State estimation for nonlinear system diagnosis using multiple models. Application to wastewater treatment plants

A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

CRAN

Nancy-Université

Centre de Recherche en Automatique de Nancy
UMR 7039 CNRS – Nancy-Université
France

CRAN

A. M. Nagy Kiss,
B. Marx,
G. Mourot,
J. Ragot

Objectives

1. Fault diagnosis for complex nonlinear systems
2. Application to the model of an activated sludge bioreactor of a Waste Water Treatment Plant
3. Need for state estimation of environmental plant with limited sensors

Objectives and context

CRAN

A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

Objectives

1. Fault diagnosis for complex nonlinear systems
2. Application to the model of an activated sludge bioreactor of a Waste Water Treatment Plant
3. Need for state estimation of environmental plant with limited sensors

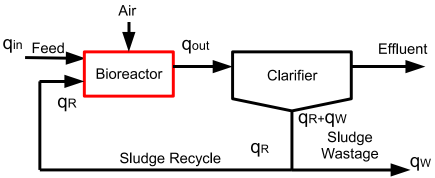
Context and tools

1. **modeling complexity** of nonlinear process
   $\longrightarrow$ Multiple Model approach
   $\longrightarrow$ Model uncertainties
2. **corrupted measurement** of the input or output
   $\longrightarrow$ observer based diagnosis
3. **limited number of sensors**
   $\longrightarrow$ unknown input observer design

Outline of the presentation

CRAN

A. M. Nagy Kiss,
B. Marx,
G. Mourot,
J. Ragot



The **wastewater treatment plant (WWTP)** is composed by :

- a bioreactor
  - polluted water and bacteria are put in contact and aerated
  - the bacterial biomass degrades the organic pollution
- a clarifier :
  - clear water and bacterial biomass are separated
  - clear water is rejected in the environment
  - a fraction of the biomass is recycled

The presented study only concerns the bioreactor

- the biorector is modelled with the Activated Sludge Model (ASM1)
- the clarifier is supposed to be ideal

Nonlinear dynamic system equations :

$$\dot{X}_{DCO}(t) = -\frac{1}{Y_H}(\varphi_1(t) + \varphi_2(t)) + (1 - f_P)(\varphi_4(t) + \varphi_5(t)) + D_1(t)$$

$$\dot{S}_O(t) = \frac{Y_H - 1}{Y_H}\varphi_1(t) + \frac{Y_A - 4.57}{Y_A}\varphi_3(t) + D_2(t)$$

$$\dot{S}_{NH}(t) = -i_{XB}(\varphi_1(t) + \varphi_2(t)) - (i_{XB} + \frac{1}{Y_A})\varphi_3(t) + (i_{XB} - f_P i_{XP})(\varphi_4(t) + \varphi_5(t)) + D_3(t)$$

$$\dot{S}_{NO}(t) = \frac{Y_H - 1}{2.86Y_H}\varphi_2(t) + \frac{1}{Y_A}\varphi_3(t) + D_4(t)$$

$$\dot{X}_{BH}(t) = \varphi_1(t) + \varphi_2(t) - \varphi_4(t) + D_5(t)$$

$$\dot{X}_{BA}(t) = \varphi_3(t) - \varphi_5(t) + D_6(t)$$

The state variables denote the following concentrations :

$X_{DCO}(t)$: demand in chemical oxygen
$S_O(t)$: dissolved oxygen
$S_{NH}(t)$: dissolved ammonia
$S_{NO}(t)$: dissolved nitrate
$X_{BH}(t)$: heterotrophic biomass
$X_{BA}(t)$: autotrophic biomass

