State estimation of two-time scale multiple models. Application to wastewater treatment plant

Nagy Kiss, A. M.[a,b,*], Marx, B.[a,*], Mourot, G.[a,*], Schutz, G.[b,**], Ragot, J.[a,*]

[a]*Centre de Recherche en Automatique de Nancy, INPL, UMR 7039 – Nancy-Université, CNRS*
[b]*Centre de Recherche Public "Henri Tudor", Department of Advanced Material and Structures*

Abstract

This paper addresses the state estimation of two-time scale nonlinear systems with an unknown input multi-observer (UIMO). In order to design such an observer, the nonlinear system presented in a singular perturbed form is transformed into an equivalent multiple model with unmeasurable premise variables (UPV) affected by unknown inputs (UI). Then an observer is built and the stability analysis of the state reconstruction error is performed by using the Lyapunov method that leads to the resolution of linear matrix inequalities (LMIs). The performances of the proposed estimation method are highlighted through the application to a wastewater treatment plant model (WWTP).

Keywords: multiple model (MM), unmeasurable premise variables (UPV), state estimation, descriptor systems, activated sludge process

*CRAN, INPL, 2, avenue de la Forêt de Haye, 54516, Vandœuvre-lès-Nancy, France
**CRP "Henri Tudor", 29, Avenue John F. Kennedy, L-1855 Luxembourg - Kirchberg
Email addresses: anca-maria.nagy@ensem.inpl-nancy.fr (Nagy Kiss, A. M.), benoit.marx@ensem.inpl-nancy.fr (Marx, B.), gilles.mourot@ensem.inpl-nancy.fr (Mourot, G.), georges.schutz@tudor.lu (Schutz, G.), jose.ragot@ensem.inpl-nancy.fr (Ragot, J.)

1. Introduction

Nonlinear models are often needed to represent real system behaviors. As a consequence, there is a need to extend linear methods -such as the observer and controller synthesis- to nonlinear systems. In order to achieve this difficult task, the multiple model (MM) approach, proposed by [1], appears to be a powerful tool to deal with nonlinear models. The MM structure allows to represent nonlinear models by aggregating linear submodels with nonlinear weighting functions. Since [1], the MM has received a lot of attention, because it is an efficient way to address nonlinear problems by slightly adapting linear techniques. Moreover, as mentioned in chap. 14 of [2], every nonlinear system can be written as a MM on a compact set of the state space.

A MM can also be obtained by linearization of an existing nonlinear model around one (or several) operating point(s), or around a state trajectory [3]. It can also be derived from system identification, using experimental data [4]. Some drawbacks of these techniques are: the loss of information, the delicate choices of different operating points or trajectories. Finally, the sector nonlinearity approach proposed in [2] allows to exactly rewrite a nonlinear system into a MM form. Nevertheless, the choice of the premise variables has not been systematically realized. A systematic multimodelling procedure with a motivated choice of the premise variable is presented in [5, 6].

With regard to the observer/controller synthesis, the MM approach does not need the Lipschitz hypothesis like [7] (and the references in) does. In [8] (and the references in) some structural constraints for the nonlinear system are requested since for MM approach it is not the case. For the sliding mode observers [9], the chattering effect is an important inconvenience, since a high frequency oscillation is produced. Also, some Lipschitz structural constraints are needed in order to overcome the mentioned inconvenience.

Most of the existing works on MM are dedicated to MM with measurable premise variables (depending on the inputs and/or outputs). Although MM with unmeasurable premise variables (UPV) appears when using the nonlinear sector transformation to obtain a MM from a nonlinear system, only a few works are devoted to the case of unmeasurable premise variables [10, 11, 12]. This lack of result is a motivation for the present work.

Real systems can have multiple time scale dynamics. The theory of singular perturbed systems is often used to highlight the systematic decomposition of the system into various time scales by identifying fast and slow dynamics [13, 14].

In this paper we consider a two-time scale nonlinear system affected by unknown input (UI). Two-time scale systems are represented by a singularly perturbed model, and in the limit case by a descriptor system. Nonlinearities are dealt with by a MM with unmeasurable premise variables. Then an unknown input observer (UIO) for descriptor MM with unmeasurable premise variables is proposed. In [15] a state estimation method for singular MM affected by UI has been presented, but the premise variables were supposed to be measured. The proposed observer was not a singular MM in order to simplify the implementation. Here, the idea of a nonsingular observer is retained, but an extension to MM descriptor with UPV is proposed, which is the main theoretical contribution. In fact many works deal with observer design for singular systems (see the reference books [16, 17] and the references in), some of them are dedicated to singular MM [15], and a few work are devoted to MM with UPV [10, 11, 12], but to the authors' knowledge no work address the observer design for singular MM with UPV. Obviously, state estimation of MM with UPV is not trivial, since the weighting functions used to synthesize the observer cannot depend on the state variables and will involve their estimates. The existence conditions of the observer are expressed through LMIs by using the Lyapunov method and the $\mathcal{L}_2$ approach.

Since the environment protection and biological wastewater treatment are essential, the modeling of wastewater treatment process recently became an active research area. In order to fulfill the requirement of the European Union concerning environmental protection, quality control of the water rejected by the wastewater treatment plants in the nature became an obligation. A Benchmark [18] has been proposed by the European program COST 624 for the evaluation of control strategies in wastewater treatment plants.

The practical contribution of the present paper is to apply the proposed multi-modeling method and observer design to a realistic model of a WWTP. The activated sludge wastewater treatment is a complex chemical and biological process. The variations in wastewater flow rate and its composition, combined with time-varying reactions in a mixed culture of micro-organisms, makes this process nonlinear. Due to the complexity of several proposed models (ASM1 - Activate Sludge Model 1 [19], ASM2 [20], ASM3 [21]), different reduced models have been proposed during the last decades: [22], [23], [24]. In this article, a nonlinear reduced model, inspired by [25] with six states and two time scales, is chosen and equivalently written as a descriptor MM using the method proposed in [5, 6] allowing to avoid the linearization and its drawbacks.

Recently [26] proposed an observer design applied to a reduced nonlinear model of an activated sludge wastewater treatment plant (WWTP). This result is done for Lipschitz nonlinear systems using the LPV approach with one measurable scheduling variable. In the present paper, no Lipschitz assumption is needed and the premise variables are unmeasurable. Moreover, from the practical point of view, some constancy approximations used in [26] -such as autotrophic and heterotrophic biomass concentrations- are not supposed in this article.

The paper is organized as follows. Section 2 presents the essential tools for modeling nonlinear systems with multiple-time scales by using MMs. Section 3 proposes a state estimation method for systems represented by a MM with UPV and UI. Before ending with some conclusions, the real application to a reduced form of the ASM1 model describing the WWTP is detailed in section 4.

