Contribution to the constrained output feedback

S. Bezzaoucha, B. Marx, D. Maquin, J. Ragot

Research Centre for Automatic Control, Nancy, France
(Centre de Recherche en Automatique de Nancy)
–
Université de Lorraine

Outline of the talk

Bezzaoucha, Marx, Maquin, Ragot

The overall objective is

- the stabilization of a dynamic nonlinear system

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = g(x(t), u(t))$$

Bezzaoucha,
Marx,
Maquin,
Ragot

Problem statement

The overall objective is

- the stabilization of a dynamic nonlinear system

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = g(x(t), u(t))$$

- by output feedback

"static" control — *or* — *dynamic control*

$$u(t) = K(t)y(t)$$

$$\dot{x}_c(t) = A^c x_c(t) + B^c y(t)$$
$$u(t) = C^c x_c(t) + D^c y(t)$$

Problem statement

Bezzaoucha, Marx, Maquin, Ragot

The overall objective is

- the stabilization of a dynamic nonlinear system

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = g(x(t), u(t))$$

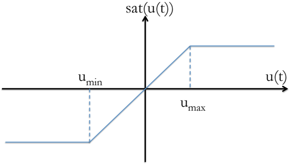
- by output feedback

"static" control or dynamic control

$$u(t) = K(t)y(t)$$

$$\dot{x}_c(t) = A^c x_c(t) + B^c y(t)$$
$$u(t) = C^c x_c(t) + D^c y(t)$$

- despite a saturated input control



$$sat(u(t)) = \begin{cases} u_{max}, & u_{max} \le u(t) \\ u(t), & u_{min} \le u(t) \le u_{max} \\ u_{min}, & u(t) \le u_{min} \end{cases}$$

Bezzaoucha, Marx, Maquin, Ragot

- Any dynamic nonlinear system

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = g(x(t), u(t))$$

with bounded nonlinearities or with $x(t)$ lying in a compact set of $\mathbb{R}^n$

- Any dynamic nonlinear system

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = g(x(t), u(t))$$

with bounded nonlinearities or with $x(t)$ lying in a compact set of $\mathbb{R}^n$

- can be written as a Takagi-Sugeno (T-S) system

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \left(A_i x(t) + B_i u(t)\right)$$

$$y(t) = \sum_{i=1}^{r} h_i(z(t)) \left(C_i x(t) + D_i u(t)\right)$$

where – $z(t)$ is the decision variable
– $h_i(z(t))$ are the activating functions

- Any dynamic nonlinear system

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = g(x(t), u(t))$$

with bounded nonlinearities or with $x(t)$ lying in a compact set of $\mathbb{R}^n$

- can be written as a Takagi-Sugeno (T-S) system

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\left(A_i x(t) + B_i u(t)\right)$$

$$y(t) = \sum_{i=1}^{r} h_i(z(t))\left(C_i x(t) + D_i u(t)\right)$$

where – $z(t)$ is the decision variable
– $h_i(z(t))$ are the activating functions

- The decision variable is assumed to be measurable

Bezzaoucha, Marx, Maquin, Ragot

- Any dynamic nonlinear system

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = g(x(t), u(t))$$

with bounded nonlinearities or with $x(t)$ lying in a compact set of $\mathbb{R}^n$

- can be written as a Takagi-Sugeno (T-S) system

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \left(A_i x(t) + B_i u(t)\right)$$
$$y(t) = \sum_{i=1}^{r} h_i(z(t)) \left(C_i x(t) + D_i u(t)\right)$$

where – $z(t)$ is the decision variable
– $h_i(z(t))$ are the activating functions

- The decision variable is assumed to be measurable
- The activating functions $h_i(z(t))$ satisfy the convex sum properties