CRAN
A. M. Nagy Kiss,
B. Marx,
G. Mourot,
J. Ragot

Nonlinear dynamic system equations :

$$\dot{X}_{DCO}(t) = -\frac{1}{Y_H}(\varphi_1(t) + \varphi_2(t)) + (1 - f_P)(\varphi_4(t) + \varphi_5(t)) + D_1(t)$$

$$\dot{S}_O(t) = \frac{Y_H - 1}{Y_H}\varphi_1(t) + \frac{Y_A - 4.57}{Y_A}\varphi_3(t) + D_2(t)$$

$$\dot{S}_{NH}(t) = -i_{XB}(\varphi_1(t) + \varphi_2(t)) - (i_{XB} + \frac{1}{Y_A})\varphi_3(t) + (i_{XB} - f_P i_{XP})(\varphi_4(t) + \varphi_5(t)) + D_3(t)$$

$$\dot{S}_{NO}(t) = \frac{Y_H - 1}{2.86 Y_H}\varphi_2(t) + \frac{1}{Y_A}\varphi_3(t) + D_4(t)$$

$$\dot{X}_{BH}(t) = \varphi_1(t) + \varphi_2(t) - \varphi_4(t) + D_5(t)$$

$$\dot{X}_{BA}(t) = \varphi_3(t) - \varphi_5(t) + D_6(t)$$

with the process kinetics :

$$\varphi_1(t) = \mu_H \frac{X_{DCO}(t)}{K_{DCO} + X_{DCO}(t)} \frac{S_O(t)}{K_{OH} + S_O(t)} X_{BH}(t)$$

$$\varphi_2(t) = \mu_H \eta_{NOg} \frac{X_{DCO}(t)}{K_{DCO} + X_{DCO}(t)} \frac{S_{NO}(t)}{K_{NO} + S_{NO}(t)} \frac{K_{OH}}{K_{OH} + S_O(t)} X_{BH}(t)$$

$$\vdots = \vdots$$

$$\varphi_5(t) = b_A X_{BA}(t)$$

Nonlinear dynamic system equations :

$$\dot{X}_{DCO}(t) = -\frac{1}{Y_H}(\varphi_1(t) + \varphi_2(t)) + (1 - f_P)(\varphi_4(t) + \varphi_5(t)) + D_1(t)$$

$$\dot{S}_O(t) = \frac{Y_H - 1}{Y_H}\varphi_1(t) + \frac{Y_A - 4.57}{Y_A}\varphi_3(t) + D_2(t)$$

$$\dot{S}_{NH}(t) = -i_{XB}(\varphi_1(t) + \varphi_2(t)) - (i_{XB} + \frac{1}{Y_A})\varphi_3(t) + (i_{XB} - f_P i_{XP})(\varphi_4(t) + \varphi_5(t)) + D_3(t)$$

$$\dot{S}_{NO}(t) = \frac{Y_H - 1}{2.86 Y_H}\varphi_2(t) + \frac{1}{Y_A}\varphi_3(t) + D_4(t)$$

$$\dot{X}_{BH}(t) = \varphi_1(t) + \varphi_2(t) - \varphi_4(t) + D_5(t)$$

$$\dot{X}_{BA}(t) = \varphi_3(t) - \varphi_5(t) + D_6(t)$$

and the input/output balances :

$$D_1(t) = \frac{q_{in}(t)}{V}(X_{DCO,in}(t) - X_{DCO}(t))$$

$$D_2(t) = -\frac{q_{in}(t)}{V}S_O(t) + K_{qa}(t)(S_{O,sat}(t) - S_O(t))$$

$$\vdots = \vdots$$

$$D_6(t) = -\frac{q_{in}(t)}{V}\frac{f_W(1 + f_R)}{f_W + f_R}X_{BA}(t)$$

- **Any nonlinear system can be equivalently written as a Multiple Model** (MM) on a compact set of the state space

$$\begin{cases} \dot{x} = f(x,u) \\ y = g(x,u) \\ nonlinear \end{cases} \Rightarrow \begin{cases} \dot{x} = A(x,u)x + B(x,u)u \\ y = C(x,u)x + D(x,u)u \\ Quasi - LPV \end{cases} \Rightarrow \begin{cases} \dot{x} = \sum_{i=1}^{r} \mu_i(x,u)(A_i x + B_i u) \\ y = \sum_{i=1}^{r} \mu_i(x,u)(C_i x + D_i u) \\ Multiple\ Model \end{cases}$$

