Reference Model Tracking Control for Nonlinear Systems Described by Takagi-Sugeno Structure

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Outline of the talk

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Let us consider:

- a TS nonlinear system

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))(A_i x(t) + B_i u(t))$$
$$y(t) = Cx(t)$$

- a reference model

$$\dot{x}_r(t) = A_r x_r(t) + B_r u_r(t)$$

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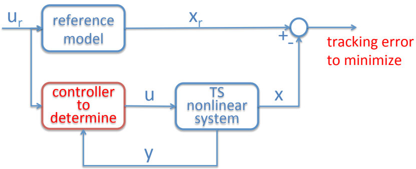
- a reference model

$$\dot{x}_r(t) = A_r x_r(t) + B_r u_r(t)$$

The objective is:

- to find the control law $u(t)$
- such that $x(t)$ is closed to $x_r(t)$



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- The considered Takagi-Sugeno (T-S) system is defined by

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t)) \left(A_i x(t) + B_i u(t) \right)$$

$$y(t) = Cx(t)$$

where – $z(t)$ is the decision variable
– $\mu_i(z(t))$ are the activating functions

Problem statement: TS systems with unmeasurable decision variable

- The considered Takagi-Sugeno (T-S) system is defined by

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))\,(A_i x(t) + B_i u(t))$$
$$y(t) = Cx(t)$$

where – $z(t)$ is the decision variable
– $\mu_i(z(t))$ are the activating functions

- The decision variable depend on the unmeasurable state

$$\mu_i(z(x(t))) = h_i(x(t))$$

- The activating functions $h_i(x(t))$ satisfy the convex sum properties

$$0 \leq h_i(x(t)) \leq 1 \quad \text{and} \quad \sum_{i=1}^{r} h_i(x(t)) = 1$$

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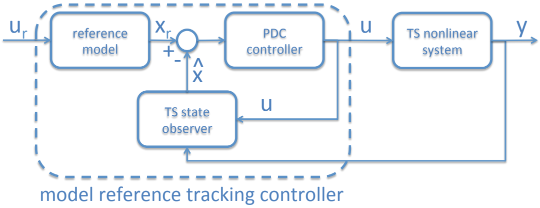
The proposed controller structure:



- PDC controller

$$u(t) = -\sum_{i=1}^{r} h_i(\hat{x}(t))K_i(x_r(t) - \hat{x}(t))$$

- TS state observer

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\hat{x}(t))(A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t)))$$

$$\hat{y}(t) = C\hat{x}(t)$$

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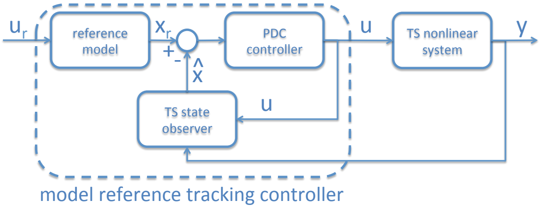

- PDC controller

$$u(t) = -\sum_{i=1}^{r} h_i(\hat{x}(t))K_i(x_r(t) - \hat{x}(t))$$

- TS state observer

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\hat{x}(t))(A_i\hat{x}(t)+B_i u(t)+L_i(y(t)-\hat{y}(t)))$$

$$\hat{y}(t) = C\hat{x}(t)$$

- The gains K_i and L_i are determined to minimize the $\mathcal{L}_2$-gain from $u_r(t)$ to the tracking error $e_r(t) = x(t) - x_r(t)$

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- The main difficulty in the observer design is that
  – the activating functions of the system depend on $x(t)$
  – the activating functions of the observer depend on $\hat{x}(t)$
- The system is then re-written as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\hat{x}(t))(A_i x(t) + B_i u(t)) + (h_i(x(t)) - h_i(\hat{x}(t)))(A_i x(t) + B_i u(t))$$

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- or equivalently as an uncertain TS system