Notations. For any square matrix M, $\mathbb{S}(M)$ is defined by $\mathbb{S}(M) = M + M^T$. For any matrix $W \in \mathbb{R}^{m\times n}$, its Moore-Penrose pseudo inverse is denoted W^+ (and satisfies $WW^+W = W$, $W^+WW^+ = W^+$, $(WW^+)^* = WW^+$ and $(W^+W)^* = W^+W$, where W^* represents the conjugate transpose of W), for details on its computation see [27]. The orthogonal of W (verifying $W^\perp W = 0$) is denoted $W^\perp = I - WW^+$ and $\otimes$ is the Kronecker product.

2. Modelling two-time scale nonlinear systems using multiple models

Generally, a dynamic nonlinear system with two-time scales and affected by unknown input can be expressed, using the standard form of the singular perturbed systems, as follows:

$$\epsilon\dot{x}_F(t) = f_F(x_S(t), x_F(t), u(t), d(t), \epsilon) \tag{1a}$$

$$\dot{x}_S(t) = f_S(x_S(t), x_F(t), u(t), d(t), \epsilon) \tag{1b}$$

$$y(t) = g(x(t), u(t), d(t)) \tag{1c}$$

where $x = [x_F,\ x_S]^T \in \mathbb{R}^n$, $x_S \in \mathbb{R}^{n_s}$ and $x_F \in \mathbb{R}^{n_f}$ are respectively the slow and fast state variables, $u \in \mathbb{R}^m$ is the input vector, $y \in \mathbb{R}^\ell$ the output vector,

$d \in \mathbb{R}^q$ is the unknown input, $f_F \in \mathbb{R}^{n_f}$, $f_S \in \mathbb{R}^{n_s}$, $g \in \mathbb{R}^\ell$ and $\epsilon > 0$ the *singular perturbed parameter*.

In the limit case ($\epsilon \to 0$), the degree of the system (1) degenerates from $n_f + n_s$ to n_s and the system is approximated by the following reduced system:

$$\bar{E}\,\dot{x}(t) = f(x,u,d) \tag{2}$$

with:

$$\bar{E} = \begin{bmatrix} 0_{n_f} & 0 \\ 0 & I_{n_s} \end{bmatrix}, \quad f(x,u,d) = \begin{bmatrix} f_F(x_F, x_S, u, d, 0) \\ f_S(x_F, x_S, u, d, 0) \end{bmatrix} \tag{3}$$

The MM allows to represent nonlinear dynamic systems into a convex combination of linear submodels. Let us consider the singularly perturbed system presented under a MM form with partially UPVs and affected by UI:

$$\bar{E}\,\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t), u(t))\,[A_i x(t) + B_i u(t) + E_i d(t)] \tag{4a}$$

$$y(t) = C\,x(t) + G\,d(t) \tag{4b}$$

The matrices A_i, B_i, E_i, C and G are known real and constant matrices of appropriate dimensions. The matrix $\bar{E}$ is a singular matrix (*i.e.* $rank(\bar{E}) \leq n$). In most practical situations, the measurement equation is generally linear and time invariant. The functions $\mu_i(x,u)$ represent the weights of the linear submodels $\{A_i, B_i, E_i\}$ and they have the following convexity properties:

$$\sum_{i=1}^{r} \mu_i(x,u) = 1 \qquad \mu_i(x,u) \geq 0, \qquad \forall (x,u) \in \mathbb{R}^n \times \mathbb{R}^m \tag{5}$$

Remark that the weighting functions $\mu_i(x,u)$ of the MM (4) depend on unmeasurable variables - states $x \in \mathbb{R}^n$ - and on input variables $u \in \mathbb{R}^m$.

In order to obtain the MM form (4), a method giving an equivalent rewriting of the nonlinear system (2) is used as follows (see [5, 6] for further details).

First, (2) is written in a quasi-Linear Parameter Varying (QLPV) form:

$$\bar{E}\dot{x}(t) = A(x,u)x(t) + B(x,u)u(t) + E(x,u)d(t) \tag{6}$$

Second, some nonlinear entries of the matrices A and / or B are considered as "premise variables" and denoted $z_j(x,u)(j = 1, ..., p)$. Several choices of the premise variables are possible due to the existence of different equivalent QLPV forms (for details in the selection procedure see [6]).

Third, a convex polytopic transformation is performed for all p premise variables:

$$z_j(x,u) = F_{j,1}(z_j(x,u))z_{j,1} + F_{j,2}(z_j(x,u))z_{j,2} \tag{7}$$

where

$$\begin{aligned} z_{j,1} &= \max_{x,u} \{z_j(x,u)\} \\ z_{j,2} &= \min_{x,u} \{z_j(x,u)\} \end{aligned} \tag{8}$$

and

$$\begin{aligned} F_{j,1}(z_j(x,u)) &= \frac{z_j(x,u) - z_{j,2}}{z_{j,1} - z_{j,2}} \\ F_{j,2}(z_j(x,u)) &= \frac{z_{j,1} - z_j(x,u)}{z_{j,1} - z_{j,2}} \end{aligned} \tag{9}$$

Due to the structure (7), for p premise variables, $r = 2^p$ submodels will be obtained. By multiplying between themselves the functions F_{j,σ_i^j} the weighting functions are obtained:

$$\mu_i(x,u) = \prod_{j=1}^{r} F_{j,\sigma_i^j}(z_j(x,u)) \tag{10}$$

The indexes σ_i^j ($i = 1, ..., 2^p$ and $j = 1, ..., p$) are equal to 1 or 2 and indicates which partition ($F_{j,1}(z_j(x, u))$ or $F_{j,2}(z_j(x, u))$) of the j^{th} premise variable z_j is involved in the i^{th} submodel. The constant matrices A_i ($i = 1, ..., r$) are obtained by replacing the premise variables z_j in the matrices A with the scalars from (8):

$$A_i \quad = \quad A(z_{1,\sigma_i^1}, ..., z_{p,\sigma_i^p}) \tag{11}$$

The matrices B_i and E_i (for $i = 1, ..., 2^p$) are defined similarly as A_i.

3. Unknown input observer design for singular MM with UPV and UI

The nonlinear system is under the singularly perturbed form (2) or (4a) and the MM interpolation mechanism depends on the UPV. The observer is chosen to be a nonsingular MM, in order to simplify its implementation.

The observer is taken under the following form:

$$\begin{cases} \dot{z}(t) &= \sum\limits_{i=1}^{r} \mu_i(\hat{x}, u) \left[N_i z(t) + G_i u(t) + L_i y(t)\right] \\ \hat{x}(t) &= z(t) + T_2\, y(t) \end{cases} \tag{12}$$

where $\hat{x}(t)$ denotes the state estimate. The state estimation error is given by

$$e(t) \quad = \quad x(t) - \hat{x}(t) \tag{13}$$

The observer design reduces to finding the gains N_i, G_i, L_i and T_2 such that the state estimation error obey to a stable generating system. In order to allow the observer design, a structural condition is needed, which is the analog of the well known UI decoupling condition in the linear case.

