State estimation for nonlinear system diagnosis using multiple models. Application to wastewater treatment plants

A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

Centre de Recherche en Automatique de Nancy
UMR 7039 CNRS – Nancy-Université
France
Objectives and context

Objectives

1. Fault diagnosis for complex nonlinear systems
2. Application to the model of an activated sludge bioreactor of a Waste Water Treatment Plant
3. Need for state estimation of environmental plant with limited sensors
Objectives and context

Objectives

1. Fault diagnosis for complex nonlinear systems
2. Application to the model of an activated sludge bioreactor of a Waste Water Treatment Plant
3. Need for state estimation of environmental plant with limited sensors

Context and tools

1. *modeling complexity* of nonlinear process
   → Multiple Model approach
   → Model uncertainties
2. *corrupted measurement* of the input or output
   → observer based diagnosis
3. *limited number of sensors*
   → unknown input observer design
Outline of the presentation

- Process description
- Multiple model approach
- Proportional Integral Unknown input Observer design
- Observer based diagnosis
- Conclusions and future works
Presentation of the wastewater treatment process

The **wastewater treatment plant (WWTP)** is composed by:

- a bioreactor
  - polluted water and bacteria are put in contact and aerated
  - the bacterial biomass degrades the organic pollution
- a clarifier:
  - clear water and bacterial biomass are separated
  - clear water is rejected in the environment
  - a fraction of the biomass is recycled

**The presented study only concerns the bioreactor**

- the bioreactor is modelled with the Activated Sludge Model (ASM1)
- the clarifier is supposed to be ideal
Nonlinear dynamic system equations:

\[ \dot{X}_{DCO}(t) = -\frac{1}{Y_H}(\varphi_1(t) + \varphi_2(t)) + (1 - f_P)(\varphi_4(t) + \varphi_5(t)) + D_1(t) \]

\[ \dot{S}_O(t) = \frac{Y_H - 1}{Y_H} \varphi_1(t) + \frac{Y_A - 4.57}{Y_A} \varphi_3(t) + D_2(t) \]

\[ \dot{S}_{NH}(t) = -i_{XB}(\varphi_1(t) + \varphi_2(t)) - \left(\frac{1}{Y_A}\right)\varphi_3(t) + (i_{XB} - f_Pi_{XP})(\varphi_4(t) + \varphi_5(t)) + D_3(t) \]

\[ \dot{S}_{NO}(t) = \frac{Y_H - 1}{2.86Y_H} \varphi_2(t) + \frac{1}{Y_A} \varphi_3(t) + D_4(t) \]

\[ \dot{X}_{BH}(t) = \varphi_1(t) + \varphi_2(t) - \varphi_4(t) + D_5(t) \]

\[ \dot{X}_{BA}(t) = \varphi_3(t) - \varphi_5(t) + D_6(t) \]

The state variables denote the following concentrations:

- \( X_{DCO}(t) \): demand in chemical oxygen
- \( S_O(t) \): dissolved oxygen
- \( S_{NH}(t) \): dissolved ammonia
- \( S_{NO}(t) \): dissolved nitrate
- \( X_{BH}(t) \): heterotrophic biomass
- \( X_{BA}(t) \): autotrophic biomass
Biorector description: the Activated Sludge Model (AMS1)

Nonlinear dynamic system equations:

\[
\dot{X}_{DCO}(t) = -\frac{1}{Y_H}(\varphi_1(t) + \varphi_2(t)) + (1 - f_P)(\varphi_4(t) + \varphi_5(t)) + D_1(t)
\]

\[
\dot{S}_O(t) = \frac{Y_H - 1}{Y_H} \varphi_1(t) + \frac{Y_A - 4.57}{Y_A} \varphi_3(t) + D_2(t)
\]

\[
\dot{S}_{NH}(t) = -i_{XB}(\varphi_1(t) + \varphi_2(t)) - (i_{XB} + \frac{1}{Y_A}) \varphi_3(t) + (i_{XB} - f_Pi_XP)(\varphi_4(t) + \varphi_5(t)) + D_3(t)
\]

