Stabilization of nonlinear systems subject to actuator saturation

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Outline of the talk

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Marx,
Maquin,
Ragot

Problem statement and some background

The Takagi-Sugeno modeling of the saturated control

Saturated PDC control

Numerical example

Possible improvements and extensions

Conclusion & perspectives

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- the stabilization of a dynamic nonlinear system

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = g(x(t), u(t))$$

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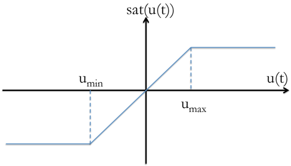
- despite a saturated input control



$$sat(u(t)) = \begin{cases} u_{max}, & u_{max} \leq u(t) \\ u(t), & u_{min} \leq u(t) \leq u_{max} \\ u_{min}, & u(t) \leq u_{min} \end{cases}$$

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$$\dot{x}(t) = f(x(t), u(t))$$
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with bounded nonlinearities or with $x(t)$ lying in a compact set of $\mathbb{R}^n$

The Takagi-Sugeno modeling of a nonlinear system

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- can be written as a Takagi-Sugeno (T-S) system

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\,(A_i x(t) + B_i u(t))$$

$$y(t) = \sum_{i=1}^{r} h_i(z(t))\,(C_i x(t) + D_i u(t))$$

where – $z(t)$ is the decision variable
– $h_i(z(t))$ are the activating functions

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where – $z(t)$ is the decision variable
– $h_i(z(t))$ are the activating functions

- The decision variable is assumed to be measurable
- The activating functions $h_i(z(t))$ satisfy the convex sum properties

$$0 \leq h_i(z(t)) \leq 1 \quad \text{and} \quad \sum_{i=1}^{r} h_i(z(t)) = 1$$

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A scalar saturated input

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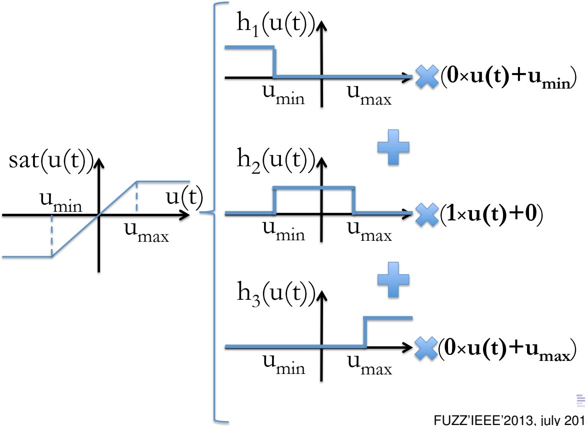
can be put in a T-S (or polytopic) form:



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can be put in a T-S (or polytopic) form:

$$sat(u(t)) = \sum_{i=1}^{3} h_i(u(t))(\lambda_i u(t) + \gamma_i)$$

with

$$\begin{cases} \lambda_1 = 0 \\ \lambda_2 = 1 \\ \lambda_3 = 0 \end{cases} \quad \begin{cases} \gamma_1 = u_{min} \\ \gamma_2 = 0 \\ \gamma_3 = u_{max} \end{cases} \quad \begin{cases} h_1(u(t)) = \frac{1-sign(u(t)-u_{min})}{2} \\ h_2(u(t)) = \frac{sign(u(t)-u_{min})-sign(u(t)-u_{max})}{2} \\ h_3(u(t)) = \frac{1+sign(u(t)-u_{max})}{2} \end{cases}$$

where the $h_i(u(t))$ functions satisfy the convex sum properties

$$0 \leq h_i(u(t)) \leq 1 \quad and \quad \sum_{i=1}^{3} h_i(u(t)) = 1$$

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- The T-S modeling can be generalized to a saturated vector input

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \begin{pmatrix} \sum_{i=1}^{3} h_i^1(u^1(t))(\lambda_i^1 u^1(t) + \gamma_i^1) \\ \sum_{j=1}^{3} h_j^2(u^2(t))(\lambda_j^2 u^2(t) + \gamma_j^2) \end{pmatrix}$$