Hypothesis 3.1. *The system* (4) *satisfies the following rank condition*

$$rank(W) = rank\left(\begin{bmatrix} W \\ Y \end{bmatrix}\right) \tag{14}$$

where $W \in \mathbb{R}^{(n+l(r+1))\times(n+q(r+1))}$ *and* $Y \in \mathbb{R}^{n\times(n+q(r+1))}$ *are defined by*

$$W = \left[\begin{array}{cc|ccc} \bar{E} & 0_{n\times q} & E_1 & \cdots & E_r \\ C & G & 0_{l\times q} & \cdots & 0_{l\times q} \\ \hline 0_{rl\times n} & 0_{rl\times q} & & I_r \otimes G & \end{array}\right] \tag{15a}$$

$$Y = \left[\begin{array}{cc|c} I_n & 0_{n\times q} & 0_{n\times rq} \end{array}\right] \tag{15b}$$

If the previous assumption is satisfied, the observer gains are obtained by LMI optimization as detailed bellow.

Theorem 3.1. *The observer* (12) *for the system* (4) *is obtained by finding a symmetric and positive definite matrix* $X \in \mathbb{R}^{n\times n}$ *and a matrix* $\tilde{Z} \in \mathbb{R}^{n\times(n+l(r+1))}$ *that minimize the positive scalar* $\bar{\gamma}$ *under the following LMI constraints:*

$$\begin{bmatrix} \Phi_i & (X\,Y\,W^+ + \tilde{Z}\,W^\perp)\,\Omega \\ \Omega^T\,(X\,Y\,W^+ + \tilde{Z}\,W^\perp)^T & -\bar{\gamma}I \end{bmatrix} < 0 \qquad i = 1, ..., r \tag{16}$$

where the matrices Ω *and* Φ_i *are defined by*

$$\Omega = [\; I_n \;\; 0 \mid 0 \;\; \cdots \;\; 0 \;]^T$$

$$\Phi_i = \mathbb{S}(XYW^+Y_i + \tilde{Z}W^\perp Y_i) + I \tag{17}$$

where W *and* Y *are given in* (15) *and the* $Y_i \in \mathbb{R}^{(n+l(r+1))\times n}$ *are defined by*

$$Y_i = \left[\begin{array}{c} A_i \\ 0_{l\times n} \\ \hline v_i \otimes C \end{array}\right], \quad i = 1, ..., r \tag{18}$$

The vector $v_i \in \mathbb{R}^{r\times 1}$ *is the column vector containing* 1 *on the* i^{th} *entry and* 0 *on all the others.*

Once X and $\tilde{Z}$ are obtained from LMI optimization (16)*, the matrices T_1, T_2 and K_i ($i = 1, ..., r$) are given by*

$$\begin{bmatrix} T_1 & T_2 & \bigg| & K_1 & \ldots & K_r \end{bmatrix} = YW^+ + X^{-1}\tilde{Z}W^\perp \tag{19}$$

Finally, the observer gains are determined by

$$N_i = T_1 A_i + K_i C \tag{20}$$

$$G_i = T_1 B_i \tag{21}$$

$$L_i = N_i T_2 - K_i \tag{22}$$

For proof details see Appendix A.

Remark 3.1. *In order to improve (or quantify) the convergence speed of the state estimates to the state variables, a decay rate denoted $\alpha > 0$, can easily be imposed during the observer design (or a posteriori checked) by adding a set of LMI constraints to the previous optimization problem.*

The optimization problem turns into finding $X = X^T > 0$, $G = G^T > 0$ and $\tilde{Z}$ that minimize $\bar{\gamma} > 0$ under the LMI constraints (16) *and*

$$\begin{bmatrix} \bar{\Phi}_i & (XYW^+ + \tilde{Z}W^\perp)\Omega \\ \Omega^T(XYW^+ + \tilde{Z}W^\perp)^T & -G \end{bmatrix} < 0 \tag{23}$$

with

$$\bar{\Phi}_i = \mathbb{S}(XYW^+Y_i + \tilde{Z}W^\perp Y_i) + G + 2\alpha X \tag{24}$$

The decay rate of the state estimation error $e(t)$ is secured by finding a Lyapunov function $V(e(t))$ satisfying $\dot{V}(e(t)) + 2\alpha V(e(t)) < 0$. With $V(e(t)) = e^T(t)Xe(t)$, the constraints (23) *are easily derived.*

4. Application to a wastewater treatment plant

In this section the proposed state estimation method is applied to a model of a wastewater treatment plant -the ASM1 model- in order to reconstruct the slow and fast states. In figure 1 presents a flow chart of the procedures to accomplish for observer design. First the wastewater treatment process is presented.

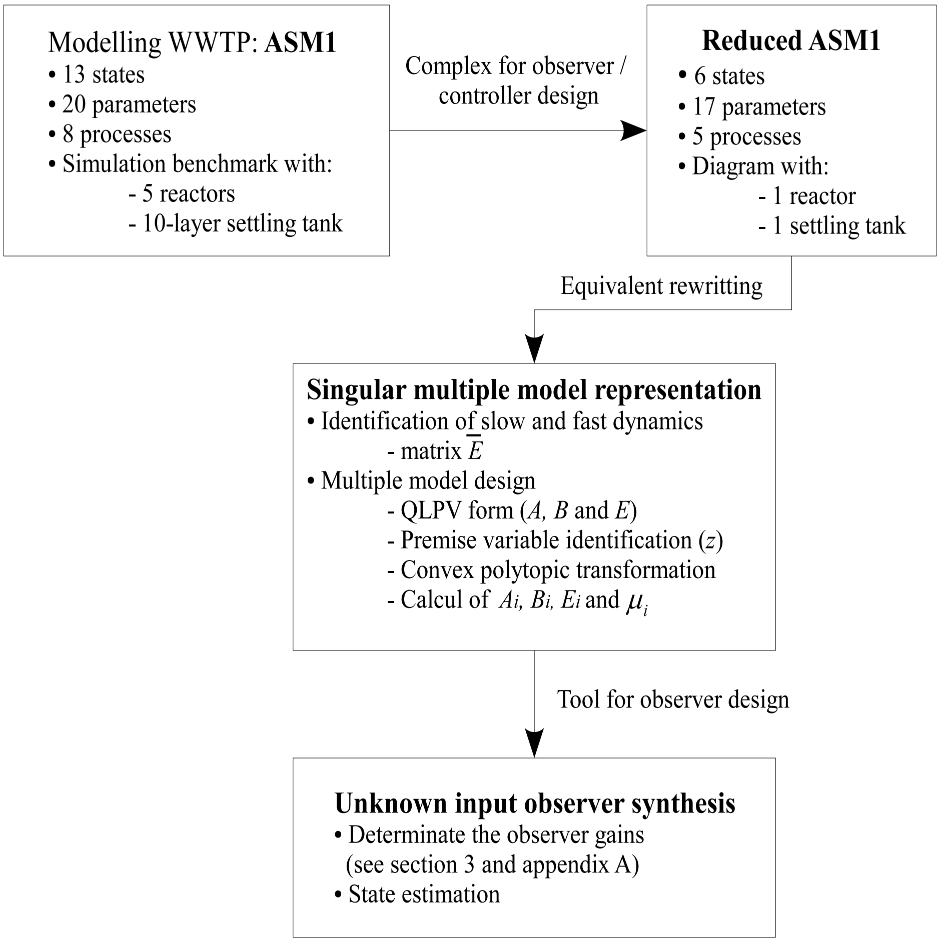

Figure 1: Flow chart resuming the procedures to realize for observer design

4.1. Process description and ASM1 model

The widely used activated sludge wastewater treatment plant consists in mixing used waters with a rich mixture of bacteria in order to degrade the organic matter [19], [28].