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\hat{x}(t))((A_i + \Delta A(t))x(t) + (B_i + \Delta B(t))u(t))$$

with time varying bounded uncertainties defined, for $X \in \{A, B\}$, by

$$\Delta X(t) = \underbrace{\begin{bmatrix} X_1 & \dots & X_r \end{bmatrix}}_{\mathcal{X}} \underbrace{\begin{bmatrix} h_1(x)-h_1(\hat{x}) & 0 & 0 \\ & \ddots & \\ 0 & 0 & h_r(x)-h_r(\hat{x}) \end{bmatrix}}_{\Sigma_X(t)} \underbrace{\begin{bmatrix} I \\ \dots \\ I \end{bmatrix}}_{E_X}$$

where $\Sigma_X(t)$ satisfies

$$\Sigma_X^T(t)\Sigma_X(t) \leq 1$$

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- Let us denote:
  - the tracking error : $e_r(t) = x(t) - x_r(t)$
  - the estimation error : $e_x(t) = x(t) - \hat{x}(t)$
  - $X_{\hat{h}} = \sum_{i=1}^{r} h_i(\hat{x}(t))$
- The equations of the closed loop system are

$$\dot{e}_r = \underbrace{(A_{\hat{h}} + \Delta A)x + (B_{\hat{h}} + \Delta B)u}_{=\dot{x}} - \underbrace{(A_r x_r + B_r u_r)}_{=\dot{x}_r}$$

$$\dot{e}_x = \underbrace{(A_{\hat{h}} + \Delta A)x + (B_{\hat{h}} + \Delta B)u}_{=\dot{x}} - \underbrace{(A_{\hat{h}}\hat{x} + B_{\hat{h}}u + L_{\hat{h}}(y - \hat{y}))}_{=\dot{\hat{x}}}$$

$$\dot{x}_r = A_r x_r + B_r u_r$$

$$u = -K_{\hat{h}} x_r + K_{\hat{h}} \hat{x}$$

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- The equations of the closed loop system are

$$\dot{e}_r = (A_{\hat{h}} + \Delta A)\underbrace{(e_r + x_r)}_{=x} + (B_{\hat{h}} + \Delta B)u - (A_r x_r + B_r u_r)$$

$$\dot{e}_x = (A_{\hat{h}} + \Delta A)\underbrace{(e_r + x_r)}_{=x} + (B_{\hat{h}} + \Delta B)u - (A_{\hat{h}}\underbrace{(e_r + x_r - e_x)}_{=\hat{x}} + B_{\hat{h}}u + L_{\hat{h}}\underbrace{Ce_x}_{(y - \hat{y})})$$

$$\dot{x}_r = A_r x_r + B_r u_r$$

$$0 = -u - K_{\hat{h}} x_r + K_{=\hat{h}}\underbrace{(e_r + x_r - e_x)}_{=\hat{x}}$$

- 3 dynamic equations and 1 static equation $\rightarrow$ descriptor system

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The closed loop system can be written as a descriptor TS system

$$E\dot{x}_a(t) = \sum_{i=1}^{r} h_i(\hat{x}(t)) \left(\bar{A}_i(t)x_a(t) + \bar{B}u_r(t)\right)$$

with

$$x_a(t) = \begin{bmatrix} e_r(t) \\ e_x(t) \\ x_r(t) \\ u(t) \end{bmatrix} \quad E = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} -B_r \\ 0 \\ B_r \\ 0 \end{bmatrix}$$

$$\bar{A}_i(t) = \begin{bmatrix} A_i + \Delta A(t) & 0 & A_i - A_r + \Delta A(t) & B_i + \Delta B(t) \\ \Delta A(t) & A_i - L_i C & \Delta A(t) & \Delta B(t) \\ 0 & 0 & A_r & 0 \\ K_i & -K_i & 0 & -I_{n_u} \end{bmatrix}$$