The Takagi-Sugeno modeling of the saturated control

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- Since $\sum_i h_i^1 = 1$ and $\sum_j h_j^2 = 1$, then $sat(u(t))$ becomes

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \begin{pmatrix} \sum_{i=1}^{3} h_i^1(u^1(t))(\lambda_i^1 u^1(t) + \gamma_i^1)\left(\sum_{j=1}^{3} h_j^2(u^2(t))\right) \\ \left(\sum_{i=1}^{3} h_i^1(u^1(t))\right) \sum_{j=1}^{3} h_j^2(u^2(t))(\lambda_j^2 u^2(t) + \gamma_j^2) \end{pmatrix}$$

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- or equivalently

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \sum_{i=1}^{3}\sum_{j=1}^{3} \underbrace{h_i^1(u^1(t))h_j^2(u^2(t))}_{\mu_i(u(t))} \left(\underbrace{\begin{pmatrix} \lambda_i^1 & 0 \\ 0 & \lambda_j^2 \end{pmatrix}}_{\Lambda_i} u(t) + \underbrace{\begin{pmatrix} \gamma_i^1 \\ \gamma_j^2 \end{pmatrix}}_{\Gamma_i} \right)$$

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- More generally, for $u(t) \in \mathbb{R}^{n_u}$, $sat(u(t))$ can be written under a TS form

$$sat(u(t)) = \sum_{i=1}^{3^{n_u}} \mu_i(u(t))(\Lambda_i u(t) + \Gamma_i)$$

- Given a saturated nonlinear system

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i sat(u(t)))$$

$$y(t) = \sum_{i=1}^{r} h_i(z(t))(C_i x(t) + D_i sat(u(t)))$$

- determine the gains K_j of the PDC state feedback controller

$$u(t) = -\sum_{j=1}^{r} h_j(z(t)) K_j x(t)$$

- in order to
  – ensure the closed loop stability
  – despite the input saturation

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- Without input saturation, the closed loop system is

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) (A_i - B_i K_j) x(t)$$

$\rightarrow$ asymptotically stable, if
$A_i P - B_i \bar{K}_j + (A_i P - B_i \bar{K}_j)^T < 0$ and $K_j = \bar{K}_j P^{-1}$

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- With the input saturation, the closed loop system is

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \mu_k(z(t)) \left((A_i - B_i \Lambda_k K_j) x(t) + B_i \Gamma_k \right)$$

$\rightarrow$ asymptotical stability is no longer ensured
$\rightarrow$ convergence in a ball, to be minimized, is sought

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- The closed-loop stability is studied with a quadratic Lyapunov function

$$V(x(t)) = x^T(t)Px(t), \quad P = P^T > 0$$

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$$V(x(t)) = x^T(t)Px(t), \quad P = P^T > 0$$

- It can be shown that:

$$\frac{dV(x(t))}{dt} \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{3^{n_u}} h_i(z(t))h_j(z(t))\mu_k(u(t))\left(x^T(t)Q_{ijk}x(t) + R_{ijk}\right)$$

with Q_{ijk} and R_{ijk} depending on P, K_j and a slack variable Σ_k.

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with Q_{ijk} and R_{ijk} depending on P, K_j and a slack variable Σ_k.

- Sufficient LMI convergence conditions into a ball are derived:

$$\begin{cases} Q_{ijk} < 0 \\ \varepsilon = \min_{i,j,k}(\underline{\lambda}(-Q_{ijk})) \\ \delta = \max_{i,j,k} R_{ijk} \end{cases} \Rightarrow \begin{cases} \frac{dV(x(t))}{dt} < 0 \\ \forall ||x(t)|| \geq \sqrt{\frac{\delta}{\varepsilon}} \end{cases} \Rightarrow x(t) \to \mathcal{B}\left(0, \sqrt{\frac{\delta}{\varepsilon}}\right)$$