In this work, a part of the COST Benchmark is considered. The COST Benchmark has been proposed by the European program COST 624 for the evaluation of control strategies in wastewater treatment plants [18]. The Benchmark is based on the most common wastewater treatment plant: a continuous flow activated sludge plant, performing nitrification and pre-nitrification. A configuration with a single tank with a settler/clarifier was developed. The objective of this study is to use the data generated by the COST 624 benchmark. Note that in order to be closed to the operating condition of the Bleesbrück wastewater plant (in Luxembourg), the measured concentrations of this station are the dissolved oxygen S_O, concentration that is routinely measured in activated sludge wastewater treatment plant, both nitrate S_{NO} and ammonia S_{NH} concentrations can be also measured online. The functioning principle of the process is briefly described after. The simplified

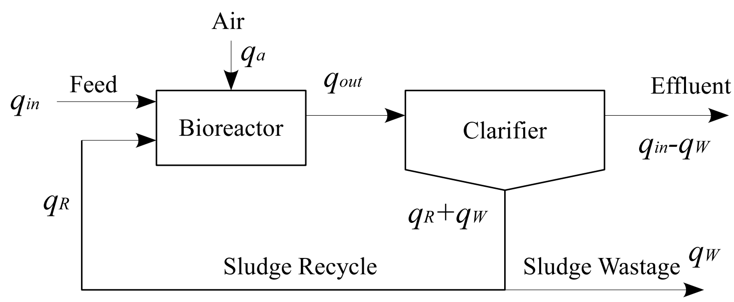

Figure 2: Wastewater treatment process diagram

diagram, given in figure 2, includes a basin of aeration (bioreactor) and a clarifier. In this figure q_{in} represents the fresh water input flow, q_{out} the output flow, q_a the air flow, q_R, q_W are respectively the recycled and the rejected flow. The reactor volume V is assumed to be constant and thus:

$$q_{out}(t) = q_{in}(t) + q_R(t) \tag{25}$$

In general, $q_R(t)$ and $q_W(t)$ represent fractions of the input flow $q_{in}(t)$:

$$q_R(t) = f_R\, q_{in}(t), \quad 1 \leq f_R \leq 2 \tag{26}$$

$$q_W(t) = f_W\, q_{in}(t), \quad 0 < f_W < 1 \tag{27}$$

The polluted water resulting from an extern source circulates in the basin of aeration in which the bacterial biomass degrades the organic pollutant. Micro-organisms bring together in colonial structures called flocs and produce sludges. The mixed liqueur is then sent to the clarifier where the bacterial separation of the purified water and the flocs is made by gravity. A fraction of settled sludges is recycled towards the bioreactor to maintain its capacity of purification. The purified water is thrown back in the natural environment.
In the bioreactor, under the homogeneity hypothesis, the equations of mass conservation for the various constituents, involving the reaction part and the input/output balance, are given by:

$$\dot{x}(t) = r(x(t)) + D(x(t), u(t)) \tag{28}$$

where

$$D(x(t), u(t)) = \frac{q_{in}(t)x_{in}(t) + q_R(t)x_R(t) - q_{out}(t)x(t)}{V} \tag{29}$$

and $x = [S_i, \ \ X_i]^T$ is the vector of soluble (S) and particular (X) constituents, $x_R = [S_{i,R}, \ \ X_{i,R}]^T$ (resp. $x_{in} = [S_{i,in}, \ \ X_{i,in}]^T$) is the vector of constituents corresponding to recycled (resp. input) sludges and i indicates in a general way a particular constituent of the state vector. The reactor homogeneity hypothesis are often represented as [19]:

$$x_{out}(t) \quad = \quad x(t) \tag{30}$$

The clarifier/settler is assumed to be perfect, i.e. no sludge leaves by the overflow the clarifier tank. In this case we can write:

$$S_{i,R}(t) = S_i(t) \tag{31}$$

$$X_{i,R}(t) = \frac{q_{in}(t) + q_R(t)}{q_R(t) + q_W(t)} X_i(t) \tag{32}$$

The reaction part of (28) can be put under the form:

$$r(x(t)) \quad = \quad C_{coef}\, \Phi(x(t)) \tag{33}$$

where the matrix $C_{coef} \in \mathbb{R}^{11\times 8}$ of stoichiometric coefficients and the vector $\Phi(x(t)) \in \mathbb{R}^8$ of process kinetics are explicitly defined in [19].

For observer/controller design, models of lower complexity are required since the full ASM1 model is too complicated and may contain unnecessary informations for control and diagnosis task. Hereafter are given a few elements providing model complexity: the model is defined by thirteen states, the number of biological parameters is large (approximately twenty), the model distinguishes eight bio-chemical processes, the activated sludge process is realized in one/cascade of aerated tanks - reactors - in series followed by one or several settling tanks, the "simulation benchmark" plant design is comprised of five reactors in series with a ten-layer secondary settling tank, etc. Accordingly, a reduced model is generally desired in order to achieve different tasks as observer synthesis.

Several simplifying assumptions have been applied for ASM1 reduction, here a simplification with respect to components is considered [25]. Thus, only the biological removal of carbon and nitrogen from wastewater are considered. It involves the six following components: soluble carbon S_S, particulate X_S, dissolved oxygen S_O, heterotrophic biomass X_{BH}, ammonia S_{NH}, nitrate S_{NO} and autotrophic biomass X_{BA}. On the other hand, the two other nitrogenous fractions - the suspended organic nitrogen (X_{ND}) and the ammonia production from organic nitrogen (S_{ND}) -describing the internal transformation of S_{NH} in the processes of hydrolysis and of ammonification- can be simplified since they are only a small part of nitrogen discharges. The used approximation is to decouple the dynamics of S_{NH} and S_{ND} by simplifying the ammonification kinetic (neglect the internal ammonification process).