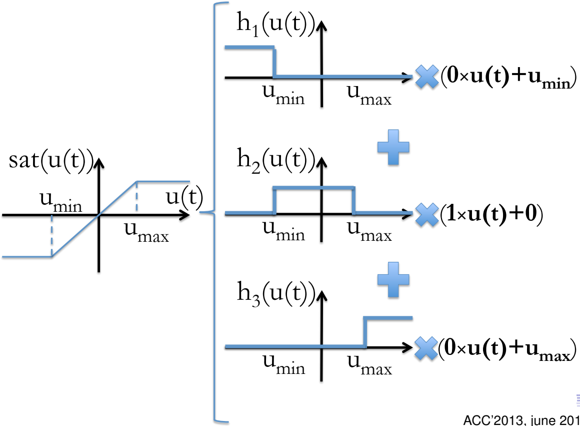
$$0 \leq h_i(z(t)) \leq 1 \quad \text{and} \quad \sum_{i=1}^{r} h_i(z(t)) = 1$$

Bezzaoucha, Marx, Maquin, Ragot

A scalar saturated input

$$sat(u(t)) = \begin{cases} u_{max}, & u_{max} \leq u(t) \\ u(t), & u_{min} \leq u(t) \leq u_{max} \\ u_{min}, & u(t) \leq u_{min} \end{cases}$$

can be put in a T-S (or polytopic) form:



The Takagi-Sugeno modeling of the saturated control

Bezzaoucha, Marx, Maquin, Ragot

A scalar saturated input

$$sat(u(t)) = \begin{cases} u_{max}, & u_{max} \leq u(t) \\ u(t), & u_{min} \leq u(t) \leq u_{max} \\ u_{min}, & u(t) \leq u_{min} \end{cases}$$

can be put in a T-S (or polytopic) form:

$$sat(u(t)) = \sum_{i=1}^{3} h_i(u(t))(\lambda_i u(t) + \gamma_i)$$

with

$$\begin{cases} \lambda_1 = 0 \\ \lambda_2 = 1 \\ \lambda_3 = 0 \end{cases} \quad \begin{cases} \gamma_1 = u_{min} \\ \gamma_2 = 0 \\ \gamma_3 = u_{max} \end{cases} \quad \begin{cases} h_1(u(t)) = \frac{1 - sign(u(t) - u_{min})}{2} \\ h_2(u(t)) = \frac{sign(u(t) - u_{min}) - sign(u(t) - u_{max})}{2} \\ h_3(u(t)) = \frac{1 + sign(u(t) - u_{max})}{2} \end{cases}$$

where the $h_i(u(t))$ functions satisfy the convex sum properties

$$0 \leq h_i(u(t)) \leq 1 \quad and \quad \sum_{i=1}^{3} h_i(u(t)) = 1$$

- The T-S modeling can be generalized to a saturated vector input

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \begin{pmatrix} \sum_{i=1}^{3} h_i^1(u^1(t))(\lambda_i^1 u^1(t) + \gamma_i^1) \\ \sum_{j=1}^{3} h_j^2(u^2(t))(\lambda_j^2 u^2(t) + \gamma_j^2) \end{pmatrix}$$

Bezzaoucha, Marx, Maquin, Ragot

- The T-S modeling can be generalized to a saturated vector input

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \begin{pmatrix} \sum_{i=1}^{3} h_i^1(u^1(t))(\lambda_i^1 u^1(t) + \gamma_i^1) \\ \sum_{j=1}^{3} h_j^2(u^2(t))(\lambda_j^2 u^2(t) + \gamma_j^2) \end{pmatrix}$$

- Since $\sum_i h_i^1 = 1$ and $\sum_j h_j^2 = 1$, then $sat(u(t))$ becomes

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \begin{pmatrix} \sum_{i=1}^{3} h_i^1(u^1(t))(\lambda_i^1 u^1(t) + \gamma_i^1)\left(\sum_{j=1}^{3} h_j^2(u^2(t))\right) \\ \left(\sum_{i=1}^{3} h_i^1(u^1(t))\right) \sum_{j=1}^{3} h_j^2(u^2(t))(\lambda_j^2 u^2(t) + \gamma_j^2) \end{pmatrix}$$

