Design of fault tolerant control for nonlinear systems subject to time varying faults

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Nancy, France

ESREL, September 18-22, 2011, Troyes France

Objective of diagnosis and fault tolerant control

- To detect, isolate and estimate the actuator fault (diagnosis)
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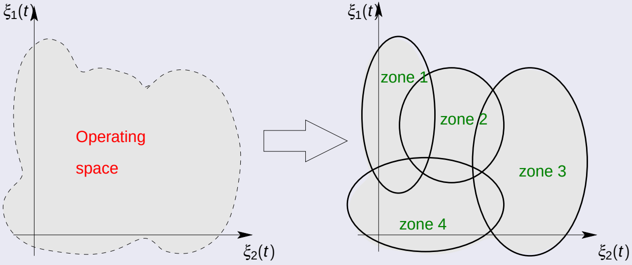
Proposed strategy

- Takagi-Sugeno representation of nonlinear systems
- Extension of the existing results on linear systems
- Observer-based fault tolerant control design
- Consideration of an *a priori* model of the fault

1. Takagi-Sugeno approach for modeling
2. Observer and FTC law structures
3. A priori considered fault models
4. Controller design
5. Simulations results
6. Conclusions

- Operating range decomposition in several local zones.
- A simple submodel represents the behavior of the system in a specific zone.
- The overall behavior of the system is obtained by aggregating the submodels with adequate weighting functions.



Nonlinear system　　Multiple Model representation

The main idea of Takagi-Sugeno approach

- Define local models $M_i, \quad i = 1..r$
- Define weighting functions $\mu_i(\xi)$, s.t. $0 \leq \mu_i \leq 1$ and $\sum_{i=1}^{r} \mu_i(\xi) = 1$

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Interests of Takagi-Sugeno approach

- The specific study of the nonlinearities is not required.
- Analysis (stability, performance, robustness, etc.) and design (controller, observer, etc.) are based on the linear submodels.

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The difficulties

- How many local models ?
- How to define the domain of influence of each local model ?
- On what variables may depend the weighting functions μ_i ?

Obtaining a Takagi-Sugeno model

- Identification approach
  - Choice of premise variables
  - Choice of the structure of the local models
  - Parameter identification
- Transformation of an *a priori* known nonlinear model
  - Linearization around some points
    - how to chose the linearization points ?
    - how to define the weighting functions, minimizing the approximation error
  - Nonlinear sector approach

Equivalent rewriting of the model in a compact set of the state space

$$\begin{cases} x(k+1) = f(x(k),u(k)) \\ y(k) = h(x(k),u(k)) \end{cases} \Rightarrow \begin{cases} x(k+1) = \sum_{i=1}^{r} \mu_i(\xi(k))(A_i x(k) + B_i u(k)) \\ y(k) = \sum_{i=1}^{r} \mu_i(\xi(k))(C_i x(k) + D_i u(k)) \end{cases}$$

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Reference model

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- Interpolation mechanism $\sum_{i=1}^{r} \mu_i(\xi(k)) = 1$ and $0 \leq \mu_i(\xi(k)) \leq 1, \forall k, \forall i \in \{1, ..., r\}$
- The premise variable $\xi(k)$ are measurable (like $u(k)$, $y(k)$).

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The faulty system

$$\begin{cases} x_f(k+1) = \sum_{i=1}^{r} \mu_i(\xi(k))(A_i x_f(k) + B_i u_f(k) + G_i f(k)) \\ y_f(k) = \sum_{i=1}^{r} \mu_i(\xi(k))(C_i x_f(k) + D_i u_f(k) + W_i f(k)) \end{cases}$$

- $f(k)$ represents the fault vector to be detected and accommodated.

