Design of fault tolerant control for nonlinear systems subject to time varying faults

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Motivations

Objective of diagnosis and fault tolerant control

- To detect, isolate and estimate the actuator fault (diagnosis)
- To modify the control law to accommodate the fault (FTC)
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Difficulties

- Taking into account the system complexity in a large operating range
- Nonlinear behavior of the system
- The faults are time varying
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- Nonlinear behavior of the system
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### Proposed strategy

- Takagi-Sugeno representation of nonlinear systems
- Extension of the existing results on linear systems
- Observer-based fault tolerant control design
- Consideration of an \textit{a priori} model of the fault
1. Takagi-Sugeno approach for modeling
2. Observer and FTC law structures
3. A priori considered fault models
4. Controller design
5. Simulations results
6. Conclusions
Multiple models principle

- Operating range decomposition in several local zones.
- A simple submodel represents the behavior of the system in a specific zone.
- The overall behavior of the system is obtained by aggregating the submodels with adequate weighting functions.

\[ \xi_1(t) \]

\[ \xi_2(t) \]

Operating space

Nonlinear system

Multiple Model representation

zone 1

zone 2

zone 3

zone 4
The main idea of Takagi-Sugeno approach

- Define local models $M_i, \quad i = 1..r$
- Define weighting functions $\mu_i(\xi)$, s.t. $0 \leq \mu_i \leq 1$ and $\sum_{i=1}^{r} \mu_i(\xi) = 1$

→ the global model is obtained by aggregation: $M = \sum_{i=1}^{r} \mu_i(\xi)M_i$
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$\rightarrow$ the global model is obtained by aggregation: $M = \sum_{i=1}^{r} \mu_i(\xi)M_i$

Interests of Takagi-Sugeno approach

- The specific study of the nonlinearities is not required.
- Analysis (stability, performance, robustness, etc.) and design (controller, observer, etc.) are based on the linear submodels.

$\rightarrow$ Possible extension of the theoretical LTI tools for nonlinear systems.
Takagi-Sugeno approach for modeling

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The difficulties

- How many local models?
- How to define the domain of influence of each local model?
- On what variables may depend the weighting functions $\mu_i$?
Takagi-Sugeno approach for modeling

Obtaining a Takagi-Sugeno model

- **Identification approach**
  - Choice of premise variables
  - Choice of the structure of the local models
  - Parameter identification

- **Transformation of an *a priori* known nonlinear model**
  - Linearization around some points
    - how to chose the linearization points?
    - how to define the weighting functions, minimizing the approximation error
  - Nonlinear sector approach

Equivalent rewriting of the model in a compact set of the state space

\[
\begin{align*}
\begin{cases}
x(k+1) &= f(x(k), u(k)) \\
y(k) &= h(x(k), u(k))
\end{cases}
\Rightarrow
\begin{cases}
x(k+1) &= \sum_{i=1}^{r} \mu_i(\xi(k))(A_ix(k) + B_iu(k)) \\
y(k) &= \sum_{i=1}^{r} \mu_i(\xi(k))(C_ix(k) + D_iu(k))
\end{cases}
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\]
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Bouarar et. al. (CRAN)
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\end{cases}
\end{aligned}
\end{equation}

Bouarar \textit{et. al.} (CRAN) Fault tolerant control for nonlinear systems
Takagi-Sugeno system

Reference model

\[
\begin{aligned}
x(k+1) &= \sum_{i=1}^{r} \mu_i(\xi(k)) (A_i x(k) + B_i u(k)) \\
y(k) &= \sum_{i=1}^{r} \mu_i(\xi(k)) (C_i x(k) + D_i u(k))
\end{aligned}
\]

- Interpolation mechanism: \(\sum_{i=1}^{r} \mu_i(\xi(k)) = 1\) and \(0 \leq \mu_i(\xi(k)) \leq 1\), \(\forall k, \forall i \in \{1, ..., r\}\)
- The premise variable \(\xi(k)\) are measurable (like \(u(k), y(k)\)).
Takagi-Sugeno system

Reference model

\[
\begin{aligned}
x(k + 1) &= \sum_{i=1}^{r} \mu_i(\xi(k)) (A_i x(k) + B_i u(k)) \\
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- The premise variable \( \xi(k) \) are measurable (like \( u(k), y(k) \)).