The following components are not considered: the inert components (S_I, X_I, X_P), the alkalinity (S_{alk}). Since in practical situation the measurement of the chemical oxygen demand (COD) does not make possible to distinguish between the soluble part S_S and the particulate part X_S [25, 23], a single organic compound (denoted X_{DCO}) will be considered by adding the two compound concentrations. The following state vector is thus considered:

$$x(t) = [X_{DCO}(t),\, S_O(t),\, S_{NH}(t),\, S_{NO}(t),\, X_{BH}(t),\, X_{BA}(t)]^T \qquad (34)$$

The matrix $C_{coef} \in \mathbb{R}^{6\times 5}$ associated to the state vector (34) is:

$$C_{coef} = \begin{bmatrix} -\frac{1}{Y_H} & -\frac{1}{Y_H} & 0 & 1-f_P & 1-f_P \\ \frac{Y_H-1}{Y_H} & 0 & \frac{Y_A-4.57}{Y_A} & 0 & 0 \\ -i_{XB} & -i_{XB} & -i_{XB}-\frac{1}{Y_A} & i_{XB}-f_P\, i_{XP} & i_{XB}-f_P\, i_{XP} \\ 0 & \frac{Y_H-1}{2.86\, Y_H} & \frac{1}{Y_A} & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix} \qquad (35)$$

where $Y_A,\ Y_H,\ f_P,\ i_{XB},\ i_{XP}$ are constant coefficients and the vector $\Phi(x(t)) = [\varphi_1(t), \cdots, \varphi_5(t)]^T \in \mathbb{R}^5$ is given by:

$$
\begin{aligned}
\varphi_1(t) &= \mu_H \frac{X_{DCO}(t)}{K_{DCO} + X_{DCO}(t)} \frac{S_O(t)}{K_{OH} + S_O(t)} X_{BH}(t) \\
\varphi_2(t) &= \mu_H \eta_{NOg} \frac{X_{DCO}(t)}{K_{DCO} + X_{DCO}(t)} \frac{S_{NO}(t)}{K_{NO} + S_{NO}(t)} \frac{K_{OH}}{K_{OH} + S_O(t)} X_{BH}(t) \\
\varphi_3(t) &= \mu_A \frac{S_{NH}(t)}{K_{NH,A} + S_{NH}(t)} \frac{S_O(t)}{K_{O,A} + S_O(t)} X_{BA}(t) \\
\varphi_4(t) &= b_H X_{BH}(t) \\
\varphi_5(t) &= b_A X_{BA}(t)
\end{aligned}
\tag{36}
$$

where $K_{DCO} = \frac{K_S}{f_{SS}}$.

It is supposed that the dissolved oxygen concentration at the reactor input ($S_{O,in}$) is null. In the same time, it can also be supposed that $S_{NO,in} \cong 0$ and $X_{BA,in} \cong 0$, which is in conformity with the European benchmark COST 624 [18].

In practice, and in particular in the Bleesbrück station in Luxembourg, the concentrations $X_{DCO,in}$, $S_{NH,in}$ and $X_{BH,in}$ are not measured online. Thus, a frequently used approximation is to replace these concentrations with their respective daily mean values. A daily mean value will be considered for $X_{DCO,in}$ and the concentrations $X_{BH,in}$ and $S_{NH,in}$ will be considered as unknown inputs. The measurements of the four concentrations in the reactor output (X_{DCO}, S_O, S_{NH} and S_{NO}) are available online. Consequently, the output y, the known input u and the unknown input d vectors are:

$$y(t) = [X_{DCO}(t),\ S_O(t),\ S_{NH}(t),\ S_{NO}(t)]^T \tag{37}$$

$$u(t) = [X_{DCO,in}(t),\ q_a(t)]^T \tag{38}$$

$$d(t) = [S_{NH,in}(t),\ X_{BH,in}(t)]^T \tag{39}$$

Let us consider the explicit form of the ASM1 model (28) and (33) characterized by the reduced state vector (34) and the stoichiometric matrix (35) as follows:

$$
\begin{aligned}
\dot{X}_{DCO}(t) &= -\frac{1}{Y_H}[\varphi_1(t)+\varphi_2(t)] + (1-f_P)(\varphi_4(t)+\varphi_5(t)) + D_1(t)\\
\dot{S}_O(t) &= \frac{Y_H-1}{Y_H}\varphi_1(t) + \frac{Y_A-4.57}{Y_A}\varphi_3(t) + D_2(t)\\
\dot{S}_{NH}(t) &= -i_{XB}[\varphi_1(t)+\varphi_2(t)] - \left(i_{XB}+\frac{1}{Y_A}\right)\varphi_3(t)\\
&\quad +(i_{XB}-f_P\, i_{XP})[\varphi_4(t)+\varphi_5(t)] + D_3(t)\\
\dot{S}_{NO}(t) &= \frac{Y_H-1}{2.86Y_H}\varphi_2(t) + \frac{1}{Y_A}\varphi_3(t) + D_4(t)\\
\dot{X}_{BH}(t) &= \varphi_1(t)+\varphi_2(t)-\varphi_4(t)+D_5(t)\\
\dot{X}_{BA}(t) &= \varphi_3(t)-\varphi_5(t)+D_6(t)
\end{aligned}
\tag{40}
$$

The matrix $D(x(t), u(t))$ (29) expressing the input/output balance is defined by:

$$
\begin{aligned}
D_1(t) &= \tfrac{q_{in}(t)}{V}\left[X_{DCO,in}(t) - X_{DCO}(t)\right]\\
D_2(t) &= \tfrac{q_{in}(t)}{V}\left(-S_O(t)\right) + Kq_a(t)\left[S_{O,sat} - S_O(t)\right]\\
D_3(t) &= \tfrac{q_{in}(t)}{V}\left[S_{NH,in}(t) - S_{NH}(t)\right]\\
D_4(t) &= \tfrac{q_{in}(t)}{V}\left[-S_{NO}(t)\right]\\
D_5(t) &= \tfrac{q_{in}(t)}{V}\left[X_{BH,in}(t) - X_{BH}(t) + f_R\tfrac{1-f_W}{f_R+f_W}X_{BH}(t)\right]\\
D_6(t) &= \tfrac{q_{in}(t)}{V}\left[-X_{BA}(t) + f_R\tfrac{1-f_W}{f_R+f_W}X_{BA}(t)\right]
\end{aligned}
\tag{41}
$$

For numerical applications, the following heterotrophic growth and decay kinetic parameters are used [19]: $\mu_H = 3.733[1/24h]$, $\mu_A = 0.3[1/24h]$, $K_S = 20[g/m^3]$, $f_{SS} = 0.79$, $K_{OH} = 0.2[g/m^3]$, $K_{OA} = 0.4[g/m^3]$, $K_{NO} = 0.5[g/m^3]$, $K_{NH,A} = 1[g/m^3]$, $b_H = 0.3[1/24h]$, $b_A = 0.05[1/24h]$, $\eta_{NOg} = 0.8$. The stoichiometric parameters are $Y_H = 0.6$[g cell formed], $Y_A = 0.24$[g cell formed], $i_{XB} = 0.086$[g N in biomass], $i_{XP} = 0.06$[g N in endogenous mass], $f_P = 0.1$ and the oxygen saturation concentration is $S_{O,sat} = 10[g/m^3]$. The following numerical values are considered here for the fractions f_R and f_W: $f_R = 1.1$ and $f_W = 0.04$. The volume of the tank is $1333[m^3]$.