$\rightarrow$ MM with unmeasurable premise variable are generally obtained

1. Nagy, Mourot, Marx, Ragot, Schutz, Systematic multi-modeling methodology applied to an activated sludge reactor model, Industrial & Engineering Chemistry Research, Vol. 46(6), pp. 2790-2799, 2010

Multiple model approach for nonlinear system

CRAN

A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

- **Any nonlinear system can be equivalently written as a Multiple Model** (MM) on a compact set of the state space

$$\begin{cases} \dot{x} = f(x,u) \\ y = g(x,u) \\ nonlinear \end{cases} \Rightarrow \begin{cases} \dot{x} = A(x,u)x + B(x,u)u \\ y = C(x,u)x + D(x,u)u \\ Quasi - LPV \end{cases} \Rightarrow \begin{cases} \dot{x} = \sum_{i=1}^{r} \mu_i(x,u)(A_i x + B_i u) \\ y = \sum_{i=1}^{r} \mu_i(x,u)(C_i x + D_i u) \\ Multiple\ Model \end{cases}$$

$\rightarrow$ MM with unmeasurable premise variable are generally obtained

- The nonlinearities are rejected in the activating functions $\mu_i(x(t), u(t))$
- System analysis and design are based on the linear submodels (A_i, B_i, C_i, D_i) with classical tools (Lyapunov functions, LMI, ...)
- Different equivalent rewrittings may not lead to the same results in terms of controller/observer design [1]

1. Nagy, Mourot, Marx, Ragot, Schutz, Systematic multi-modeling methodology applied to an activated sludge reactor model, Industrial & Engineering Chemistry Research, Vol. 46(6), pp. 2790-2799, 2010

Summing up, the bioreactor is described by a MM with 16 submodels :

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x,u)\,[A_i x(t) + B_i u(t)]$$
$$y(t) = Cx(t)$$

- state vector and measured output :

$$x^T(t) = [X_{DCO}(t)\ \ S_O(t)\ \ S_{NH}(t)\ \ S_{NO}(t)\ \ X_{BH}(t)\ \ X_{BA}(t)]$$
$$y^T(t) = [X_{DCO}(t)\ S_O(t)\ S_{NH}(t)\ S_{NO}(t)]$$

CRAN
A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

Summing up, the bioreactor is described by a MM with 16 submodels :

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x,u)\,[A_i x(t) + B_i u(t) + E_i d(t)]$$
$$y(t) = Cx(t) + Gd(t)$$

- state vector and measured output :

$$x^T(t) = [X_{DCO}(t)\;\; S_O(t)\;\; S_{NH}(t)\;\; S_{NO}(t)\;\; X_{BH}(t)\;\; X_{BA}(t)]$$
$$y^T(t) = [X_{DCO}(t)\; S_O(t)\; S_{NH}(t)\; S_{NO}(t)]$$

- some incoming pollutants cannot be measured
  $\rightarrow$ **unknown inputs** $d(t)$:

$$d(t)^T = [S_{NH_in}(t)\; X_{BH,in}(t)]$$

Summing up, the bioreactor is described by a MM with 16 submodels :

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x,u) \left[(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + E_i d(t) \right]$$
$$y(t) = Cx(t) + Gd(t)$$

- state vector and measured output :

$$x^T(t) = [X_{DCO}(t) \;\; S_O(t) \;\; S_{NH}(t) \;\; S_{NO}(t) \;\; X_{BH}(t) \;\; X_{BA}(t)]$$
$$y^T(t) = [X_{DCO}(t) \; S_O(t) \; S_{NH}(t) \; S_{NO}(t)]$$

- some incoming pollutants cannot be measured
  → **unknown inputs** $d(t)$:

$$d(t)^T = [S_{NH_in}(t) \; X_{BH,in}(t)]$$

- some model parameters are time varying and not perfectly known
  → **model uncertainties** $\Delta A_i(t)$ **and** $\Delta B_i(t)$:

$$b_A(t) = b_A + \Delta b_A(t) \text{ and } b_H(t) = b_H + \Delta b_H(t)$$

- The state and unknown input of the uncertain MM :

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x,u)\left[(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + E_i d(t)\right]$$

$$y(t) = Cx(t) + Gd(t)$$

CRAN

A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

- The state and unknown input of the uncertain MM :