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There exists a PDC controller for a saturated input system such that the system state converges toward an origin-centered ball of radius bounded by β if there exists matrices $P_1 = P_1^T > 0$, R_j, $\Sigma_k = (\Sigma_k)^T > 0$, solutions of

$$\min_{P_1, R_j, \Sigma_k, \beta} \beta$$

under the LMI constraints (for $i, j = 1, \ldots, n$ and $k = 1, \ldots, 3^{n_u}$)

$$\left(\begin{array}{cc|cc} A_iP_1 - B_i\Lambda_kR_j + (A_iP_1 - B_i\Lambda_kR_j)^T & I & I & 0 \\ I & -\Sigma_k & 0 & I \\ \hline I & 0 & -\beta I & 0 \\ 0 & I & 0 & -\beta I \end{array}\right) < 0$$

$$\Gamma_k^T B_i^T \Sigma_k B_i \Gamma_k < \beta$$

The gains of the controller $u(t) = -\sum_{j=1}^{r} h_j(z(t))K_jx(t)$ are given by

$$K_j = P_1^{-1} R_j$$

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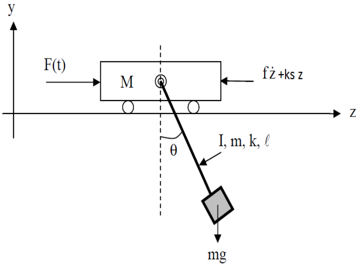

- $z(t)$: cart position
- $\theta(t)$: angle between vertical and pendulum
- M and m: cart and pendulum masses
- l and I_m: length and inertia moment of the pendulum
- f, k_s and k: friction coefficients
- $F(t)$: saturated control input

The system is described by:

$$(m+M)\ddot{z}(t) + k_s z(t) + f\dot{z}(t) - ml\ddot{\theta}(t)\cos(\theta(t)) + ml\dot{\theta}^2(t)\sin(\theta(t)) = F(t)$$

$$-ml\ddot{z}(t)\cos(\theta(t)) + (ml^2 + I_m)\ddot{\theta}(t) + k\dot{\theta}(t) + mgl\sin(\theta(t)) = 0$$

with a saturated control input: $F(t) \in [0\ 3]$

- With $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1$, it becomes

$$(m+M)\ddot{z}(t) + k_s z(t) + f\dot{z}(t) - ml\ddot{\theta}(t) + ml\dot{\theta}^2(t)\theta(t) = F(t)$$
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- Defining the premisse variable by $\xi(t) = \dot{\theta}^2(t)$, with $\xi(t) \in [\underline{\xi}\ \overline{\xi}]$

$$\xi(t) = h_1(\xi(t))\overline{\xi} + h_2(\xi(t))\underline{\xi}, \quad \textit{with} \quad \begin{cases} h_1(\xi(t)) = \frac{\xi(t)-\underline{\xi}}{\overline{\xi}-\underline{\xi}} \\ h_2(\xi(t)) = \frac{\overline{\xi}-\xi(t)}{\overline{\xi}-\underline{\xi}} \end{cases}$$

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$$
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\end{aligned}
$$

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$$

- the system becomes

$$
\begin{pmatrix} \dot{z}(t) \\ \ddot{z}(t) \\ \dot{\theta}(t) \\ \ddot{\theta}(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_s}{m+M} & \frac{-f-lma}{m+M} & \frac{-ml}{m+M} & ka \\ 0 & 0 & 0 & 1 \\ -k_s a & -fa & -mla\xi(t)-(m+M)ga & -k_s a \end{pmatrix} \begin{pmatrix} z(t) \\ \dot{z}(t) \\ \theta(t) \\ \dot{\theta}(t) \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1+mla}{m+M} \\ 0 \\ a \end{pmatrix} F(t)
$$

with $a = \frac{1}{(l+I_m/(ml))(m+M)-ml}$

- Using the nonlinear sector transformation, a TS system with $r=2$ submodels is derived.