The faulty system

\[
\begin{aligned}
x_f(k + 1) &= \sum_{i=1}^{r} \mu_i(\xi(k)) (A_i x_f(k) + B_i u_f(k) + G_i f(k)) \\
y_f(k) &= \sum_{i=1}^{r} \mu_i(\xi(k)) (C_i x_f(k) + D_i u_f(k) + W_i f(k))
\end{aligned}
\]

- \( f(k) \) represents the fault vector to be detected and accommodated.
Fault tolerant control design

Objectives: estimation + diagnosis + FTC

- estimate the faulty system state \( x_f(k) \)
- estimate the occurring fault \( f(k) \)
- reconfigure the control law for trajectory tracking \( x_f(k) \rightarrow x(k) \)
Fault tolerant control design

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Fault tolerant control scheme

---

Bouarar et. al. (CRAN)
Observer and FTC law structures

Faulty system

\[
\begin{align*}
x_f(k+1) &= \sum_{i=1}^{r} \mu_i(\xi(k)) \left( A_i x_f(k) + B_i u_f(k) + G_i f(k) \right) \\
y_f(k) &= \sum_{i=1}^{r} \mu_i(\xi(k)) \left( C_i x_f(k) + D_i u_f(k) + W_i f(k) \right)
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\end{align*}
\]

PI Observer

\[
\begin{align*}
\hat{x}_f(k+1) &= \sum_{i=1}^{r} \mu_i(\xi(k)) \left( A_i \hat{x}_f(k) + B_i u_f(k) + G_i \hat{f}(k) + H_i^1 (y_f(k) - \hat{y}_f(k)) \right) \\
\hat{f}(k+1) &= \sum_{i=1}^{r} \mu_i(\xi(k)) \left( H_i^2 (y_f(k) - \hat{y}_f(k)) + \hat{f}(k) \right) \\
\hat{y}_f(k) &= \sum_{i=1}^{r} \mu_i(\xi(k)) \left( C_i \hat{x}_f(k) + D_i u_f(k) + W_i \hat{f}(k) \right)
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\end{aligned}
\]

FTC law

\[
u_f(k) = u(k) + \sum_{i=1}^{r} \mu_i(\xi(k)) \left( K_i (x(k) - \hat{x}_f(k)) - \hat{f}(k) \right)\]
Considered faults

**Exponential faults**

\[ f_i(k) = e^{\alpha_i k + \beta_i}, \text{ with } \alpha_i, \beta_i \in \mathbb{R}, i = 1, ..., q \]

\[ \alpha_i = \alpha_{0,i} + \Delta \alpha_i \]

where \( \alpha_{0,i} \) and \( \Delta \alpha_i \) are respectively the nominal and the uncertain parts of \( \alpha_i \)

Let us define:

\[ \alpha = \text{diag}(\alpha_1, ..., \alpha_q) \]

\[ \alpha_0 = \text{diag}(\alpha_{0,1}, ..., \alpha_{0,q}) \]

\[ \Delta \alpha = \text{diag}(\Delta \alpha_1, ..., \Delta \alpha_q) \]

The uncertain part can be bounded as:

\[ (\Delta \alpha)^T \Delta \alpha \leq \lambda \]

where \( \lambda \in \mathbb{R}^{q \times q} \) is a known diagonal positive definite matrix.
Controller design – the exponential fault case

**Estimation errors**

\[
\begin{align*}
  e_p(k) &= x(k) - x_f(k) : \text{state tracking error} \\
  e_s(k) &= x_f(k) - \hat{x}_f(k) : \text{state estimation error} \\
  e_d(k) &= f(k) - \hat{f}(k) : \text{fault estimation error}
\end{align*}
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Controller design – the exponential fault case

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\]

Notation and hypothesis

\[
X_{\mu} = \sum_{i=1}^{r} \mu_i(\xi(k))X_i \\
X_{\mu\mu} = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\xi(k))\mu_j(\xi(k))X_{ij} \\
f_i(k + 1) = e^{\alpha_i} f_i(k)
\]
## Controller design – the exponential fault case