\[
\dot{S}_{NO}(t) = \frac{Y_H - 1}{2.86Y_H} \varphi_2(t) + \frac{1}{Y_A} \varphi_3(t) + D_4(t)
\]

\[
\dot{X}_{BH}(t) = \varphi_1(t) + \varphi_2(t) - \varphi_4(t) + D_5(t)
\]

\[
\dot{X}_{BA}(t) = \varphi_3(t) - \varphi_5(t) + D_6(t)
\]

with the process kinetics:

\[
\varphi_1(t) = \mu_H \frac{X_{DCO}(t)}{K_{DCO} + X_{DCO}(t)} \frac{S_O(t)}{K_{OH} + S_O(t)} X_{BH}(t)
\]

\[
\varphi_2(t) = \mu_H \eta_{NOg} \frac{X_{DCO}(t)}{K_{DCO} + X_{DCO}(t)} \frac{S_{NO}(t)}{K_{NO} + S_{NO}(t)} \frac{K_{OH}}{K_{OH} + S_O(t)} X_{BH}(t)
\]

\[\vdots = \vdots\]

\[
\varphi_5(t) = b_A X_{BA}(t)
\]
Nonlinear dynamic system equations:

\[ \dot{X}_{DCO}(t) = -\frac{1}{Y_H}(\varphi_1(t) + \varphi_2(t)) + (1 - f_P)(\varphi_4(t) + \varphi_5(t)) + D_1(t) \]

\[ \dot{S}_O(t) = \frac{Y_H - 1}{Y_H} \varphi_1(t) + \frac{Y_A - 4.57}{Y_A} \varphi_3(t) + D_2(t) \]

\[ \dot{S}_{NH}(t) = -i_{XB}(\varphi_1(t) + \varphi_2(t)) - (i_{XB} + \frac{1}{Y_A})\varphi_3(t) + (i_{XB} - f_P i_{XP})(\varphi_4(t) + \varphi_5(t)) + D_3(t) \]

\[ \dot{S}_{NO}(t) = \frac{Y_H - 1}{2.86Y_H} \varphi_2(t) + \frac{1}{Y_A} \varphi_3(t) + D_4(t) \]

\[ \dot{X}_{BH}(t) = \varphi_1(t) + \varphi_2(t) - \varphi_4(t) + D_5(t) \]

\[ \dot{X}_{BA}(t) = \varphi_3(t) - \varphi_5(t) + D_6(t) \]

and the input/output balances:

\[ D_1(t) = \frac{q_{in}(t)}{V}(X_{DCO,in}(t) - X_{DCO}(t)) \]

\[ D_2(t) = -\frac{q_{in}(t)}{V} S_O(t) + K_{qa}(t)(S_{O,sat}(t) - S_O(t)) \]

\[ \vdots = \vdots \]

\[ D_6(t) = -\frac{q_{in}(t)}{V} \frac{f_W(1 + f_R)}{f_W + f_R} X_{BA}(t) \]
Any nonlinear system can be equivalently written as a Multiple Model (MM) on a compact set of the state space

$$\begin{align*}
\dot{x} &= f(x, u) \\
y &= g(x, u) \\
\text{nonlinear} \\
\Rightarrow \\
\dot{x} &= A(x, u)x + B(x, u)u \\
y &= C(x, u)x + D(x, u)u
\end{align*}$$

\text{Quasi – LPV}

$$\begin{align*}
\dot{x} &= \sum_{i=1}^{r} \mu_i(x, u)(A_i x + B_i u) \\
y &= \sum_{i=1}^{r} \mu_i(x, u)(C_i x + D_i u)
\end{align*}$$