4.2. Slow and fast variables

In order to obtain the standard singularly perturbed form (2) from (40), the slow and fast dynamics identification and separation are the key points [29, 24, 30]. This is realized by using the mathematical homotopy method for the linearized system, proposed by [14]. This method is essentially based on the eigenvalue analysis of the linearized system and will be briefly presented. Note that the linearized system is only used to identify the slow and fast dynamics, but neither for observer design nor for simulation (for these two points, the nonlinear system will be used).

Let us consider the linearization of the nonlinear system (40) around various equilibrium points (x_0, u_0):

$$\dot{x}(t) = A_0 x(t) + B_0 u(t) \tag{42}$$

where $A_0 = \frac{\partial f(x,u)}{\partial x}\Big|_{(x_0,u_0)}$ and $B_0 = \frac{\partial f(x,u)}{\partial u}\Big|_{(x_0,u_0)}$.

The homotopy method consist in analysing the following homotopy matrix:

$$H(r) = (1-r)A_{0\,D} + rA_0, \qquad 0 \le r \le 1$$

where r is the homotopy parameter, $A_{0\,D}$ is the diagonal matrix of A_0. For $r = 0$, $H(r) = A_{0\,D}$ represents the decoupled system and for $r = 1$, $H(r) = A_0$ represents the coupled system. Considering $\lambda_1 \le \lambda_2 \le ... \le \lambda_n$ the ordered eigenvalues of $H(r)$, the biggest (resp. smallest) eigenvalue corresponds to the slowest (resp. fastest) dynamic. This separation will be made by setting a separation threshold of the time scales, τ, such as: $\lambda_1 \le ... \le \lambda_{n_f} << \tau \le \lambda_{n_f+1} \le ... \le \lambda_n$. The separation of two time scale dynamics of the considered ASM1 (40) is confirmed, using the homotopy method, by the eigenvalues of the homotopy matrix Hr depicted on figure 3 where forty operating points are considered. Five eigenvalues are included between -65 and -1 and another one is around -250.

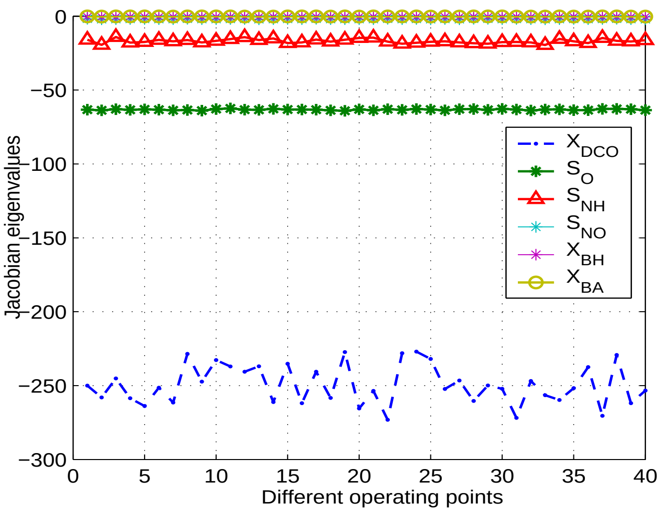

Figure 3: The eigenvalues of the linearized decoupled system

Setting a threshold at $\tau = -90$, it can be deduced that the system has one fast dynamic and five slow dynamics:

$$x_F(t) = X_{DCO}(t) \tag{43a}$$

$$x_S(t) = [S_O(t) \;\; S_{NH}(t) \;\; S_{NO}(t) \;\; X_{BH}(t) \;\; X_{BA}(t)]^T \tag{43b}$$

This separation will be considered for the reduced form of the ASM1 model. The MM design is proposed in the following for state estimation purpose.

4.3. Singular multiple model design

In this section, the nonlinear system is rewritten as a descriptor MM with unmeasurable premise variables (4). Considering the process equations (36), (40) and (41), it is natural to define the following premise variables:

$$
\begin{aligned}
z_1(x,u) &= \frac{q_{in}(t)}{V} && \text{(44a)}\\
z_2(x,u) &= \frac{X_{DCO}(t)}{K_{DCO}+X_{DCO}(t)}\frac{S_O(t)}{K_{OH}+S_O(t)} && \text{(44b)}\\
z_3(x,u) &= \frac{X_{DCO}(t)}{K_{DCO}+X_{DCO}(t)}\frac{S_{NO}(t)}{K_{NO}+S_{NO}(t)}\frac{K_{OH}}{K_{OH}+S_O(t)} && \text{(44c)}\\
z_4(x,u) &= \frac{1}{K_{O,A}+S_O(t)}\frac{S_{NH}(t)}{K_{NH,A}+S_{NH}(t)}X_{BA}(t) && \text{(44d)}
\end{aligned}
$$

The matrices involved in the QLPV form having a similar form as (6) (with matrices $A(x,u)$ - state part, $B(x,u)$ - control part and $E(x,u)$ - unknown inputs part) are expressed by using the premise variables previously defined:

$$
A(x,u) = \begin{bmatrix} a_{11} & 0 & 0 & 0 & a_{15} & a_{16} \\ 0 & a_{21} & 0 & 0 & a_{25} & 0 \\ 0 & a_{32} & -z_1(u) & 0 & a_{35} & a_{36} \\ 0 & a_{42} & 0 & -z_1(u) & a_{45} & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 \\ 0 & a_{62} & 0 & 0 & 0 & a_{66} \end{bmatrix} \tag{45}
$$

$$
B(u) = \begin{bmatrix} z_1(u) & 0 \\ 0 & K\,S_{O,sat} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad E(u) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ z_1(u) & 0 \\ 0 & 0 \\ 0 & z_1(u) \\ 0 & 0 \end{bmatrix} \tag{46}
$$

where:

$$
\begin{aligned}
a_{11}(x,u) &= -z_1(u) \\
a_{15}(x,u) &= -\frac{\mu_H}{Y_H} z_2(x,u) + (1-f_P)\, b_H - \frac{\mu_H\, \eta_{NOg}}{Y_H} z_3(x,u) \\
a_{16}(x,u) &= (1-f_P)\, b_A \\
a_{21}(x,u) &= -z_1(u) - K\, q_a - \frac{4.57 - Y_A}{Y_A} \mu_A\, z_4(x,u) \\
a_{25}(x,u) &= \frac{(Y_H - 1)\mu_H}{Y_H} z_2(x,u) \\
a_{32}(x,u) &= -(i_{XB} + \frac{1}{Y_A})\mu_A\, z_4(x,u) \\
a_{35}(x,u) &= (i_{XB} - f_P\, i_{XP}) b_H - i_{XB}\, \mu_H\, z_2(x,u) - i_{XB}\, \mu_H\, \eta_{NOg}\, z_3(x,u) \\
a_{36}(x,u) &= (i_{XB} - f_P\, i_{XP})\, b_A \\
a_{42}(x,u) &= \frac{1}{Y_A} \mu_A z_4(x,u)
\end{aligned}
$$

$$
\begin{aligned}
a_{45}(x,u) &= \frac{Y_H - 1}{2.86\, Y_H} \mu_H\, \eta_{NOg}\, z_3(x,u) \\
a_{55}(x,u) &= \mu_H\, z_2(x,u) - b_H + z_1(u) \left[\frac{f_W(1+f_R)}{f_R + f_W} - 1 \right] + \mu_H\, \eta_{NOg}\, z_3(x,u) \\
a_{62}(x,u) &= \mu_A\, z_4(x,u) \\
a_{66}(x,u) &= z_1(u) \left[\frac{f_W(1+f_R)}{f_R + f_W} - 1 \right] - b_A
\end{aligned} \tag{47}
$$