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x,u)\left[(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + E_i d(t)\right]$$
$$y(t) = Cx(t) + Gd(t)$$

- are reconstructed with the following observer :

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x},u)\left(A_i\hat{x}(t) + B_i u(t) + E_i\hat{d}(t) + L_i^P(y(t) - \hat{y}(t))\right)$$
$$\dot{\hat{d}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x},u) L_i^I (y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C\hat{x}(t) + G\hat{d}(t)$$

- **Objective :**
  find the observer gains : L_i^P and L_i^I minimizing the $\mathcal{L}_2$-gain from the inputs $\begin{bmatrix} u(t) \\ d(t) \end{bmatrix}$ to the estimation errors $\begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} = \begin{bmatrix} x(t) - \hat{x}(t) \\ d(t) - \hat{d}(t) \end{bmatrix}$

CRAN

A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

- The state and unknown input of the uncertain MM :

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x,u)\left[(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + E_i d(t)\right]$$
$$y(t) = Cx(t) + Gd(t)$$

- are reconstructed with the following observer :

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x},u)\left(A_i\hat{x}(t) + B_i u(t) + E_i\hat{d}(t) + L_i^P(y(t) - \hat{y}(t))\right)$$
$$\dot{\hat{d}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x},u) L_i^I(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C\hat{x}(t) + G\hat{d}(t)$$

- **Assumptions :**
  - constant unknown input : $\dot{d}(t) = 0$
  - bounded uncertainties : $\Delta A_i(t) = M_i^a F_a(t) N_i^a$, with $F_a^T(t)F_a(t) \leq I$
  - bounded uncertainties : $\Delta A_i(t) = M_i^a F_a(t) N_i^a$, with $F_a^T(t)F_a(t) \leq I$

CRAN

A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

- The state and unknown input of the uncertain MM :

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x,u) \left[(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t) + E_i d(t)\right]$$

$$y(t) = Cx(t) + Gd(t)$$

- are reconstructed with the following observer :

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x},u) \left(A_i\hat{x}(t) + B_i u(t) + E_i\hat{d}(t) + L_i^P(y(t) - \hat{y}(t))\right)$$

$$\dot{\hat{d}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x},u) L_i^I (y(t) - \hat{y}(t))$$

$$\hat{y}(t) = C\hat{x}(t) + G\hat{d}(t)$$

- **Difficulties :**
  the activating functions of the system depend on $x(t)$, while these of the observer depend on $\hat{x}(t)$
  the presence of unknown inputs

- Find a positive definite Lyapunov function $V(e_x, e_d, x)$ such that :

$$\dot{V}(e_x, e_d, x) - \gamma^2 \begin{bmatrix} u(t) \\ d(t) \end{bmatrix}^T \begin{bmatrix} u(t) \\ d(t) \end{bmatrix} + \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} < 0 \tag{1}$$

CRAN
A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

- Find a positive definite Lyapunov function $V(e_x, e_d, x)$ such that :

$$\dot{V}(e_x, e_d, x) - \gamma^2 \begin{bmatrix} u(t) \\ d(t) \end{bmatrix}^T \begin{bmatrix} u(t) \\ d(t) \end{bmatrix} + \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} < 0 \qquad (1)$$

- The proposed quadratic Lyapunov function is :

$$V(e_x, e_d, x) = \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T P_1 \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} + x^T(t) P_2 x(t)$$

with $P_1 = P_1^T > 0$ and $P_2 = P_2^T > 0$

CRAN
A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

- Find a positive definite Lyapunov function $V(e_x, e_d, x)$ such that :

$$\dot{V}(e_x, e_d, x) - \gamma^2 \begin{bmatrix} u(t) \\ d(t) \end{bmatrix}^T \begin{bmatrix} u(t) \\ d(t) \end{bmatrix} + \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} < 0 \qquad (1)$$