Multiple Model

\rightarrow MM with unmeasurable premise variable are generally obtained

**Multiple model approach for nonlinear system**

- **Any nonlinear system** can be equivalently written as a **Multiple Model** (MM) on a compact set of the state space

\[
\begin{align*}
\dot{x} &= f(x, u) \\
y &= g(x, u)
\end{align*}
\]

\[
\begin{align*}
\dot{x} &= A(x, u)x + B(x, u)u \\
y &= C(x, u)x + D(x, u)u
\end{align*}
\]

\[
\begin{align*}
\dot{x} &= \sum_{i=1}^{r} \mu_i(x, u)(A_ix + B_iu) \\
y &= \sum_{i=1}^{r} \mu_i(x, u)(C_ix + D_iu)
\end{align*}
\]

- MM with unmeasurable premise variable are generally obtained

- The nonlinearities are rejected in the activating functions \(\mu_i(x(t), u(t))\)

- System analysis and design are based on the linear submodels \((A_i, B_i, C_i, D_i)\) with classical tools (Lyapunov functions, LMI, . . .)

- Different equivalent rewritings may not lead to the same results in terms of controller/observer design \(^1\)

---

Summing up, the bioreactor is described by a MM with 16 submodels:

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x, u) [A_i x(t) + B_i u(t)] \\
y(t) = C x(t)
\]

- state vector and measured output:

\[
x^T(t) = [X_{DCO}(t) \quad S_O(t) \quad S_{NH}(t) \quad S_{NO}(t) \quad X_{BH}(t) \quad X_{BA}(t)] \\
y^T(t) = [X_{DCO}(t) \quad S_O(t) \quad S_{NH}(t) \quad S_{NO}(t)]
\]
Multiple Model of the Activated Sludge Model (AMS1)

Summing up, the bioreactor is described by a MM with 16 submodels:

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x, u) [A_i x(t) + B_i u(t) + E_i d(t)]
\]
\[
y(t) = Cx(t) + Gd(t)
\]

- state vector and measured output:
  \[
x^T(t) = [X_{DCO}(t) \ S_O(t) \ S_{NH}(t) \ S_{NO}(t) \ X_{BH}(t) \ X_{BA}(t)]
\]
  \[
y^T(t) = [X_{DCO}(t) \ S_O(t) \ S_{NH}(t) \ S_{NO}(t)]
\]

- some incoming pollutants cannot be measured
  → **unknown inputs** \(d(t)\):
  \[
d(t)^T = [S_{NH,in}(t) \ X_{BH,in}(t)]
\]
Multiple Model of the Activated Sludge Model (AMS1)

Summing up, the bioreactor is described by a MM with 16 submodels:

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x, u) [(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + E_i d(t)]
\]

\[
y(t) = Cx(t) + Gd(t)
\]

- state vector and measured output:

\[
x^T(t) = [X_{DCO}(t) \ S_O(t) \ S_{NH}(t) \ S_{NO}(t) \ X_{BH}(t) \ X_{BA}(t)]
\]

\[
y^T(t) = [X_{DCO}(t) \ S_O(t) \ S_{NH}(t) \ S_{NO}(t)]
\]

- some incoming pollutants cannot be measured
  → **unknown inputs** \(d(t)\):

\[
d(t)^T = [S_{NH,in}(t) \ X_{BH,in}(t)]
\]

- some model parameters are time varying and not perfectly known
  → **model uncertainties** \(\Delta A_i(t)\) and \(\Delta B_i(t)\):

\[
b_A(t) = b_A + \Delta b_A(t) \text{ and } b_H(t) = b_H + \Delta b_H(t)
\]
The state and unknown input of the uncertain MM:

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x, u) [(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + E_i d(t)]
\]

\[
y(t) = Cx(t) + Gd(t)
\]
The state and unknown input of the uncertain MM:

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x, u) [(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + E_i d(t)]
\]

\[
y(t) = Cx(t) + Gd(t)
\]

are reconstructed with the following observer:

\[
\dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}, u) \left( A_i \dot{\hat{x}}(t) + B_i u(t) + E_i \dot{\hat{d}}(t) + L_i^P (y(t) - \hat{y}(t)) \right)
\]

\[
\dot{\hat{d}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}, u) L_i^I(y(t) - \hat{y}(t))
\]

\[
\hat{y}(t) = C\hat{x}(t) + G\hat{d}(t)
\]

