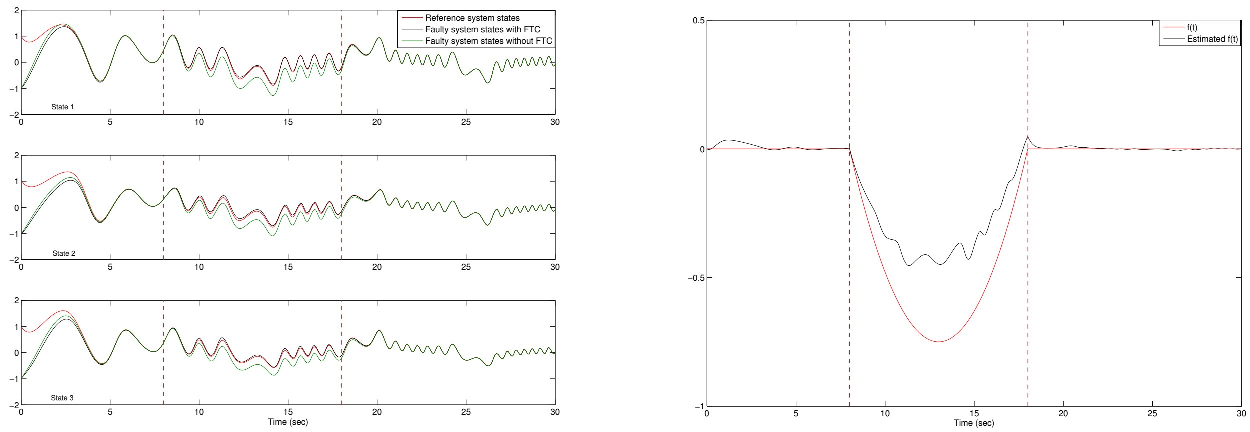

Fig. 2. Second order polynomial fault. Reference model states, faulty system states with and without active FTC controller (left) ; fault and its estimation (right)

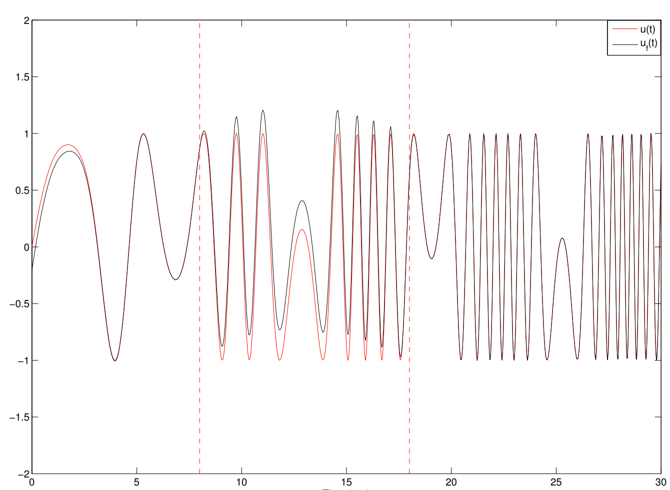

Fig. 3. Second order polynomial fault. Nominal and active FTC control signals

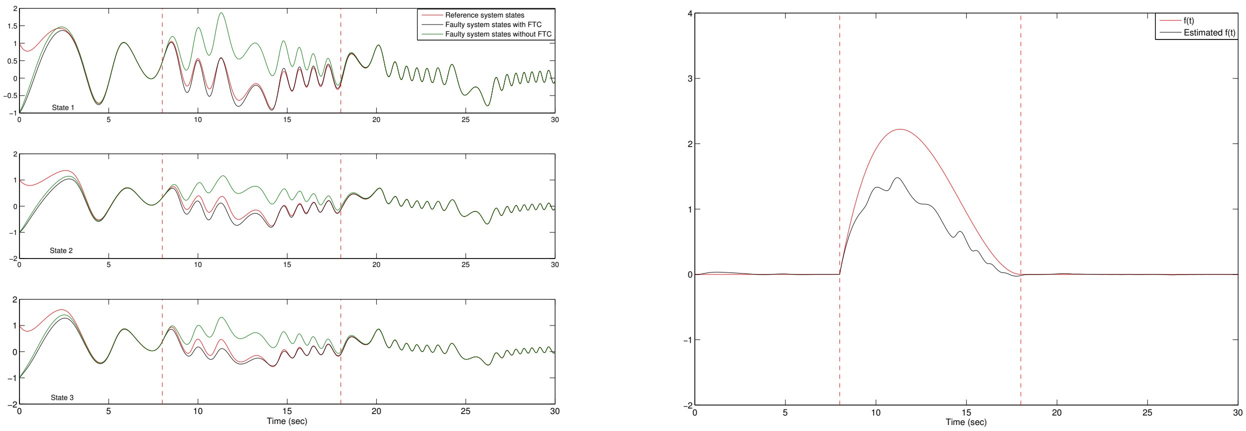

Fig. 4. Third order polynomial fault. Reference model states, faulty system states with and without active FTC controller (left) ; fault and its estimation (right)