- The proposed quadratic Lyapunov function is :

$$V(e_x, e_d, x) = \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T P_1 \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} + x^T(t) P_2 x(t)$$

with $P_1 = P_1^T > 0$ and $P_2 = P_2^T > 0$

- LMI optimization allows to find
  - $\rightarrow$ the Lyapunov matrices P_1, P_2
  - $\rightarrow$ the observer gains L_i^P and L_i^I
  - $\rightarrow$ that minimize the $\mathcal{L}_2$-gain γ under (1)

- Find a positive definite Lyapunov function $V(e_x, e_d, x)$ such that :

$$\dot{V}(e_x, e_d, x) - \gamma^2 \begin{bmatrix} u(t) \\ d(t) \end{bmatrix}^T \begin{bmatrix} u(t) \\ d(t) \end{bmatrix} + \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} < 0 \qquad (1)$$

- The proposed quadratic Lyapunov function is :

$$V(e_x, e_d, x) = \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix}^T P_1 \begin{bmatrix} e_x(t) \\ e_d(t) \end{bmatrix} + x^T(t) P_2 x(t)$$

with $P_1 = P_1^T > 0$ and $P_2 = P_2^T > 0$

- LMI optimization allows to find
  - $\longrightarrow$ the Lyapunov matrices P_1, P_2
  - $\longrightarrow$ the observer gains L_i^P and L_i^I
  - $\longrightarrow$ that minimize the $\mathcal{L}_2$-gain γ under (1)
- **sufficient LMI conditions are obtained**

CRAN
A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

Find matrices $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $\overline{P}_j$, and scalars $\varepsilon_{1i} > 0$ and ε_{2i} minimizing the scalar $\overline{\gamma} > 0$ under the following LMI constraints

$$\begin{bmatrix} \Phi_{ij}^{11} & \Phi_{ij}^{12} & 0 & \Phi_{ij}^{14} & P_1\overline{M}_i^a & P_1\overline{M}_i^b \\ * & \Phi_{ij}^{22} & P_2B_i & P_2E_i & P_2M_i^a & P_2M_i^b \\ * & * & \Phi_{ij}^{33} & 0 & 0 & 0 \\ * & * & * & -\overline{\gamma}I & 0 & 0 \\ * & * & * & * & -\varepsilon_{1i}I & 0 \\ * & * & * & * & * & -\varepsilon_{2i}I \end{bmatrix} < 0, \quad i,j = 1,\dots,r$$

with

$$\Phi_{ij}^{11} = I + \mathbb{S}(P_1\overline{A}_j - \overline{P}_j\overline{C}), \qquad \Phi_{ij}^{12} = P_1(\tilde{A}_i - \tilde{A}_j), \qquad \Phi_{ij}^{14} = P_1(\tilde{E}_i - \tilde{E}_j),$$

$$\Phi_{ij}^{22} = \varepsilon_{1i}N_i^{aT}N_i^a + \mathbb{S}(P_2A_i), \qquad \Phi_{ij}^{33} = \varepsilon_{2i}N_i^{bT}N_i^b - \overline{\gamma}I$$

$$\overline{C}^T = \begin{bmatrix} C^T \\ G^T \end{bmatrix}, \ \overline{A}_i = \begin{bmatrix} A_i & E_i \\ 0 & 0 \end{bmatrix}, \ \tilde{A}_i = \begin{bmatrix} A_i \\ 0 \end{bmatrix}, \ \tilde{E}_i = \begin{bmatrix} E_i \\ 0 \end{bmatrix}, \ \overline{M}_i^a = \begin{bmatrix} M_i^a \\ 0 \end{bmatrix}, \ \overline{M}_i^b = \begin{bmatrix} M_i^b \\ 0 \end{bmatrix}$$

The observer gains L_j^P and L_j^I, and the $\mathcal{L}_2$-gain γ are obtained by :

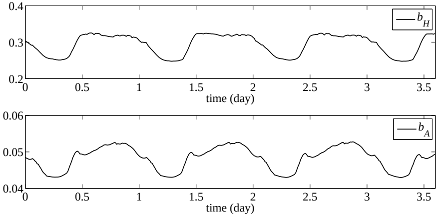
$$\begin{bmatrix} L_j^P \\ L_j^I \end{bmatrix} = P_1^{-1}\overline{P}_j \qquad \gamma = \sqrt{\overline{\gamma}}$$

CRAN

A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

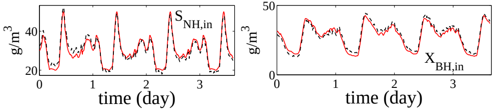
The proposed observer for MM with unmeasurable premise variables affected by time varying uncertainties and unknown inputs is designed.