The Takagi-Sugeno modeling of the saturated control

- The T-S modeling can be generalized to a saturated vector input

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \begin{pmatrix} \sum_{i=1}^{3} h_i^1(u^1(t))(\lambda_i^1 u^1(t) + \gamma_i^1) \\ \sum_{j=1}^{3} h_j^2(u^2(t))(\lambda_j^2 u^2(t) + \gamma_j^2) \end{pmatrix}$$

- Since $\sum_i h_i^1 = 1$ and $\sum_j h_j^2 = 1$, then $sat(u(t))$ becomes

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \begin{pmatrix} \sum_{i=1}^{3} h_i^1(u^1(t))(\lambda_i^1 u^1(t) + \gamma_i^1)\left(\sum_{j=1}^{3} h_j^2(u^2(t))\right) \\ \left(\sum_{i=1}^{3} h_i^1(u^1(t))\right) \sum_{j=1}^{3} h_j^2(u^2(t))(\lambda_j^2 u^2(t) + \gamma_j^2) \end{pmatrix}$$

- or equivalently

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \sum_{i=1}^{3}\sum_{j=1}^{3} \underbrace{h_i^1(u^1(t))h_j^2(u^2(t))}_{\mu_i(u(t))} \left(\underbrace{\begin{pmatrix} \lambda_i^1 & 0 \\ 0 & \lambda_j^2 \end{pmatrix}}_{\Lambda_i} u(t) + \underbrace{\begin{pmatrix} \gamma_i^1 \\ \gamma_j^2 \end{pmatrix}}_{\Gamma_i} \right)$$

Bezzaoucha, Marx, Maquin, Ragot

- The T-S modeling can be generalized to a saturated vector input

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \begin{pmatrix} \sum_{i=1}^3 h_i^1(u^1(t))(\lambda_i^1 u^1(t) + \gamma_i^1) \\ \sum_{j=1}^3 h_j^2(u^2(t))(\lambda_j^2 u^2(t) + \gamma_j^2) \end{pmatrix}$$

- Since $\sum_i h_i^1 = 1$ and $\sum_j h_j^2 = 1$, then $sat(u(t))$ becomes

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \begin{pmatrix} \sum_{i=1}^3 h_i^1(u^1(t))(\lambda_i^1 u^1(t) + \gamma_i^1)\left(\sum_{j=1}^3 h_j^2(u^2(t))\right) \\ \left(\sum_{i=1}^3 h_i^1(u^1(t))\right) \sum_{j=1}^3 h_j^2(u^2(t))(\lambda_j^2 u^2(t) + \gamma_j^2) \end{pmatrix}$$

- or equivalently

$$sat\left(\begin{pmatrix} u^1(t) \\ u^2(t) \end{pmatrix}\right) = \sum_{i=1}^3 \sum_{j=1}^3 \underbrace{h_i^1(u^1(t)) h_j^2(u^2(t))}_{\mu_i(u(t))} \left(\underbrace{\begin{pmatrix} \lambda_i^1 & 0 \\ 0 & \lambda_j^2 \end{pmatrix}}_{\Lambda_i} u(t) + \underbrace{\begin{pmatrix} \gamma_i^1 \\ \gamma_j^2 \end{pmatrix}}_{\Gamma_i} \right)$$

- More generally, for $u(t) \in \mathbb{R}^{n_u}$, $sat(u(t))$ can be written as

$$sat(u(t)) = \sum_{i=1}^{3^{n_u}} \mu_i(u(t))(\Lambda_i u(t) + \Gamma_i)$$

Bezzaoucha, Marx, Maquin, Ragot

- Given a saturated nonlinear system

$$\dot{x}(t) = \sum_{j=1}^{r} h_j(z(t))(A_j x(t) + B_j sat(u(t)))$$

$$y(t) = \sum_{j=1}^{r} h_j(z(t))(C_j x(t) + D_j sat(u(t)))$$

- determine the gains K_j of the static output feedback controller

$$u(t) = \sum_{j=1}^{r} h_j(z(t)) K_j y(t)$$

- in order to
  – ensure the closed loop stability
  – despite the input saturation