The decomposition of the four premise variables (44) is realized by using the convex polytopic transformation (7). The scalars $z_{j,1}$ and $z_{j,2}$ are defined as in (8) and the functions $F_{j,1}(z_j(x,u))$ and $F_{j,2}(z_j(x,u))$ are given by (9) for $j = 1, ..., 4$. By multiplying between themselves the functions $F_{j,\sigma_i^j}(z_j(x,u))$, the $r = 16$ weighting functions $\mu_i(z(x,u))$ are obtained:

$$
\mu_i(z(x,u)) = F_{1,\sigma_i^1}(z_1(u)) F_{2,\sigma_i^2}(z_2(x,u)) F_{3,\sigma_i^3}(z_3(x,u)) F_{4,\sigma_i^4}(z_4(x,u)) \tag{48}
$$

The constant matrices A_i, B_i and E_i (4a) representing the 16 submodels, are defined by using the matrices A, B and E and the scalars z_{j,σ_i^j} $(i = 1, ..., 16,\ j = 1, ..., 4)$:

$$\begin{aligned} A_i &= A(z_{1,\sigma_i^1}, z_{2,\sigma_i^2}, z_{3,\sigma_i^3}, z_{4,\sigma_i^4}) \\ B_i &= B(z_{1,\sigma_i^1}) \\ E_i &= E(z_{1,\sigma_i^1}) \end{aligned} \tag{49}$$

In conformity with (43), the matrix $\bar{E}$ from (4a) is defined by:

$$\bar{E} = \mathbf{diag}(0\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1) \tag{50}$$

In the output $y(t) = C\,x(t) + G\,d(t)$, we have the following definitions:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \qquad G = O_{4\times 2} \tag{51}$$

where $O_{4\times 2}$ is the zero matrix. A zero mean random signal $\delta(t)$ is added on the output (X_{DCO}, S_O, S_{NH} and S_{NO}) to model noise measurements; thus $y(t) = C\,x(t)+\delta(t)$. In conclusion, the reduced ASM1 model having the initial form (40) is rewritten under the singularly perturbed MM form with partially unmeasurable premise variables, as described in (4). The state estimation method, proposed in section 3, can now be applied.

4.4. *Unknown input observer design and state estimation*

As seen is section 3, a classic observer structure (12) based on MM is proposed for the singularly perturbed MM (4). Since the ASM1 model (40) is rewritten into such a form, a similar observer structure is designed. The matrices $\bar{E}$, A_i, B_i, E_i,

C and G involved in (4) and corresponding to the ASM1 model are defined in the previous section 4.3, with equations (49), (50) and (51). In the same time, the weighting functions are defined in (48). After the observer synthesis, with respect to the theorem 3.1, the observer matrices N_i, G_i, L_i and T_2 are deduced.
It should be highlighted that the data used for simulation are generated with the complete ASM1 model with $n = 13$ [28, 22], in order to represent a realistic behavior of a WWTP, whereas the model used for observer design is the reduced one ($n = 6$). Even if the observer design is based on a reduced ASM1 model ($n = 6$) it will be seen that the estimation results are satisfactory. In figure In figure 5 the state variables and their estimates are presented. The $\mathcal{L}_2$ gain of the transfer from $\omega(t)$ to $e(t)$ is bounded by $\gamma = 4.5$. The reconstructed output $\hat{y}(t) = C\,\hat{x}(t)$ of the system is presented in figure 6. One can see that although a noise is added on the output measurements, the output and state estimation are of good quality. A criteria for estimation error is chosen: the VAF (Variance Accounted For) coefficient between two signals. The VAF between y and $\hat{y}$ for the i^{th} component is defined as: $VAF_{y_i} = \left[1 - \frac{var(y_i - \hat{y}_i)}{var(y_i)}\right] 100\%$. The VAF of two signals that are the same is 100%. If they differ, the VAF will be lower. For the state and output estimations it is obtained:

$$\begin{aligned} VAF_x &= [96.23;\;\; 99.63;\;\; 87.95;\;\; 98.88;\;\; 83.94;\;\; 96.12]\% \\ VAF_y &= [92.75;\;\; 96.25;\;\; 71.05;\;\; 95.46]\% \end{aligned}$$

5. Conclusion

In this paper a state estimation method is proposed for nonlinear systems with two time scales. Rewriting the initial nonlinear system, a MM with unmeasurable premise variables is obtained with no information loss. After slow and fast

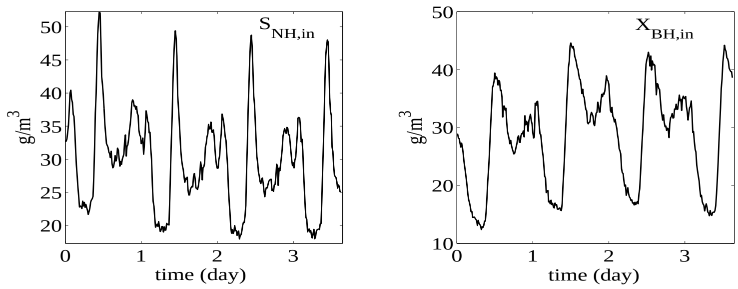

Figure 4: Unknown input $d(t)$ for wastewater treatment plant

dynamics identification, the classical MM form is slightly modified in order to separate the slow and the fast dynamics. The fast dynamics are taken into account as algebraic constraints, then a singular MM is obtained. In order to estimate the state of this kind of system, even if all the inputs are not known, an observer for singular MM with unmeasurable premise variables and affected by unknown inputs is proposed. The observer design is formulated as an LMI optimization problem. An application to a realistic reduced model of a WWTP has been exposed and gives good results even if the observer is designed for a reduced system (with $n = 6$ state variables) and applied to the complete ASM1 (with $n = 13$).