The time varying uncertain parameters :



The proposed observer for MM with unmeasurable premise variables affected by time varying uncertainties and unknown inputs is designed.

- The unknown inputs and their estimates

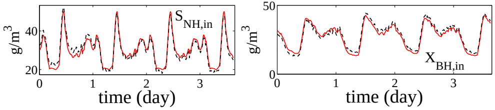


CRAN

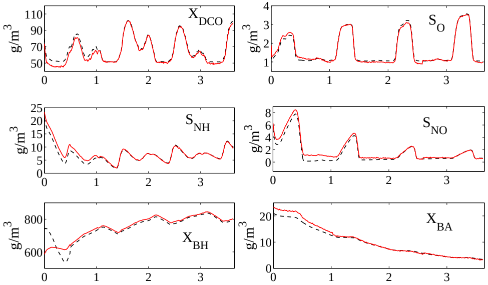
A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

The proposed observer for MM with unmeasurable premise variables affected by time varying uncertainties and unknown inputs is designed.

- The unknown inputs and their estimates



- The states variables and their estimates



- different faults are to be detected and isolated :
  - output measurement fault (OMF) : $y(t) = Cx(t) + \delta(t)$
  - input measurement fault (IMF) : $u(t) + \eta(t)$

- residual generation :
  $\rightarrow$ by comparing measured and estimated output $r = y - \hat{y}$

CRAN

A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

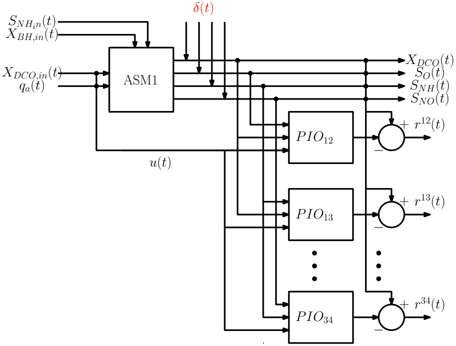
- different faults are to be detected and isolated :
  - output measurement fault (OMF) : $y(t) = Cx(t) + \delta(t)$
  - input measurement fault (IMF) : $u(t) + \eta(t)$

- residual generation :
  $\rightarrow$ by comparing measured and estimated output $r = y - \hat{y}$

- residual structuration by observer banks
  - OMF detection and isolation :
    $\rightarrow$ each observer is fed with $u(t)$ and a subset of $y(t)$
    $\rightarrow$ the estimated output is insensitive to the faults affecting the other outputs

- different faults are to be detected and isolated :
  - output measurement fault (OMF) : $y(t) = Cx(t) + \delta(t)$
  - input measurement fault (IMF) : $u(t) + \eta(t)$
- residual generation :
  $\rightarrow$ by comparing measured and estimated output $r = y - \hat{y}$
- residual structuration by observer banks
  - OMF detection and isolation :
    $\rightarrow$ each observer is fed with $u(t)$ and a subset of $y(t)$
    $\rightarrow$ the estimated output is insensitive to the faults affecting the other outputs
  - IMF detection and isolation :
    $\rightarrow$ each observer is fed with $y(t)$ and a subset subset of $u(t)$
    $\rightarrow$ the others inputs are considered as unknown inputs
    $\rightarrow$ the output estimation error is insensitive to the faults affecting the other inputs

Strucured residual generation scheme for OMFDI



The residual signals $r_i^j(t)$ are :

$$r_i^j(t) = y_i(t) - \hat{y}_i^j(t)$$

where j is the observer number and i the component number

CRAN

A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

- Residual signals : $r_i^j(t) = y_i(t) - \hat{y}_i^j(t)$
- The i^{th} sensor fault, $\delta_i(t)$, ...
  - affects $y_i(t)$
  - affects $\hat{y}_k^j(t)$ $(k = 1, \ldots, n_y)$ if PIO^j is fed with $y_i(t)$