### Estimation errors

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\begin{align*}
e_p(k) &= x(k) - x_f(k) : \text{state tracking error} \\
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f_i(k+1) = e^{\alpha_i}f_i(k)
\]

### Dynamics of the tracking and estimation errors

\[
\begin{pmatrix}
e_p(k+1) \\
e_s(k+1) \\
e_d(k+1)
\end{pmatrix} = \\
\begin{pmatrix}
A_{\mu\mu} - B_\mu K_\mu & -B_\mu K_\mu & -B_\mu \\
0 & A_{\mu} - H_1^{\mu} C_{\mu} & G_{\mu} - H_1^{\mu} W_{\mu} \\
0 & -H_2^{\mu} C_{\mu} & I - H_2^{\mu} W_{\mu}
\end{pmatrix} \\
\begin{pmatrix}
e_p(k) \\
e_s(k) \\
e_d(k)
\end{pmatrix} + \\
\begin{pmatrix}
B_{\mu} - G_{\mu} \\
0 \\
\alpha - I
\end{pmatrix} \begin{pmatrix}
\bar{e}(k) \\
f(k)
\end{pmatrix}
\]
Controller design – the exponential fault case

The tracking, state estimation and fault estimation errors are ruled by:

\[ \bar{e}(k+1) = \bar{A}_{\mu \mu} \bar{e}(k) + \bar{B}_{\mu} f(k) \]

The FTC design reduces to find the controller and observer gains: \( K_i, H_i^1 \) and \( H_i^2 \) satisfying the two main objectives.

### Tracking, state and fault estimation error convergence in the fault free case

Find a positive definite Lyapunov function such that

\[ \Delta V(k) = V(k+1) - V(k) < 0 \]

Here, a quadratic Lyapunov function is chosen:

\[ V(k) = \bar{e}^T(k) X \bar{e}(k), \quad \text{with} \quad X = X^T > 0 \]
The tracking, state estimation and fault estimation errors are ruled by:

\[ \bar{e}(k+1) = \bar{A}_{\mu,\mu} \bar{e}(k) + \bar{B}_\mu f(k) \]

The FTC design reduces to find the controller and observer gains: \( K_i, H^1_i \) and \( H^2_i \)
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**Tracking, state and fault estimation error convergence in the fault free case**

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**Attenuation of the fault effect**

The \( \mathcal{L}_2 \)-gain from the fault \( f(k) \) to the errors \( \bar{e}(k) \) is bounded by a positive \( \gamma \)

\[ \sum_{k=1}^{N} \bar{e}^T(k)Q\bar{e}(k) \leq \gamma^2 \sum_{k=1}^{N} f^T(k)f(k) \]
Summary

The tracking error $e_p(k)$, state and fault estimation errors $e_s(k)$ and $e_d(k)$ must therefore satisfy the following inequality:

$$\bar{e}^T(k+1)X\bar{e}(k+1) - \bar{e}^T(k)X\bar{e}(k) + \bar{e}^T(k)Q\bar{e}(k) - \gamma^2f^T(k)f(k) < 0$$

This inequality is fulfilled if:

$$\begin{pmatrix} Q - X & 0 \\ 0 & -\gamma^2 I \end{pmatrix} + \begin{pmatrix} A^T_{\mu\mu} \\ B_{\mu}^T \end{pmatrix}X\begin{pmatrix} A_{\mu\mu} & B_{\mu} \end{pmatrix} < 0$$
Controller design – the exponential fault case

**Summary**

The tracking error $e_p(k)$, state and fault estimation errors $e_s(k)$ and $e_d(k)$ must therefore satisfy the following inequality:

$$\bar{e}^T(k + 1)X\bar{e}(k + 1) - \bar{e}^T(k)X\bar{e}(k) + \bar{e}^T(k)Q\bar{e}(k) - \gamma^2f^T(k)f(k) < 0$$

This inequality is fulfilled if:

$$\begin{pmatrix} Q - X & 0 \\ 0 & -\gamma^2I \end{pmatrix} + \begin{pmatrix} \bar{A}\mu & \bar{B}\mu \\ B\mu & \end{pmatrix}X\begin{pmatrix} \bar{A}\mu & \bar{B}\mu \end{pmatrix} < 0$$