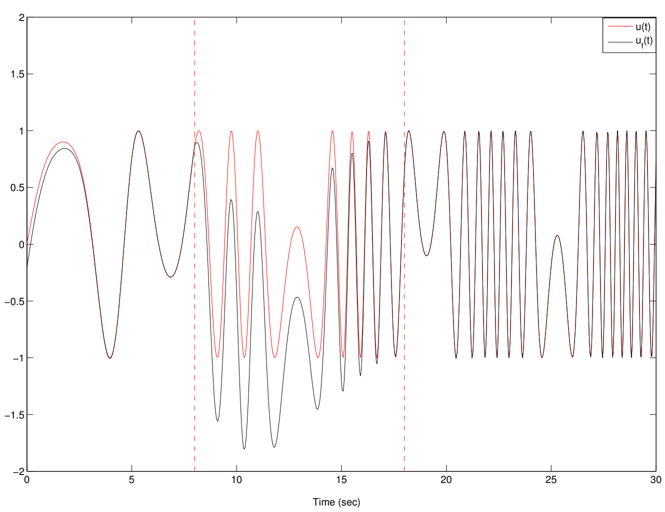

Fig. 5. Third order polynomial fault: nominal and active FTC control signals

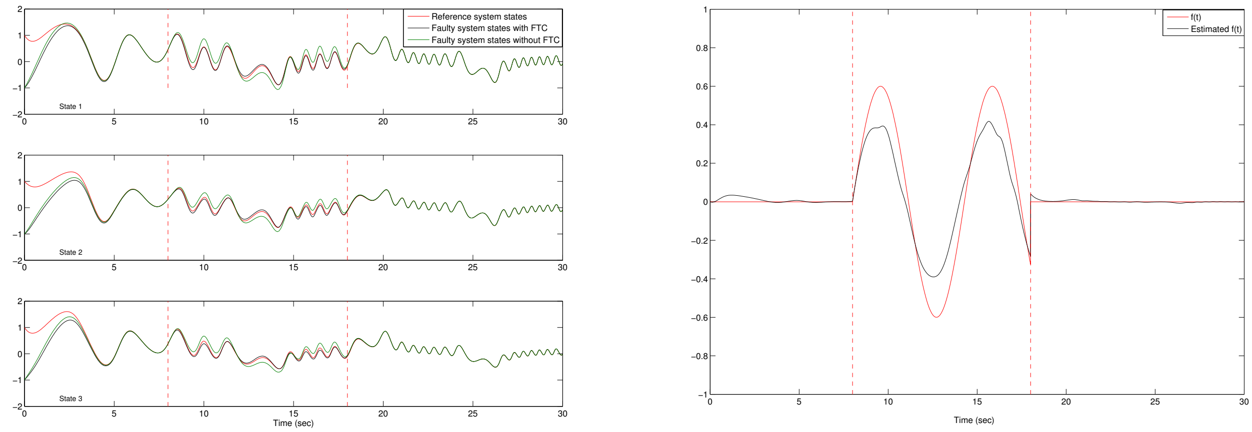

Fig. 6. Sinusoidal fault. Reference model states, faulty system states with and without active FTC controller (left) ; fault and its estimation (right)

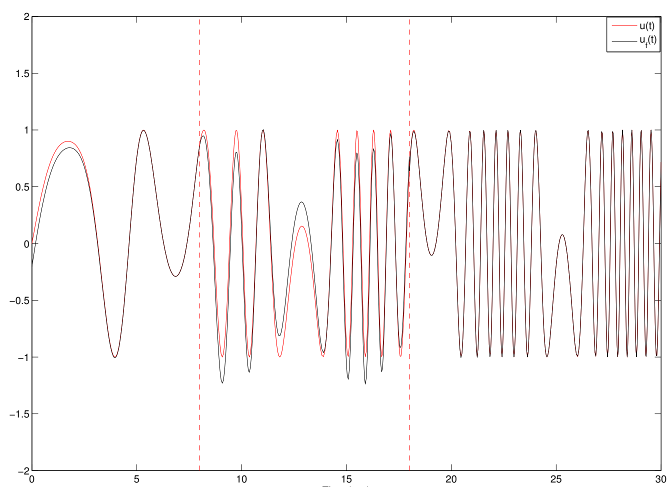

Fig. 7. Sinusoidal fault: nominal and active FTC control signals