Bezzaoucha, Marx, Maquin, Ragot

- Without saturation and with $D_i = 0$, the closed loop system is

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \mu_i(z(t))\mu_j(z(t))\mu_k(z(t))(A_i + B_i K_j C_k)x(t)$$

- Without saturation and with $D_i = 0$, the closed loop system is

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\mu_i(z(t))\mu_j(z(t))\mu_k(z(t))(A_i + B_iK_jC_k)x(t)$$

- In order to avoid
  - triple summations ($i, j, k = 1, \ldots, n$)
  - coupled terms in the Lyapunov inequalities (i.e. $PB_iK_jC_k$)

the closed loop system is put in the descriptor form

$$\begin{pmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{x}(t) \\ \dot{u}(t) \\ \dot{y}(t) \end{pmatrix} = \sum_{j=1}^{r} h_j(z(t))\begin{pmatrix} A_j & B_j & 0 \\ 0 & -I & K_j \\ C_j & D_j & -I \end{pmatrix}\begin{pmatrix} x(t) \\ u(t) \\ y(t) \end{pmatrix}$$

Static output feedback (descriptor approach)

- Without saturation and with $D_i = 0$, the closed loop system is

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r} \mu_i(z(t))\mu_j(z(t))\mu_k(z(t))(A_i + B_i K_j C_k)x(t)$$

- In order to avoid
  - triple summations ($i, j, k = 1, \ldots, n$)
  - coupled terms in the Lyapunov inequalities (i.e. $PB_iK_jC_k$)

the closed loop system is put in the descriptor form

$$\begin{pmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x}(t) \\ \dot{u}(t) \\ \dot{y}(t) \end{pmatrix} = \sum_{j=1}^{r} h_j(z(t)) \begin{pmatrix} A_j & B_j & 0 \\ 0 & -I & K_j \\ C_j & D_j & -I \end{pmatrix} \begin{pmatrix} x(t) \\ u(t) \\ y(t) \end{pmatrix}$$

- Because of the input saturation, the closed loop system is

$$\underbrace{\begin{pmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{E} \underbrace{\begin{pmatrix} \dot{x}(t) \\ \dot{u}(t) \\ \dot{y}(t) \end{pmatrix}}_{\dot{x}_a(t)} = \sum_{j=1}^{r}\sum_{i=1}^{3^{n_u}} h_j(z)\mu_i(u) \left(\underbrace{\begin{pmatrix} A_j & B_j\Lambda_i & 0 \\ 0 & -I & K_j \\ C_j & D_j\Lambda_i & -I \end{pmatrix}}_{A_{ij}} \underbrace{\begin{pmatrix} x(t) \\ u(t) \\ y(t) \end{pmatrix}}_{x_a(t)} + \underbrace{\begin{pmatrix} B_j\Gamma_i \\ 0 \\ D_j\Gamma_i \end{pmatrix}}_{B_{ij}} \right)$$

Bezzaoucha, Marx, Maquin, Ragot

- The closed-loop stability of

$$E\dot{x}_a(t) = \sum_{j=1}^{r} \sum_{i=1}^{3^{nu}} h_j(z(t))\mu_i(u(t))(A_{ij}x_a(t) + B_{ij}u(t))$$

is studied with a quadratic Lyapunov function

$$V(x_a(t)) = x_a^T(t)E^T P x_a(t), \quad E^T P = P^T E \geq 0$$

- The closed-loop stability of

$$E\dot{x}_a(t) = \sum_{j=1}^{r}\sum_{i=1}^{3^{n_u}} h_j(z(t))\mu_i(u(t))(A_{ij}x_a(t) + B_{ij}u(t))$$

is studied with a quadratic Lyapunov function

$$V(x_a(t)) = x_a^T(t)E^T P x_a(t), \quad E^T P = P^T E \geq 0$$

- It can be shown that:

$$\frac{dV(x_a(t))}{dt} \leq \sum_{i=1}^{3^{n_u}}\sum_{j=1}^{r} \mu_i(u(t))h_j(z(t))\left(x_a^T(t)Q_{ij}x_a(t) + R_{ij}\right)$$

with Q_{ij} and R_{ij} linearly depending on P, K_j and other slack variables.