Objective:
find the observer gains: \(L_i^P\) and \(L_i^I\) minimizing the \(L_2\)-gain from the inputs \([u(t)\; d(t)]\) to the estimation errors \([e_x(t)\; e_d(t)] = [x(t) - \hat{x}(t)\; d(t) - \hat{d}(t)]\)
The state and unknown input of the uncertain MM:

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x, u) [(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + E_i d(t)]
\]

\[
y(t) = Cx(t) + Gd(t)
\]

are reconstructed with the following observer:

\[
\dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}, u) (A_i \hat{x}(t) + B_i u(t) + E_i \hat{d}(t) + L_i^P(y(t) - \hat{y}(t)))
\]

\[
\hat{d}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}, u) L_i^I(y(t) - \hat{y}(t))
\]

\[
\hat{y}(t) = C\hat{x}(t) + G\hat{d}(t)
\]

**Assumptions:**
- constant unknown input: \(\dot{d}(t) = 0\)
- bounded uncertainties: \(\Delta A_i(t) = M_i^a F_a(t) N_i^a\), with \(F_a^T(t) F_a(t) \leq I\)
- bounded uncertainties: \(\Delta A_i(t) = M_i^a F_a(t) N_i^a\), with \(F_a^T(t) F_a(t) \leq I\)
The state and unknown input of the uncertain MM:

\[ \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x, u) [(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + E_i d(t)] \]

\[ y(t) = Cx(t) + Gd(t) \]

are reconstructed with the following observer:

\[ \dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}, u) \left( A_i \hat{x}(t) + B_i u(t) + E_i \hat{d}(t) + L_i^p(y(t) - \hat{y}(t)) \right) \]

\[ \dot{\hat{d}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}, u) L_i^l(y(t) - \hat{y}(t)) \]

\[ \hat{y}(t) = C\hat{x}(t) + G\hat{d}(t) \]

**Difficulties:**

the activating functions of the system depend on \( x(t) \), while these of the observer depend on \( \hat{x}(t) \)

the presence of unknown inputs
Find a positive definite Lyapunov function $V(e_x, e_d, x)$ such that:

$$\dot{V}(e_x, e_d, x) - \gamma^2 \begin{bmatrix} u(t) \\ d(t) \end{bmatrix}^T \begin{bmatrix} u(t) \\ d(t) \end{bmatrix} + \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} < 0 \quad (1)$$
Find a positive definite Lyapunov function $V(e_x, e_d, x)$ such that:

$$\dot{V}(e_x, e_d, x) - \gamma^2 \begin{bmatrix} u(t) \\ d(t) \end{bmatrix}^T \begin{bmatrix} u(t) \\ d(t) \end{bmatrix} + \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} < 0 \quad (1)$$

The proposed quadratic Lyapunov function is:

$$V(e_x, e_d, x) = \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T P_1 \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} + x^T(t)P_2 x(t)$$

with $P_1 = P_1^T > 0$ and $P_2 = P_2^T > 0$
Find a positive definite Lyapunov function $V(e_x, e_d, x)$ such that:

$$\dot{V}(e_x, e_d, x) - \gamma^2 \begin{bmatrix} u(t) \\ d(t) \end{bmatrix}^T \begin{bmatrix} u(t) \\ d(t) \end{bmatrix} + \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} < 0 \quad (1)$$

The proposed quadratic Lyapunov function is:

$$V(e_x, e_d, x) = \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T P_1 \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} + x^T(t) P_2 x(t)$$

with $P_1 = P_1^T > 0$ and $P_2 = P_2^T > 0$

LMI optimization allows to find

- the Lyapunov matrices $P_1, P_2$
- the observer gains $L^P_i$ and $L^I_i$
- that minimize the $\mathcal{L}_2$-gain $\gamma$ under (1)
Find a positive definite Lyapunov function $V(e_x, e_d, x)$ such that:

$$\dot{V}(e_x, e_d, x) - \gamma^2 \begin{bmatrix} u(t) \\ d(t) \end{bmatrix}^T \begin{bmatrix} u(t) \\ d(t) \end{bmatrix} + \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} < 0 \quad (1)$$