Objectives : estimation + diagnosis + FTC

- estimate the faulty system state $x_f(k)$
- estimate the occurring fault $f(k)$
- reconfigure the control law for trajectory tracking $x_f(k) \rightarrow x(k)$

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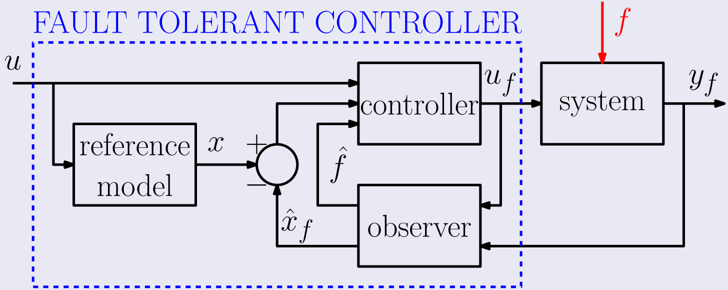
Fault tolerant control scheme



Faulty system

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PI Observer

$$\begin{cases} \hat{x}_f(k+1) = \sum_{i=1}^{r} \mu_i(\xi(k)) \left(A_i \hat{x}_f(k) + B_i u_f(k) + G_i \hat{f}(k) + H_i^1 (y_f(k) - \hat{y}_f(k)) \right) \\ \hat{f}(k+1) = \sum_{i=1}^{r} \mu_i(\xi(k)) \left(H_i^2 (y_f(k) - \hat{y}_f(k)) + \hat{f}(k) \right) \\ \hat{y}_f(k) = \sum_{i=1}^{r} \mu_i(\xi(k)) \left(C_i \hat{x}_f(k) + D_i u_f(k) + W_i \hat{f}(k) \right) \end{cases}$$

Observer and FTC law structures

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FTC law

$$u_f(k) = u(k) + \sum_{i=1}^{r} \mu_i(\xi(k)) \left(K_i (x(k) - \hat{x}_f(k)) - \hat{f}(k) \right)$$

Exponential faults

$$f_i(k) = e^{\alpha_i k + \beta_i}, \text{ with } \alpha_i, \beta_i \in \mathbb{R}, i = 1, \dots, q$$

$$\alpha_i = \alpha_{0,i} + \Delta\alpha_i$$

where $\alpha_{0,i}$ and $\Delta\alpha_i$ are respectively the nominal and the uncertain parts of α_i

Let us define :

$$\alpha = \mathrm{diag}(\alpha_1, \dots, \alpha_q)$$

$$\alpha_0 = \mathrm{diag}(\alpha_{0,1}, \dots, \alpha_{0,q})$$

$$\Delta\alpha = \mathrm{diag}(\Delta\alpha_1, \dots, \Delta\alpha_q)$$

The uncertain part can be bounded as :

$$(\Delta\alpha)^T \Delta\alpha \leq \lambda$$

where $\lambda \in \mathbb{R}^{q \times q}$ is a known diagonal positive definite matrix.

Estimation errors

$$
\begin{cases}
e_p(k) = x(k) - x_f(k) & : \text{state tracking error} \\
e_s(k) = x_f(k) - \hat{x}_f(k) & : \text{state estimation error} \\
e_d(k) = f(k) - \hat{f}(k) & : \text{fault estimation error}
\end{cases}
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Notation and hypothesis

$$X_\mu = \sum_{i=1}^{r} \mu_i(\xi(k)) X_i \qquad X_{\mu\mu} = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\xi(k)) \mu_j(\xi(k)) X_{ij} \qquad f_i(k+1) = e^{\alpha_i} f_i(k)$$