▶ Chosing the Lyapunov matrix structure: $X = \begin{pmatrix} X_1 & 0 & 0 \\ 0 & X_2 & 0 \\ 0 & 0 & X_3 \end{pmatrix}$

▶ knowing that $\mu_i(\hat{\xi}(k)) \geq 0$

▶ with some matrix manipulations (Schur complement, S-procedure)

→ **sufficient LMI conditions are derived**
Theorem 1

The tracking and estimation errors asymptotically converge to zero in the fault free case and the $L_2$-gain from $f$ to $\bar{e}$ is bounded by $\gamma$, if there exists matrices $X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$, $K_i$, $L^1_i$ and $L^2_i$ and scalars $\gamma$ and $\tau$ such that, for $i = 1, 2, \ldots, r$

$$\begin{pmatrix}
Q_1 - X_1 & 0 & 0 & 0 & \ast & 0 & 0 & 0 & 0 & \ast & 0 \\
0 & Q_2 - X_2 & 0 & 0 & 0 & 0 & \ast & 0 & 0 & \ast & 0 \\
0 & 0 & Q_3 - X_3 & 0 & \ast & \ast & \ast & 0 & 0 & \ast & 0 \\
0 & 0 & 0 & \tau^{-1} \lambda - \bar{\gamma} I & 0 & \ast & \ast & 0 & 0 & \ast & 0 \\
X_1 A_i & 0 & -X_1 B_i & X_1 (B_i - G_i) & -X_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & X_2 A_i - L^1_i C_i & X_2 G_i - L^1_i W_i & 0 & 0 & -X_2 & 0 & 0 & 0 & 0 & 0 \\
0 & L^2_i C_i & X_3 - L^2_i W_i & -X_3 & 0 & 0 & -X_3 & \ast & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & X_1 & 0 & 0 & 0 & -2 I & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & X_1 & 0 & 0 & 0 & -I \\
B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & \ast & 0 & \ast & 0 \\
0 & B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & \ast & \ast & 0 \\
\end{pmatrix} < 0$$

The observer gains and the attenuation level are obtained by:

$$H^1_i = X_2^{-1} L^1_i, \quad H^2_i = X_3^{-1} L^2_i \quad \text{and} \quad \gamma = \sqrt{\bar{\gamma}}$$
Polynomial faults

\[ f_i(k) = a_i k + b_i, \text{ with } a_i, b_i \in \mathbb{R}, i = 1, \ldots, q \]

As well as for exponential function, defining different diagonal matrices, \( a = a_0 + \Delta a \), with \( \Delta a \) verifying:

\[(\Delta a)^T \Delta a \leq \delta\]

where \( \delta \in \mathbb{R}^{q \times q} \) is a known diagonal positive definite matrix.
Controller design – the polynomial fault case

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where \( \delta \in \mathbb{R}^{q \times q} \) is a known diagonal positive definite matrix.

Dynamics of the tracking and estimation errors

Defining \( \bar{e}^T(k) = [e_p^T(k) \ e_s^T(k) \ e_d^T(k)] \), it follows

\[
\bar{e}(k+1) = \begin{pmatrix}
A_{\mu \mu} - B_{\mu} K_{\mu} & -B_{\mu} K_{\mu} & -B_{\mu} \\
0 & A_{\mu} - H_{\mu}^1 C_{\mu} & G_{\mu} - H_{\mu}^1 W_{\mu} \\
0 & -H_{\mu}^2 C_{\mu} & I - H_{\mu}^2 W_{\mu}
\end{pmatrix} \begin{pmatrix}
e_p(k) \\
e_s(k) \\
e_d(k)
\end{pmatrix} + \begin{pmatrix}
B_{\mu} - G_{\mu} \\
0 \\
0
\end{pmatrix} f(k) + \begin{pmatrix}
0 \\
0 \\
a
\end{pmatrix}
\]
Theorem 2