The proposed quadratic Lyapunov function is:

$$V(e_x, e_d, x) = \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T P_1 \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} + x^T(t) P_2 x(t)$$

with $P_1 = P_1^T > 0$ and $P_2 = P_2^T > 0$

LMI optimization allows to find
- the Lyapunov matrices $P_1, P_2$
- the observer gains $L_P^i$ and $L_I^i$
- that minimize the $\mathcal{L}_2$-gain $\gamma$ under (1)

sufficient LMI conditions are obtained
Proportional Integral Unknown input Observer design

Find matrices $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $\overline{P}_j$, and scalars $\varepsilon_{1i} > 0$ and $\varepsilon_{2i}$ minimizing the scalar $\overline{\gamma} > 0$ under the following LMI constraints

$$\begin{bmatrix}
\Phi_{ij}^{11} & \Phi_{ij}^{12} & 0 & \Phi_{ij}^{14} & P_1 \overline{M}_i^a & P_1 \overline{M}_i^b \\
* & \Phi_{ij}^{22} & P_2 B_i & P_2 E_i & P_2 M_i^a & P_2 M_i^b \\
* & * & \Phi_{ij}^{33} & 0 & 0 & 0 \\
* & * & * & -\overline{\gamma} I & 0 & 0 \\
* & * & * & * & -\varepsilon_{1i} I & 0 \\
* & * & * & * & * & -\varepsilon_{2i} I \\
\end{bmatrix} < 0, \quad i, j = 1, \ldots, r$$

with

$$\Phi_{ij}^{11} = I + \mathbb{S}(P_1 \overline{A}_j - \overline{P}_j \overline{C}), \quad \Phi_{ij}^{12} = P_1 (\overline{A}_i - \overline{A}_j), \quad \Phi_{ij}^{14} = P_1 (\overline{E}_i - \overline{E}_j),$$

$$\Phi_{ij}^{22} = \varepsilon_{1i} N_i^{aT} N_i^a + \mathbb{S}(P_2 A_i), \quad \Phi_{ij}^{33} = \varepsilon_{2i} N_i^{bT} N_i^b - \overline{\gamma} I$$

$$\overline{C}^T = \begin{bmatrix} C^T \\ G^T \end{bmatrix}, \quad \overline{A}_i = \begin{bmatrix} A_i & E_i \\ 0 & 0 \end{bmatrix}, \quad \tilde{A}_i = \begin{bmatrix} A_i \\ 0 \end{bmatrix}, \quad \tilde{E}_i = \begin{bmatrix} E_i \\ 0 \end{bmatrix}, \quad \overline{M}_i^a = \begin{bmatrix} M_i^a \\ 0 \end{bmatrix}, \quad \overline{M}_i^b = \begin{bmatrix} M_i^b \\ 0 \end{bmatrix}$$

The observer gains $L_j^P$ and $L_j^I$, and the $\mathcal{L}_2$-gain $\gamma$ are obtained by:

$$\begin{bmatrix} L_j^P \\ L_j^I \end{bmatrix} = P_1^{-1} \overline{P}_j \gamma = \sqrt{\overline{\gamma}}$$
State and unknown input estimation of the ASM1

The proposed observer for MM with unmeasurable premise variables affected by time varying uncertainties and unknown inputs is designed.

The time varying uncertain parameters:

\[ b_d \]

\[ b_A \]

![Graph showing time varying uncertain parameters](image-url)
The proposed observer for MM with unmeasurable premise variables affected by time varying uncertainties and unknown inputs is designed.

- The unknown inputs and their estimates

![Graph showing NH3 and XrH.](image-url)
The proposed observer for MM with unmeasurable premise variables affected by time varying uncertainties and unknown inputs is designed.