- The closed-loop stability of

$$E\dot{x}_a(t) = \sum_{j=1}^{r} \sum_{i=1}^{3^{n_u}} h_j(z(t))\mu_i(u(t))(A_{ij}x_a(t) + B_{ij}u(t))$$

is studied with a quadratic Lyapunov function

$$V(x_a(t)) = x_a^T(t)E^T P x_a(t), \quad E^T P = P^T E \geq 0$$

- It can be shown that:

$$\frac{dV(x_a(t))}{dt} \leq \sum_{i=1}^{3^{n_u}} \sum_{j=1}^{r} \mu_i(u(t))h_j(z(t)) \left(x_a^T(t)Q_{ij}x_a(t) + R_{ij} \right)$$

with Q_{ij} and R_{ij} linearly depending on P, K_j and other slack variables.

- Sufficient LMI convergence conditions into a ball are derived:

$$\begin{cases} Q_{ij} < 0 \\ \varepsilon = \min_{i,j}(\underline{\lambda}(-Q_{ij})) \\ \delta = \max_{i,j} R_{i,j} \end{cases} \Rightarrow \begin{cases} \frac{dV(x_a(t))}{dt} < 0 \\ \forall ||x_a(t)|| \geq \sqrt{\frac{\delta}{\varepsilon}} \end{cases} \Rightarrow x_a(t) \to \mathcal{B}\left(0, \sqrt{\frac{\delta}{\varepsilon}}\right)$$

Bezzaoucha, Marx, Maquin, Ragot

There exists a static output feedback controller for a saturated input system such that the system state converges toward an origin-centered ball of radius bounded by β if there exists matrices $P_1 = P_1^T > 0$, $P_2 > 0$, P_3, R_j, $\Sigma_{ij}^1 = (\Sigma_{ij}^1)^T > 0$, $\Sigma_{ij}^3 = (\Sigma_{ij}^3)^T > 0$, solutions of the following minimization problem under LMI constraints (for $i = 1, \ldots, 3^{n_u}$ and $j = 1, \ldots, r$)

$$\min_{P_1,\, P_2,\, P_3,\, R_j, \Sigma_{ij}^1, \Sigma_{ij}^3, \beta} \beta$$

$$\left(\begin{array}{ccc|cc|ccc} A_j^T P_1 + P_1 A_j & P_1 B_j \Lambda_i & C_j^T P_3 & P_1 & 0 & I & 0 & 0 \\ * & -P_2 - P_2^T & R_j + \Lambda_i D_j^T P_3 & 0 & 0 & 0 & I & 0 \\ * & * & -P_3 - P_3^T & 0 & P_3 & 0 & 0 & I \\ \hline * & * & * & -\Sigma_{ij}^1 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\Sigma_{ij}^3 & 0 & 0 & 0 \\ \hline * & * & * & * & * & -\beta I & 0 & 0 \\ * & * & * & * & * & * & -\beta I & 0 \\ * & * & * & * & * & * & * & -\beta I \end{array}\right) < 0$$

$$\Gamma_i^T B_j^T \Sigma_{ij}^1 B_j \Gamma_i + \Gamma_i^T D_j^T \Sigma_{ij}^3 D_j \Gamma_i < \beta$$

The gains of the controller are given by

$$K_j = (P_2^T)^{-1} R_j$$

Saturated static output feedback

$$\textit{system} : \begin{cases} \dot{x}(t) = \sum_i h_i(z(t))(A_i x(t) + B_i sat(u(t))) \\ y(t) = \sum_i h_i(z(t))(C_i x(t) + D_i sat(u(t))) \end{cases}$$