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Dynamics of the tracking and estimation errors

$$\underbrace{\begin{pmatrix} e_p(k+1) \\ e_s(k+1) \\ e_d(k+1) \end{pmatrix}}_{\bar{e}(k+1)} = \underbrace{\begin{pmatrix} A_{\mu\mu} - B_\mu K_\mu & -B_\mu K_\mu & -B_\mu \\ 0 & A_\mu - H_\mu^1 C_\mu & G_\mu - H_\mu^1 W_\mu \\ 0 & -H_\mu^2 C_\mu & I - H_\mu^2 W_\mu \end{pmatrix}}_{\bar{A}_\mu} \underbrace{\begin{pmatrix} e_p(k) \\ e_s(k) \\ e_d(k) \end{pmatrix}}_{\bar{e}(k)} + \underbrace{\begin{pmatrix} B_\mu - G_\mu \\ 0 \\ \alpha - I \end{pmatrix}}_{\bar{B}_\mu} f(k)$$

Controller design – the exponential fault case

The tracking, state estimation and fault estimation errors are ruled by :

$$\overline{e}(k+1) = \overline{A}_{\mu\mu}\overline{e}(k) + \overline{B}_{\mu}f(k)$$

The FTC design reduces to find the controller and observer gains : K_i, H_i^1 and H_i^2 satisfying the two main objectives.

Tracking, state and fault estimation error convergence in the fault free case

Find a positive definite Lyapunov function such that

$$\Delta V(k) = V(k+1) - V(k) < 0$$

Here, a quadratic Lyapunov function is chosen :

$$V(k) = \overline{e}^T(k)X\overline{e}(k), \quad \text{with} \quad X = X^T > 0$$

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Attenuation of the fault effect

The $\mathscr{L}_2$-gain from the fault $f(k)$ to the errors $\overline{e}(k)$ is bounded by a positive γ

$$\sum_{k=1}^{N} \overline{e}^T(k) Q \overline{e}(k) \leq \gamma^2 \sum_{k=1}^{N} f^T(k) f(k)$$

Summary

The tracking error $e_p(k)$, state and fault estimation errors $e_s(k)$ and $e_d(k)$ must therefore satisfy the following inequality :

$$\overline{e}^T(k+1)X\overline{e}(k+1)-\overline{e}^T(k)X\overline{e}(k)+\overline{e}^T(k)Q\overline{e}(k)-\gamma^2 f^T(k)f(k)<0$$

This inequality is fulfilled if :

$$\begin{pmatrix} Q-X & 0 \\ 0 & -\gamma^2 I \end{pmatrix}+\begin{pmatrix} \overline{A}_{\mu\mu}^T \\ \overline{B}_{\mu}^T \end{pmatrix} X \begin{pmatrix} \overline{A}_{\mu\mu} & \overline{B}_{\mu} \end{pmatrix}<0$$

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- Chosing the Lyapunov matrix structure : $X = \begin{pmatrix} X_1 & 0 & 0 \\ 0 & X_2 & 0 \\ 0 & 0 & X_3 \end{pmatrix}$
- knowing that $\mu_i(\xi(k)) \geq 0$
- with some matrix manipulations (Schur complement, S-procedure)

$\rightarrow$ **sufficient LMI conditions are derived**

Theorem 1

The tracking and estimation errors asymptotically converge to zero in the fault free case and the $\mathscr{L}_2$-gain from f to $\bar{e}$ is bounded by γ, if there exists matrices $X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$, K_i, L_i^1 and L_i^2 and scalars $\bar{\gamma}$ and τ such that, for $i = 1, 2, ..., r$

$$\begin{pmatrix} Q_1 - X_1 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 & * & 0 \\ 0 & Q_2 - X_2 & 0 & 0 & 0 & * & * & 0 & 0 & 0 & * \\ 0 & 0 & Q_3 - X_3 & 0 & * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau^{-1}\lambda - \bar{\gamma} I & * & 0 & * & 0 & * & 0 & 0 \\ X_1 A_i & 0 & -X_1 B_i & X_1(B_i - G_i) & -X_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & X_2 A_i - L_j^1 C_i & X_2 G_i - L_j^1 W_i & 0 & 0 & -X_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_j^2 C_i & X_3 - L_j^2 W_i & -X_3 & 0 & 0 & -X_3 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_0 X_3 & -\tau^{-1} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & X_1 & 0 & 0 & 0 & -2I & 0 & 0 \\ B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ 0 & B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{pmatrix} < 0$$