- The unknown inputs and their estimates

- The states variables and their estimates
different faults are to be detected and isolated:
- output measurement fault (OMF): $y(t) = Cx(t) + \delta(t)$
- input measurement fault (IMF): $u(t) + \eta(t)$

residual generation:
- by comparing measured and estimated output $r = y - \hat{y}$
different faults are to be detected and isolated:
- output measurement fault (OMF): $y(t) = Cx(t) + \delta(t)$
- input measurement fault (IMF): $u(t) + \eta(t)$

residual generation:
→ by comparing measured and estimated output $r = y - \hat{y}$

residual structuration by observer banks
- OMF detection and isolation:
  → each observer is fed with $u(t)$ and a subset of $y(t)$
  → the estimated output is insensitive to the faults affecting the other outputs
different faults are to be detected and isolated:
  ▶ output measurement fault (OMF): \( y(t) = Cx(t) + \delta(t) \)
  ▶ input measurement fault (IMF): \( u(t) + \eta(t) \)

residual generation:
  → by comparing measured and estimated output \( r = y - \hat{y} \)

residual structuration by observer banks
  ▶ OMF detection and isolation:
    → each observer is fed with \( u(t) \) and a subset of \( y(t) \)
    → the estimated output is insensitive to the faults affecting the other outputs
  ▶ IMF detection and isolation:
    → each observer is fed with \( y(t) \) and a subset subset of \( u(t) \)
    → the others inputs are considered as unknown inputs
    → the output estimation error is insensitive to the faults affecting the other inputs
The residual signals $r_{ji}(t)$ are:

$$r_{ji}(t) = y_i(t) - \hat{y}_i(t)$$

where $j$ is the observer number and $i$ the component number.
Output measurement fault detection and isolation

- Residual signals: $r^j_i(t) = y_i(t) - \hat{y}^j_i(t)$
- The $i^{th}$ sensor fault, $\delta_i(t)$, ...
  - affects $y_i(t)$
  - affects $\hat{y}^j_k(t)$ ($k = 1, \ldots, n_y$) if $PIO^j$ is fed with $y_i(t)$
  - nothing can be said about $r^j_k$, when $PIO^j$ is fed with $y_i(t)$
Output measurement fault detection and isolation

- Residual signals: $r^i_j(t) = y_i(t) - \hat{y}^i_j(t)$
- The $i^{th}$ sensor fault, $\delta_i(t)$, ...
  - affects $y_i(t)$
  - affects $\hat{y}^i_k(t)$ ($k = 1, \ldots, n_y$) if $PIO^i$ is fed with $y_i(t)$
    - nothing can be said about $r^i_k(t)$, when $PIO^i$ is fed with $y_i(t)$
    - does not affect $\hat{y}^i_k(t)$ ($k = 1, \ldots, n_y$) if $PIO^i$ is not fed with $y_i(t)$
    - affects $r^i_j(t)$, if $PIO^i$ is not fed with $y_i(t)$
    - does not affect $r^i_k(t)$ ($k \neq i$) if $PIO^i$ is not fed with $y_i(t)$
Residual signals: \( r^j_i(t) = y_i(t) - \hat{y}^j_i(t) \)

The \( i^{th} \) sensor fault, \( \delta_i(t) \), ...
- affects \( y_i(t) \)
- affects \( \hat{y}^j_k(t) \) (\( k = 1, \ldots, n_y \)) if \( PIO^j \) is fed with \( y_i(t) \)
  - nothing can be said about \( r^j_k(t) \), when \( PIO^j \) is fed with \( y_i(t) \)
  - does not affect \( \hat{y}^j_k(t) \) (\( k = 1, \ldots, n_y \)) if \( PIO^j \) is not fed with \( y_i(t) \)
  - affects \( r^j_i(t) \), if \( PIO^j \) is not fed with \( y_i(t) \)
  - does not affect \( r^j_k(t) \) (\( k \neq i \)) if \( PIO^j \) is not fed with \( y_i(t) \)