+

$$\textit{controller} : u(t) = \sum_i h_i(z(t)) K_i y(t)$$

Closed loop system in the descriptor form:

$$\begin{pmatrix} I & 0 & 0 \\ I & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x}(t) \\ \dot{u}(t) \\ \dot{y}(t) \end{pmatrix} = \sum_{j=1}^{r} \sum_{i=1}^{3^{n_u}} h_j(z)\mu_i(u) \left(\begin{pmatrix} A_j & B_j\Lambda_i & 0 \\ 0 & -I & K_j \\ C_j & D_j\Lambda_i & -I \end{pmatrix} \begin{pmatrix} x(t) \\ u(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} B_j\Gamma_i \\ 0 \\ D_j\Gamma_i \end{pmatrix} \right)$$

$$\Rightarrow E\dot{x}_a(t) = \Sigma_i \Sigma_j h_j(z(t))\mu_i(u(t))(A_{ij} x_a(t) + B_{ij})$$

Bezzaoucha, Marx, Maquin, Ragot

Saturated dynamic output feedback

$$system : \begin{cases} \dot{x}(t) = \sum_i h_i(z(t))(A_i x(t) + B_i sat(u(t))) \\ y(t) = \sum_i h_i(z(t))(C_i x(t) + D_i sat(u(t))) \end{cases}$$

+

$$controller : \begin{cases} \dot{x}_c(t) = \sum_i h_i(z(t))(A_i^c x_c(t) + B_i^c y(t)) \\ u(t) = \sum_i h_i(z(t))(C_i^c x_c(t) + D_i^c y(t)) \end{cases}$$

Closed loop system in the descriptor form:

$$\begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x}(t) \\ \dot{x}_c(t) \\ \dot{u}(t) \\ \dot{y}(t) \end{pmatrix} = \sum_{j=1}^{r} \sum_{i=1}^{3^{n_u}} h_j \mu_i \left(\begin{pmatrix} A_j & 0 & B_j \Lambda_i & 0 \\ 0 & A_j^c & 0 & B_j^c \\ 0 & C_j^c & -I & D_j^c \\ C_j & 0 & D_j \Lambda_i & -I \end{pmatrix} \begin{pmatrix} x(t) \\ x_c(t) \\ u(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} B_j \Gamma_i \\ 0 \\ 0 \\ D_j \Gamma_i \end{pmatrix} \right)$$

$$\Rightarrow E \dot{x}_a(t) = \Sigma_i \Sigma_j h_j(z(t)) \mu_i(u(t))(A_{ij} x_a(t) + B_{ij})$$

Bezzaoucha, Marx, Maquin, Ragot

Saturated dynamic output feedback

$$system : \begin{cases} \dot{x}(t) = \sum_i h_i(z(t))(A_i x(t) + B_i sat(u(t))) \\ y(t) = \sum_i h_i(z(t))(C_i x(t) + D_i sat(u(t))) \end{cases}$$

+

$$controller : \begin{cases} \dot{x}_c(t) = \sum_i h_i(z(t))(A_i^c x_c(t) + B_i^c y(t)) \\ u(t) = \sum_i h_i(z(t))(C_i^c x_c(t) + D_i^c y(t)) \end{cases}$$

Closed loop system in the descriptor form:

$$\begin{pmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x}(t) \\ \dot{x}_c(t) \\ \dot{u}(t) \\ \dot{y}(t) \end{pmatrix} = \sum_{j=1}^{r} \sum_{i=1}^{3^{n_u}} h_j \mu_i \left(\begin{pmatrix} A_j & 0 & B_j \Lambda_i & 0 \\ 0 & A_j^c & 0 & B_j^c \\ 0 & C_j^c & -I & D_j^c \\ C_j & 0 & D_j \Lambda_i & -I \end{pmatrix} \begin{pmatrix} x(t) \\ x_c(t) \\ u(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} B_j \Gamma_i \\ 0 \\ 0 \\ D_j \Gamma_i \end{pmatrix} \right)$$

$$\Rightarrow E\dot{x}_a(t) = \Sigma_i \Sigma_j h_j(z(t)) \mu_i(u(t))(A_{ij} x_a(t) + B_{ij})$$