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- The closed-loop stability of

$$E\dot{x}_a(t) = \sum_{i=1}^{r} h_i(\hat{x}(t)) \left(\bar{A}_i x_a(t) + \bar{B}_i u_r(t)\right)$$

is studied with a quadratic Lyapunov function

$$V(x_a(t)) = x_a^T(t)E^T P x_a(t), \quad E^T P = P^T E \geq 0$$

with $P = diag(P_1, P_2, P_3, P_4)$

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- The $\mathcal{L}_2$-gain from $u_r(t)$ to $e_r(t)$ is bounded by η if

$$\dot{V}(x_a(t)) + x_a^T(t) Q_a x_a(t) - \eta^2 u_r^T(t) u_r(t) < 0$$

with $Q_a = diag(Q, 0, 0, 0)$.
Or equivalently if

$$\begin{pmatrix} x_a(t) \\ u_r(t) \end{pmatrix}^T \begin{pmatrix} \bar{A}_{\hat{h}}^T(t) P + P^T \bar{A}_{\hat{h}}(t) + Q_a & P^T \bar{B} \\ \bar{B}^T P & -\eta^2 I \end{pmatrix} \begin{pmatrix} x_a(t) \\ u_r(t) \end{pmatrix} < 0$$

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- By bounding the time varying terms, sufficient LMI conditions are derived

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Design of the reference model tracking controller

The $\mathcal{L}_2$-gain from the reference input u_r to the tracking error e_r is bounded by η if there exists symmetric positive definite matrices P_1, P_2, P_3, matrices P_4, F_i and R_i, and positive scalars λ_1^1, λ_3^1, λ_5^1, λ_1^2, λ_3^2 and λ_4^2, minimizing $\bar{\eta} = \eta^2$ under the following LMI constraints, for $i = 1, \ldots, r$

$$\begin{pmatrix}
M_i^1 & * & * & * & * & * & * & * & * & * & * \\
0 & M_i^2 & * & * & * & * & * & * & * & * & * \\
(A_i^T - A_r^T)P_1^T & 0 & M^3 & * & * & * & * & * & * & * & * \\
R_i + B_i^T P_1^T & -R_i & 0 & M^4 & * & * & * & * & * & * & * \\
-B_r^T P_1^T & 0 & B_r^T P_3^T & 0 & -\bar{\eta} I & * & * & * & * & * & * \\
\mathcal{A}^T P_1^T & 0 & 0 & 0 & 0 & -\lambda_1^1 I & * & * & * & * & * \\
\mathcal{A}^T P_1^T & 0 & 0 & 0 & 0 & 0 & -\lambda_3^1 I & * & * & * & * \\
\mathcal{B}^T P_1^T & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_4^1 I & * & * & * \\
0 & \mathcal{A}^T P_2^T & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_1^2 I & * & * \\
0 & \mathcal{A}^T P_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_3^2 I & * \\
0 & \mathcal{B}^T P_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_4^2 I
\end{pmatrix} < 0$$

with

$$M_i^1 = A_i^T P_1 + P_1 A_i + Q + (\lambda_1^1 + \lambda_1^2) E_A^T E_A \qquad M_i^2 = P_2 A_i + A_i^T P_2 - C^T F_i^T - F_i C$$

$$M^3 = A_r^T P_3 + P_3 A_r + (\lambda_3^1 + \lambda_3^2) E_A^T E_A \qquad M^4 = -P_4 - P_4^T + (\lambda_4^1 + \lambda_4^2) E_B^T E_B$$

The observer and controller gains are given by

$$K_i = (P_4^T)^{-1} \quad \text{and} \quad L_i = P_2^{-1} F_i$$

A numerical example

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- Let consider the TS system with $r = 2$ defined by

$$A_1 = \begin{pmatrix} -1 & 1 & 0 \\ -6 & -5 & -1 \\ 3 & 0 & -1 \end{pmatrix} \qquad A_2 = \begin{pmatrix} -1 & 1 & 0 \\ -3 & -5 & -1 \\ -1 & -1 & -2 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 & 0 \\ 0.4 & 0.1 \\ 0 & 0.2 \end{pmatrix} \quad B_2 = \begin{pmatrix} 0 & 0 \\ -0.2 & -1 \\ 1 & 0.5 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

- The activating functions, depending on $x(t)$ are defined by

$$h_1(z(t)) = \frac{2 - \sin(x_1(t)) - \tanh(x_2(t))}{2} \qquad h_2(z(t)) = 1 - h_1(z(t))$$