The tracking and estimation errors asymptotically converge to zero in the fault free case and the $L_2$-gain from $f$ to $e$ is bounded by $\gamma$, if there exists matrices $X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$, $K_i$, $L_i^1$ and $L_i^2$ and scalars $\bar{\gamma}$, $\rho$ and $\tau$ such that, for $i = 1, 2, \ldots, r$

\[
\begin{bmatrix}
\Phi_{1,1} & 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & * & 0 & 0 \\
0 & \Phi_{2,2} & 0 & 0 & 0 & * & * & 0 & 0 & * & 0 & 0 \\
0 & 0 & \Phi_{3,3} & 0 & 0 & * & * & * & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\bar{\gamma}I & 0 & * & * & * & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Phi_{5,5} & 0 & 0 & * & 0 & 0 & 0 & 0 \\
X_1 A_i & 0 & -X_1 B_i & \Phi_{6,4} & 0 & -X_1 & 0 & 0 & * & 0 & 0 & 0 \\
0 & \Phi_{7,2} & \Phi_{7,3} & 0 & 0 & 0 & -X_2 & 0 & 0 & 0 & 0 & 0 \\
0 & -L_i^2 C_i & \Phi_{8,3} & 0 & X_3 a_0 & 0 & 0 & -X_3 & 0 & 0 & 0 & * \\
0 & 0 & 0 & 0 & X_1 & 0 & 0 & -2I & 0 & 0 & 0 & 0 \\
B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 & 0 \\
0 & B_i K_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\tau^{-1} I \\
\end{bmatrix} < 0
\]

The observer gains and the attenuation level are obtained by:

\[
H_i^1 = X_2^{-1} L_i^1, \quad H_i^2 = X_3^{-1} L_i^2 \quad \text{and} \quad \gamma = \sqrt{\bar{\gamma}}
\]
Simulation results

**Takagi-Sugeno model**

\[
\begin{align*}
\begin{cases}
    x(k+1) &= \sum_{i=1}^{2} \mu_i(u(k))(A_i x_f(k) + B_i u_f(k) + G_i f(k)) \\
y(k) &= \sum_{i=1}^{2} \mu_i(u(k))(C_i x_f(k) + D_i u_f(k) + W_i f(k))
\end{cases}
\end{align*}
\]

with

\[
\begin{align*}
    A_1 &= \begin{pmatrix} -0.5 & 0.1 \\ -1 & -1 \end{pmatrix} & A_2 &= \begin{pmatrix} 0 & 0.2 \\ -0.45 & -0.7 \end{pmatrix} & B_1 &= \begin{pmatrix} 0.4 \\ 0.5 \end{pmatrix} & B_2 &= \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix} \\
    G_1 &= \begin{pmatrix} 0.2 \\ 0.4 \end{pmatrix} & G_2 &= \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \\
    C_1 &= \begin{pmatrix} 0.2 \\ 0 \end{pmatrix} & C_2 &= \begin{pmatrix} 0.4 \\ 0.1 \end{pmatrix} & W_1 &= -0.3 & W_2 &= -0.4 \\
    \mu_1(u(k)) &= \frac{1 - \tanh(0.5 - u(k))}{2} & \mu_2(u(k)) &= \frac{1 + \tanh(0.5 - u(k))}{2}
\end{align*}
\]

The nominal input signal is: \( u(k) = 0.5 \cos(\sin(0.1k)0.1k) \).
The FT Controller is designed for: \( \alpha_0 = 0.1 \) and \( \lambda = 1.3 \)
The fault affecting the system is: \( f(k) = e^{0.5k-10} \), for \( 9 \leq k \leq 17 \)
Simulation results – state and fault estimation

**Figure:** Fault and its estimation

**Figure:** State estimation errors
Simulation results – trajectory tracking

**Figure:** Reference model states vs. faulty system ones with FTC

**Figure:** Nominal and FTC control inputs

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Conclusions

- Active fault tolerant control law for nonlinear systems represented by a Takagi-Sugeno structure.

Perspectives
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- Study of the unmeasurable premise variable case ($\xi(t) = x(t)$).
- Comparison with multiple integral observer approach
- Implementation of a bank of different controller each of them dedicated to a particular kind of fault and design of a switching control law depending on the measured performances.