The dynamic case is tackled as the static case

$\Rightarrow$ Lyapunov function

$\Rightarrow$ LMI conditions for state convergence in an origin centered ball

Bezzaoucha, Marx, Maquin, Ragot

- Let consider the saturated nonlinear system

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(z(t))(A_i x(t) + B_i sat(u(t)))$$

$$y(t) = \sum_{i=1}^{2} h_i(z(t))(C_i x(t) + D_i sat(u(t)))$$

Bezzaoucha, Marx, Maquin, Ragot

A numerical example

- Let consider the saturated nonlinear system

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(z(t))(A_i x(t) + B_i sat(u(t)))$$

$$y(t) = \sum_{i=1}^{2} h_i(z(t))(C_i x(t) + D_i sat(u(t)))$$

- with $z(t) = y(t)$ the measurable decision variable and

$$h_1(z(t)) = \frac{1 - tanh(y_1(t) + y_2(t))}{2} \qquad h_2(z(t)) = 1 - h_1(z(t))$$

$$A_1 = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{pmatrix} \qquad A_2 = \begin{pmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix} \quad B_2 = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \quad C_1 = C_2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad D_1 = D_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Bezzaoucha, Marx, Maquin, Ragot

- Let consider the saturated nonlinear system

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(z(t))(A_i x(t) + B_i sat(u(t)))$$

$$y(t) = \sum_{i=1}^{2} h_i(z(t))(C_i x(t) + D_i sat(u(t)))$$

- with $z(t) = y(t)$ the measurable decision variable and

$$h_1(z(t)) = \frac{1 - tanh(y_1(t) + y_2(t))}{2} \qquad h_2(z(t)) = 1 - h_1(z(t))$$

$$A_1 = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -3 & 0 \\ 2 & 1 & -8 \end{pmatrix} \qquad A_2 = \begin{pmatrix} -3 & 2 & -2 \\ 5 & -3 & 0 \\ 1 & 2 & -4 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix} \quad B_2 = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \quad C_1 = C_2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad D_1 = D_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- and the input saturation:

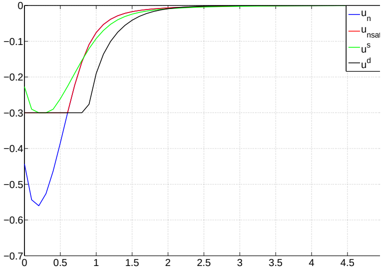
$$-0.3 \leq sat(u(t)) \leq 0.3$$

Bezzaoucha,
Marx,
Maquin,
Ragot

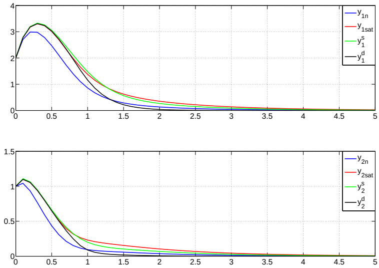
A numerical example

Let us compare, four different cases:

- nominal control of the unsaturated system
- nominal control of the saturated system
- proposed static control of the saturated system
- proposed dynamic control of the saturated system



control input



system outputs

Bezzaoucha, Marx, Maquin, Ragot

- Unified T-S representation of
  - – the nonlinear system
  - – the input saturation

Bezzaoucha, Marx, Maquin, Ragot

- Unified T-S representation of
  - the nonlinear system
  - the input saturation

- LMI formulation of the saturated output feedback control for nonlinear systems

- Unified T-S representation of
  - the nonlinear system
  - the input saturation
- LMI formulation of the saturated output feedback control for nonlinear systems
- Unified solution for both
  - static output control
  - dynamic output control of arbitrary order

- Unified T-S representation of
  – the nonlinear system
  – the input saturation
- LMI formulation of the saturated output feedback control for nonlinear systems
- Unified solution for both
  – static output control
  – dynamic output control of arbitrary order
- Perspectives
  – state or output tracking control
  – conservatism reduction of the LMI constraints

Contribution to the constrained output feedback

S. Bezzaoucha, B. Marx, D. Maquin, J. Ragot

Research Centre for Automatic Control, Nancy, France
(Centre de Recherche en Automatique de Nancy)
–
Université de Lorraine