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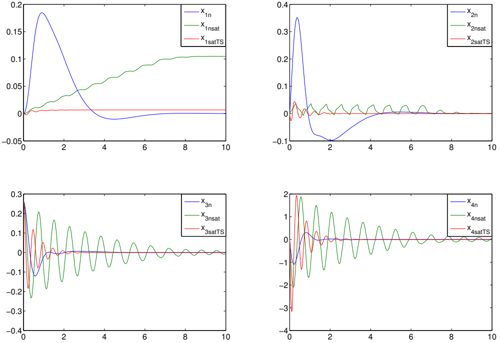
The input saturation is defined by: $F(t) \in [0\ 3]$

Applying the proposed approach, the obtained gains are:

$K_1 = [0.012 \quad -15.04 \quad 15.88 \quad 0.79]$ $\qquad K_2 = [0.008 \quad -19.03 \quad 8.77 \quad 0.53]$



- nominal control of the unsaturated system
- nominal control applied to the saturated system $\rightarrow$ **unstable !**
- proposed PDC control of the saturated system

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Possible improvements and extensions

- Relaxation of the LMI constraints by applying the relaxation scheme from [Tuan et. al., IEEE Tr. Fuzzy Syst., 2001]

$$R_{ijk} < 0 \;\Rightarrow\; \begin{cases} R_{iik} < 0 \\ \frac{2}{r-1} R_{iik} + R_{ijk} + R_{jik} < 0 \end{cases}$$

$$n^2 3^{n_u} \; LMIs \;\Rightarrow\; \frac{n(n+1)3^{n_u}}{2} \; LMIs$$

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$$n^2 3^{n_u} \; LMIs \;\Rightarrow\; \frac{n(n+1)3^{n_u}}{2} \; LMIs$$

- With the descriptor approach, the saturated closed-loop system can be written as

$$\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{x}(t) \\ \dot{u}(t) \end{pmatrix} = \sum_{i=1}^{r} \sum_{j=1}^{3^{n_u}} h_i(z(t))\mu_i(u(t))\left(\begin{pmatrix} A_i & B_i\Lambda_j \\ -K_j & -I \end{pmatrix}\begin{pmatrix} x(t) \\ u(t) \end{pmatrix} + \begin{pmatrix} B_i\Gamma_j \\ 0 \end{pmatrix}\right)$$

$$n^2 3^{n_u} \; LMIs \;\Rightarrow\; n 3^{n_u} \; LMIs$$

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- Relaxation of the LMI constraints by applying the relaxation scheme from [Tuan et. al., IEEE Tr. Fuzzy Syst., 2001]

$$R_{ijk} < 0 \;\Rightarrow\; \begin{cases} R_{iik} < 0 \\ \frac{2}{r-1} R_{iik} + R_{ijk} + R_{jik} < 0 \end{cases}$$

$$n^2 3^{n_u} \; LMIs \;\Rightarrow\; \frac{n(n+1)3^{n_u}}{2} \; LMIs$$

- With the descriptor approach, the saturated closed-loop system can be written as

$$\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x}(t) \\ \dot{u}(t) \end{pmatrix} = \sum_{i=1}^{r} \sum_{j=1}^{3^{n_u}} h_i(z(t)) \mu_i(u(t)) \left(\begin{pmatrix} A_i & B_i \Lambda_j \\ -K_j & -I \end{pmatrix} \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} + \begin{pmatrix} B_i \Gamma_j \\ 0 \end{pmatrix} \right)$$

$$n^2 3^{n_u} \; LMIs \;\Rightarrow\; n 3^{n_u} \; LMIs$$

- The descriptor approach allows to extend these results to static and dynamic output feedback, see [Bezzaoucha et. al., Contribution to the constrained output feedback, ACC 2013]

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- Perspectives
  - state or output tracking control
  - conservatism reduction of the LMI constraints

Stabilization of nonlinear systems subject to actuator saturation

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