- The reference model is defined by

$$A_r = \begin{pmatrix} -1 & 1 & 0 \\ -2 & -8 & -1 \\ -1 & -2 & -5 \end{pmatrix} \qquad B_r = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}$$

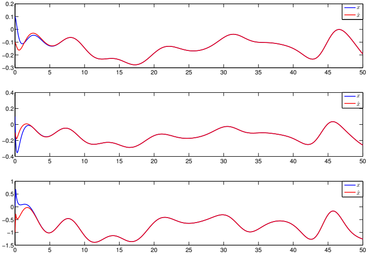
A numerical example

Applying the proposed approach, the obtained gains are:

$$K_1 = \begin{pmatrix} 20.63 & -381.4 & -47.24 \\ 10.08 & -90.82 & -137.3 \end{pmatrix} \quad K_2 = \begin{pmatrix} 13.42 & -320.8 & -859.0 \\ -56.96 & 863.6 & 66.28 \end{pmatrix}$$

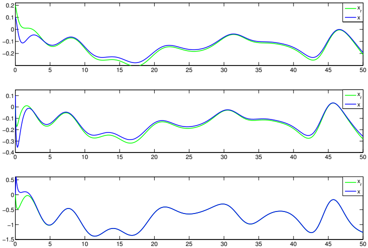
$$L_1^T = \begin{pmatrix} -0.6914 & -3.230 & 3.157 \end{pmatrix} \quad L_2^T = \begin{pmatrix} -0.6954 & -0.1777 & -0.8158 \end{pmatrix}$$

- Estimation result



system state $x(t)$
estimated state $\hat{x}(t)$

- Tracking result



system state $x(t)$
reference state $x_r(t)$

Measurement noise

- If a measurement noise $d(t)$ affects the system output

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(x(t)) \left(A_i x(t) + B_i u(t)\right)$$
$$y(t) = Cx(t) + Gd(t)$$

- the closed loop system becomes

$$E\dot{x}_a(t) = \bar{A}_{\hat{h}} x_a(t) + \underbrace{\begin{pmatrix} -B_r & 0 \\ 0 & -L_{\hat{h}} G \\ B_r & 0 \\ 0 & 0 \end{pmatrix}}_{\bar{\bar{B}}_{\hat{h}}} \underbrace{\begin{pmatrix} u_r(t) \\ d(t) \end{pmatrix}}_{\bar{u}_r(t)}$$

- The $\mathcal{L}_2$-gain from $\bar{u}_r(t)$ to $e_r(t)$ is minimized by finding $V(x_a)$ satisfying

$$\dot{V}(x_a(t)) + x_a^T(t) Q_a x_a(t) - \eta^2 \bar{u}_r^T(t) \bar{u}_r(t) < 0$$

- Solved as previously by LMI optimization

Output tracking

- The objective is to minimize the output tracking error defined by $e_y(t) = y(t) - y_r(t)$, with $y_r(t)$ the output of the reference model

$$\dot{x}_r(t) = A_r x_r(t) + B_r u_r(t)$$
$$y_r(t) = C_r x_r(t)$$

- The output tracking error is generated by

$$E\dot{x}_a(t) = \bar{A}_{\hat{h}} x_a(t) + \bar{B} u_r(t)$$
$$e_y(t) = \underbrace{\begin{pmatrix} C & 0 & (C - C_r) & 0 \end{pmatrix}}_{\bar{C}} x_a(t)$$

- The $\mathcal{L}_2$-gain from $u_r(t)$ to $e_y(t)$ is minimized by finding $V(x_a)$ satisfying

$$\dot{V}(x_a(t)) + x_a^T(t)\bar{C}^T Q_a \bar{C} x_a(t) - \eta^2 u_r^T(t) u_r(t) < 0$$

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Conclusion

- Reference model tracking controller
  - TS state observer
  - PDC controller
- Solution based on
  - descriptor approach
  - LMI formulation
- Easy extension to
  - noise measurement case
  - output tracking control

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Perspectives

- State or output tracking control of nonlinear reference model
- Conservatism reduction of the LMI constraints

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