The observer gains and the attenuation level are obtained by :

$$H_i^1 = X_2^{-1} L_i^1, \quad H_i^2 = X_3^{-1} L_i^2 \quad \text{and} \quad \gamma = \sqrt{\bar{\gamma}}$$

Polynomial faults

$$f_i(k) = a_i k + b_i, \text{ with } a_i, b_i \in \mathbb{R}, i = 1, \ldots, q$$

As well as for exponential function, defining different diagonal matrices, $a = a_0 + \Delta a$, with Δa verifying :

$$(\Delta a)^T \Delta a \leq \delta$$

where $\delta \in \mathbb{R}^{q \times q}$ is a known diagonal positive definite matrix.

Controller design – the polynomial fault case

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Dynamics of the tracking and estimation errors

Defining $\bar{e}^T(k) = [e_p^T(k)\ e_s^T(k)\ e_d^T(k)]$, it follows

$$\bar{e}(k+1) = \underbrace{\begin{pmatrix} A_{\mu\mu} - B_\mu K_\mu & -B_\mu K_\mu & -B_\mu \\ 0 & A_\mu - H_\mu^1 C_\mu & G_\mu - H_\mu^1 W_\mu \\ 0 & -H_\mu^2 C_\mu & I - H_\mu^2 W_\mu \end{pmatrix}}_{\bar{A}_\mu} \underbrace{\begin{pmatrix} e_p(k) \\ e_s(k) \\ e_d(k) \end{pmatrix}}_{\bar{e}(k)} + \underbrace{\begin{pmatrix} B_\mu - G_\mu \\ 0 \\ 0 \end{pmatrix}}_{\bar{E}_\mu} f(k) + \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix}$$

Theorem 2

The tracking and estimation errors asymptotically converge to zero in the fault free case and the $\mathscr{L}_2$-gain from f to $\bar{e}$ is bounded by γ, if there exists matrices $X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$, K_i, L_i^1 and L_i^2 and scalars $\bar{\gamma}$ ρ and τ such that, for $i = 1, 2, \ldots, r$

$$\begin{pmatrix}
\Phi^{1,1} & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & * & 0 & 0 \\
0 & \Phi^{2,2} & 0 & 0 & 0 & 0 & * & * & 0 & 0 & * & 0 \\
0 & 0 & \Phi^{3,3} & 0 & 0 & * & * & * & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\bar{\gamma} I & 0 & * & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Phi^{5,5} & 0 & 0 & * & 0 & 0 & 0 & 0 \\
X_1 A_i & 0 & -X_1 B_i & \Phi_i^{6,4} & 0 & -X_1 & 0 & 0 & * & 0 & 0 & 0 \\
0 & \Phi_{ij}^{7,2} & \Phi_{ij}^{7,3} & 0 & 0 & 0 & -X_2 & 0 & 0 & 0 & 0 & 0 \\
0 & -L_j^2 C_i & \Phi_{ij}^{8,3} & 0 & X_3 a_0 & 0 & 0 & -X_3 & 0 & 0 & 0 & * \\
0 & 0 & 0 & 0 & 0 & X_1 & 0 & 0 & -2I & 0 & 0 & 0 \\
B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 \\
0 & B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & X_3 & 0 & 0 & 0 & -\tau^{-1} I
\end{pmatrix} < 0$$

$$\Phi^{1,1} = \rho I + Q_1 - X_1 \quad \Phi^{2,2} = \rho I + Q_2 - X_2 \quad \Phi^{3,3} = \rho I + Q_3 - X_3 \quad \Phi^{5,5} = -\rho \varepsilon I + \tau^{-1} \delta I$$

$$\Phi_i^{6,4} = X_1 (B_i - G_i) \quad \Phi_{ij}^{7,2} = X_2 A_i - L_j^1 C_i \quad \Phi_{ij}^{7,3} = X_2 G_i - L_j^1 W_i \quad \Phi_{ij}^{8,3} = X_3 - L_j^2 W_i$$