$\rightarrow$ nothing can be said about r_k^j, when PIO^j is fed with $y_i(t)$

Output measurement fault detection and isolation

- Residual signals : $r_i^j(t) = y_i(t) - \hat{y}_i^j(t)$
- The i^{th} sensor fault, $\delta_i(t)$, ...
  - affects $y_i(t)$
  - affects $\hat{y}_k^j(t)$ $(k = 1, \ldots, n_y)$ if PIO^j is fed with $y_i(t)$

$\longrightarrow$ nothing can be said about r_k^j, when PIO^j is fed with $y_i(t)$

  - does not affect $\hat{y}_k^j(t)$ $(k = 1, \ldots, n_y)$ if PIO^j is not fed with $y_i(t)$

$\longrightarrow$ affects $r_i^j(t)$, if PIO^j is not fed with $y_i(t)$

$\longrightarrow$ does not affect $r_k^j(t)$ $(k \neq i)$ if PIO^j is not fed with $y_i(t)$

CRAN

A. M. Nagy Kiss, B. Marx, G. Mourot, J. Ragot

Process description

Multiple model approach

Proportional Integral Unknown input Observer design

Observer based diagnosis

Conclusions and future works

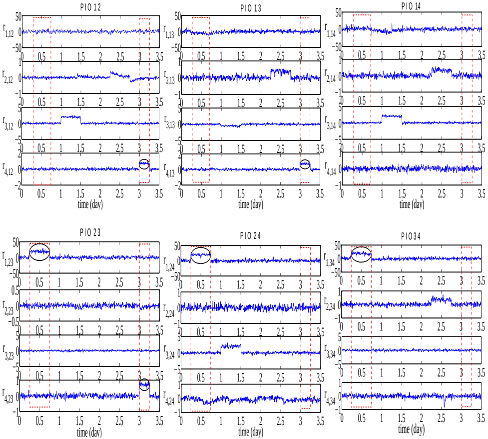
- Residual signals : $r_i^j(t) = y_i(t) - \hat{y}_i^j(t)$
- The i^{th} sensor fault, $\delta_i(t)$, ...
  - affects $y_i(t)$
  - affects $\hat{y}_k^j(t)$ ($k = 1, \ldots, n_y$) if PIO^j is fed with $y_i(t)$
  - $\rightarrow$ nothing can be said about r_k^j, when PIO^j is fed with $y_i(t)$
  - does not affect $\hat{y}_k^j(t)$ ($k = 1, \ldots, n_y$) if PIO^j is not fed with $y_i(t)$
  - $\rightarrow$ affects $r_i^j(t)$, if PIO^j is not fed with $y_i(t)$
  - $\rightarrow$ does not affect $r_k^j(t)$ ($k \neq i$) if PIO^j is not fed with $y_i(t)$
- The alarm associated to each fault is :

$$r_{b,i}^j(t) = \begin{cases} 0, & \text{if } |r_i^j(t)| \leq T \\ 1, & \text{if } |r_i^j(t)| > T \end{cases}$$

$$a_i(t) = \prod_{j \in I_{\bar{i}}} \left(r_{b,i}^j(t) \prod_{k \neq j, k=1}^{\ell} \bar{r}_{b,k}^j(t) \right)$$

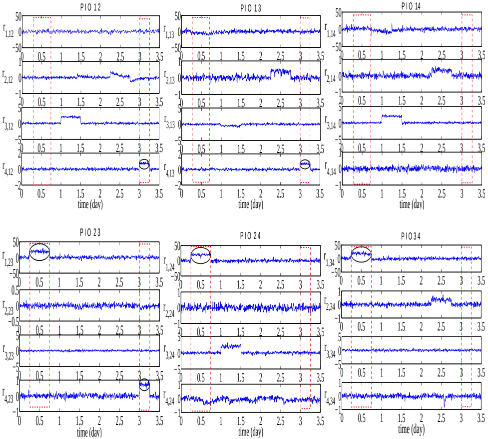
$I_{\bar{i}}$ is the index set of observers being not fed with the i^{th} output.