The alarm associated to each fault is:

\[
 r^j_{b, i}(t) = \begin{cases} 
 0, & \text{if } |r^j_i(t)| \leq T \\
 1, & \text{if } |r^j_i(t)| > T 
\end{cases}
\]

\[
 a_i(t) = \prod_{j \in l_i} \left( r^j_{b, i}(t) \prod_{k \neq j}^{\ell} \bar{r}^j_{b, k}(t) \right)
\]

\( l_i \) is the index set of observers being not fed with the \( i^{th} \) output.
Output measurement fault detection and isolation

- sensor faults
  - $\delta_1(t)$ affects $y_1$, for $t \in [0.25, 0.75][\text{days}]$
  - $\delta_2(t)$ affects $y_2$, for $t \in [2.25, 2.75][\text{days}]$
  - $\delta_3(t)$ affects $y_3$, for $t \in [1.15][\text{days}]$
  - $\delta_4(t)$ affects $y_4$, for $t \in [3, 3.25][\text{days}]$
sensor faults

- $\delta_1(t)$ affects $y_1$, for $t \in [0.25, 0.75]\,[\text{days}]$
  - $\text{PIO}_{12}^1$, $\text{PIO}_{13}^1$ and $\text{PIO}_{14}^1$ are fed with $y_1$
  - $\text{PIO}_{23}^1$, $\text{PIO}_{24}^1$ and $\text{PIO}_{34}^1$ are not fed with $y_1$
  - $r_{1}^{23}$, $r_{1}^{24}$ and $r_{1}^{34}$ are non null
  - $r_{k}^{23}$, $r_{k}^{24}$ and $r_{k}^{34}$ ($k = 2, \ldots, 4$) are null

- $\delta_2(t)$ affects $y_2$, for $t \in [2.25, 2.75]\,[\text{days}]$
- $\delta_3(t)$ affects $y_3$, for $t \in [1, 1.5]\,[\text{days}]$
- $\delta_4(t)$ affects $y_4$, for $t \in [3, 3.25]\,[\text{days}]$
Output measurement fault detection and isolation

- **sensor faults**
  - \( \delta_1(t) \) affects \( y_1 \), for \( t \in [0.25 0.75][\text{days}] \)
  - \( \delta_2(t) \) affects \( y_2 \), for \( t \in [2.25 2.75][\text{days}] \)
  - \( \delta_3(t) \) affects \( y_3 \), for \( t \in [1 1.5][\text{days}] \)
  - \( \delta_4(t) \) affects \( y_4 \), for \( t \in [3 3.25][\text{days}] \)

- **PIO**
  - \( \text{PIO}^{14}, \text{PIO}^{24} \) and \( \text{PIO}^{34} \) are fed with \( y_4 \)
  - \( \text{PIO}^{12}, \text{PIO}^{13} \) and \( \text{PIO}^{23} \) are not fed with \( y_4 \)
  - \( r_{4,12}^{1}, r_{4,13}^{13} \) and \( r_{4,23}^{23} \) are non null
  - \( r_{k,12}^{12}, r_{k,13}^{13} \) and \( r_{k,23}^{23} \) (\( k = 1, \ldots, 3 \)) are null
The residual signals \( r^j_i(t) \) are:

\[
r^j_i(t) = y_i(t) - \hat{y}_i^j(t)
\]

where \( j \) is the observer number and \( i \) the component number.

- Each observer is fed with all input except one, considered as unknown input.
- \( r^j_i(t) (i = 1, \ldots, n_y) \) is affected by \( \eta_i \) if \( PIO^j \) is fed with \( u_i \).
- \( r^j_i(t) (i = 1, \ldots, n_y) \) is not affected by \( \eta_i \) if \( PIO^j \) was designed with \( u_i \) as an UI.
Conclusions and future works

- Multiple Model representation of a nonlinear model of a bioreactor
  - unmeasurable premise variables
  - model uncertainties
  - unknown inputs

- State and unknown input estimation for uncertain MM with UI
  - proportionnal integral observer

- Application to fault diagnosis
  - sensor fault detection and isolation
  - actuator fault detection and isolation

- More complex models are under study (with \( n = 10 \))
- Estimation error stability conditions should be relaxed
Thank you for your attention