The observer gains and the attenuation level are obtained by :

$$H_i^1 = X_2^{-1} L_i^1, \quad H_i^2 = X_3^{-1} L_i^2 \quad \text{and} \quad \gamma = \sqrt{\bar{\gamma}}$$

Takagi-Sugeno model

$$\begin{cases} x(k+1) = \sum_{i=1}^{2} \mu_i(u(k))\left(A_i x_f(k) + B_i u_f(k) + G_i f(k)\right) \\ y(k) = \sum_{i=1}^{2} \mu_i(u(k))\left(C_i x_f(k) + D_i u_f(k) + W_i f(k)\right) \end{cases}$$

with

$$A_1 = \begin{pmatrix} -0.5 & 0.1 \\ -1 & -1 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & 0.2 \\ -0.45 & -0.7 \end{pmatrix} \quad B_1 = \begin{pmatrix} 0.4 \\ 0.5 \end{pmatrix} \quad B_2 = \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} 0.2 \\ 0.4 \end{pmatrix} \quad G_2 = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

$$C_1 = \begin{pmatrix} 0.2 & 0 \end{pmatrix} \quad C_2 = \begin{pmatrix} 0.4 & 0.1 \end{pmatrix} \quad W_1 = -0.3 \quad W_2 - 0.4$$

$$\mu_1(u(k)) = \frac{1 - \tanh(0.5 - u(k))}{2} \quad \mu_2(u(k)) = \frac{1 + \tanh(0.5 - u(k))}{2}$$

The nominal input signal is : $u(k) = 0.5\cos(\sin(0.1k)0.1k)$.
The FT Controller is designed for : $\alpha_0 = 0.1$ and $\lambda = 1.3$
The fault affecting the system is : $f(k) = e^{0.5k-10}$, for $9 \leq k \leq 17$

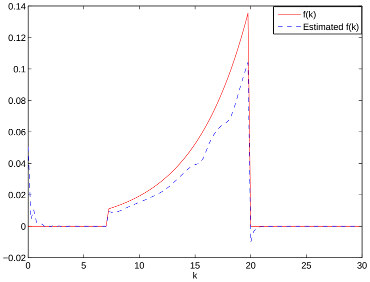

FIGURE: Fault and its estimation

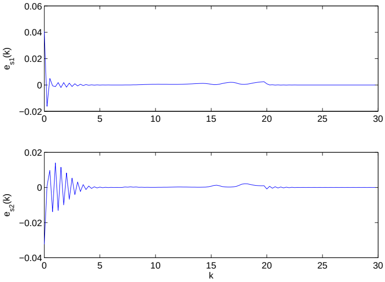

FIGURE: State estimation errors

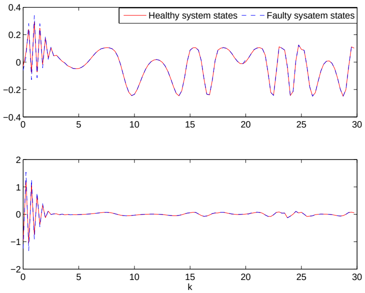

FIGURE: Reference model states vs. faulty system ones with FTC

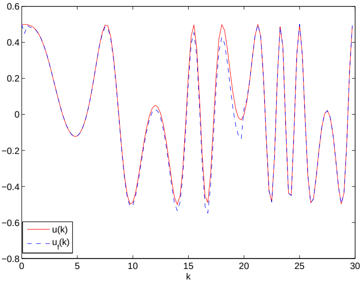

FIGURE: Nominal and FTC control inputs

Conclusions

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Perspectives

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- Implementation of a bank of different controller each of them dedicated to a particular kind of fault and design of a switching control law depending on the measured performances.