Output measurement fault detection and isolation



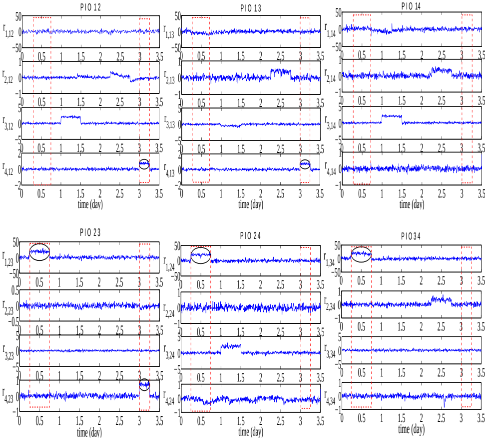
- sensor faults
  - $\delta_1(t)$ affects y_1, for $t \in [0.25\ 0.75][days]$
  - $\delta_2(t)$ affects y_2, for $t \in [2.25\ 2.75][days]$
  - $\delta_3(t)$ affects y_3, for $t \in [1\ 1.5][days]$
  - $\delta_4(t)$ affects y_4, for $t \in [3\ 3.25][days]$

Output measurement fault detection and isolation



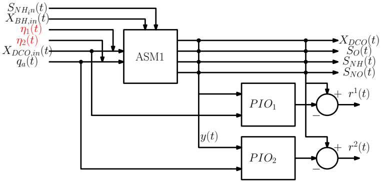
- sensor faults
  - $\delta_1(t)$ affects y_1, for $t \in [0.25\ 0.75][days]$
    - PIO^{12}, PIO^{13} and PIO^{14} are fed with y_1
    - PIO^{23}, PIO^{24} and PIO^{34} are not fed with y_1
    - r_1^{23}, r_1^{24} and r_1^{34} are non null
    - r_k^{23}, r_k^{24} and r_k^{34} $(k = 2, \ldots, 4)$ are null
  - $\delta_2(t)$ affects y_2, for $t \in [2.25\ 2.75][days]$
  - $\delta_3(t)$ affects y_3, for $t \in [1\ 1.5][days]$
  - $\delta_4(t)$ affects y_4, for $t \in [3\ 3.25][days]$

Output measurement fault detection and isolation



- sensor faults
  - $\delta_1(t)$ affects y_1, for $t \in [0.25\ 0.75][days]$
  - $\delta_2(t)$ affects y_2, for $t \in [2.25\ 2.75][days]$
  - $\delta_3(t)$ affects y_3, for $t \in [1\ 1.5][days]$
  - $\delta_4(t)$ affects y_4, for $t \in [3\ 3.25][days]$
    - PIO^{14}, PIO^{24} and PIO^{34} are fed with y_4
    - PIO^{12}, PIO^{13} and PIO^{23} are not fed with y_4
    - r_4^{12}, r_4^{13} and r_4^{23} are non null
    - r_k^{12}, r_k^{13} and r_k^{23} $(k = 1, \ldots, 3)$ are null

Input measurement fault detection and isolation



The residual signals $r_i^j(t)$ are :

$$r_i^j(t) = y_i(t) - \hat{y}_i^j(t)$$

where j is the observer number and i the component number

- each observer is fed with all input except one, considered as unknown input
- $r_i^j(t)$ $(i = 1, \ldots, n_y)$ is affected by η_i if PIO^j is fed with u_i
- $r_i^j(t)$ $(i = 1, \ldots, n_y)$ is not affected by η_i if PIO^j was designed with u_i as an UI

Conclusions and future works

- Multiple Model representation of a nonlinear model of a bioreactor
  - unmeasurable premise variables
  - model uncertainties
  - unknown inputs
- State and unknown input estimation for uncertain MM with UI
  - proportionnal integral observer
- Application to fault diagnosis
  - sensor fault detection and isolation
  - actuator fault detection and isolation
- More complex models are under study (with $n = 10$)
- Estimation error stability conditions should be relaxed

Thank you for your attention