6. Acknowledgments

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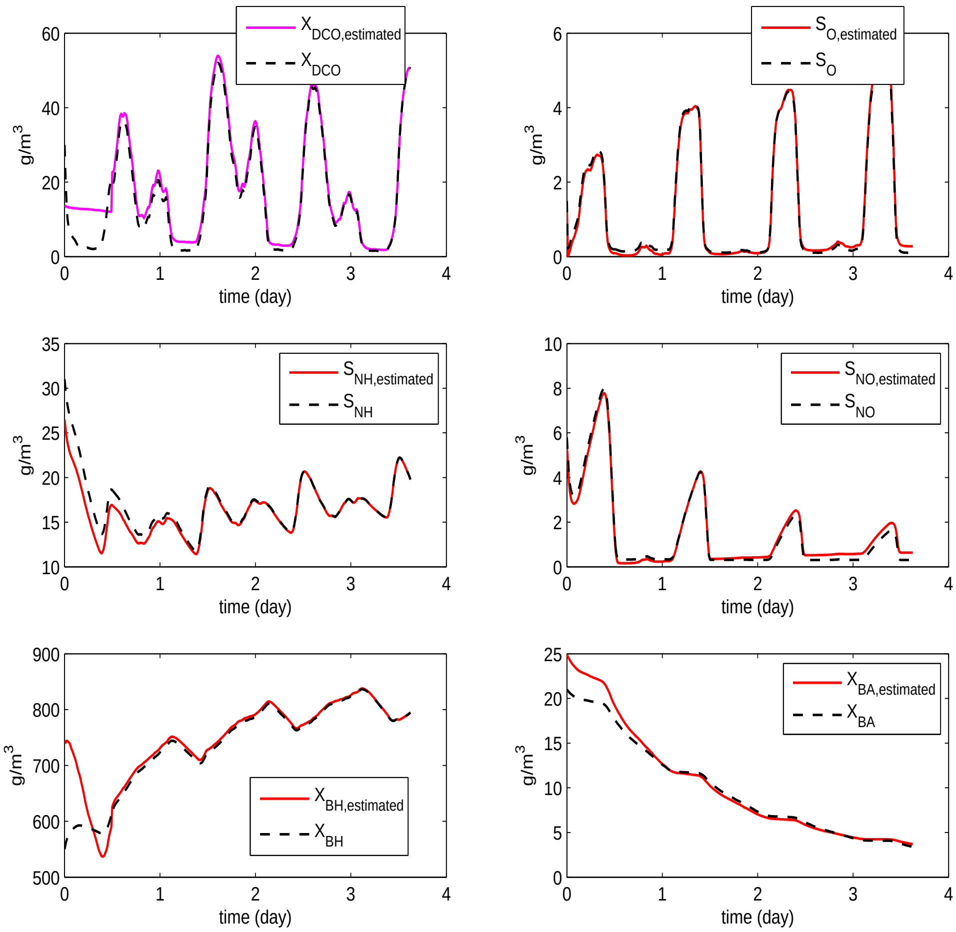

Figure 5: State estimation for the ASM1 model using the observer (12)

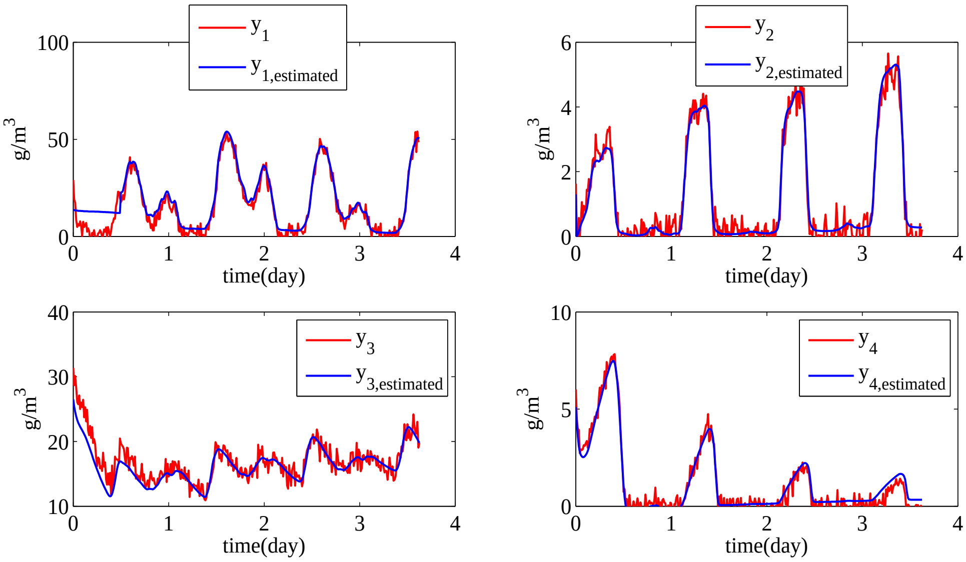

Figure 6: Estimated outputs of the ASM1 model

Appendix A. Proof of Theorem 3.1

Proof Appendix A.1. *The state equation of system* (4) *can be rewritten as a perturbed MM with unmeasurable premise variables as follows*

$$\bar{E}\,\dot{x}(t) \quad = \quad \sum_{i=1}^{r} \mu_i(\hat{x}(t), u(t))\,[A_i x(t) + B_i u(t) + E_i d(t) + \omega(t)] \quad \text{(A.1a)}$$

where the perturbation $\omega(t)$ *has the following form*

$$\omega(t) = \sum_{i=1}^{r} (\mu_i(x(t), u(t)) - \mu_i(\hat{x}(t), u(t)))\,[A_i x(t) + B_i u(t) + E_i d(t)]$$

The MMs (4) *and* (A.1) *are equivalent forms although the estimated state* $\hat{x}$ *appears in the weighting functions of* (A.1)*, like in* (12)*. Obviously, the second MM is more convenient for the observer design. The state estimation error* $e(t)$ *is expressed as*

$$e(t) \quad = \quad (I_n - T_2\,C)\,x(t) - z(t) - T_2 G\,d(t) \qquad \text{(A.2)}$$

If there exists T_1 and T_2 such that

$$I_n - T_2 C = T_1 \bar{E} \tag{A.3}$$

$$T_2 G = 0 \tag{A.4}$$

the state estimation error becomes $e(t) = T_1 \bar{E} x(t) - z(t)$. Its time derivative is:

$$\begin{aligned} \dot{e}(t) \quad = \quad & \sum_{i=1}^{r} \mu_i(\hat{x}, u) \left[N_i\, e(t) + \big(T_1 A_i - N_i T_1 \bar{E} - L_i C \big)\, x(t) \right. \\ & \left. + \left(T_1 B_i - G_i \right) u(t) + \left(T_1 E_i - L_i G \right) d(t) + T_1 \omega(t) \right] \end{aligned} \tag{A.5}$$

If the following conditions hold for $i = 1, ..., r$

$$I_n - T_2\, C = T_1\, \bar{E} \tag{A.6}$$

$$T_2\, G = 0 \tag{A.7}$$

$$0 = T_1 A_i - N_i T_1 \bar{E} - L_i C \tag{A.8}$$

$$0 = T_1 B_i - G_i \tag{A.9}$$

$$0 = T_1 E_i - L_i G \tag{A.10}$$

then, the dynamic of the state estimation error reduces to

$$\dot{e}(t) \quad = \quad \sum_{i=1}^{r} \mu_i(\hat{x}, u) \left[N_i\, e(t) + T_1\, \omega(t) \right] \tag{A.11}$$

Taking (A.11) *into account, the design objectives are, on the one hand, to guarantee the asymptotic convergence of the estimation error when $\omega = 0$ and, on the other hand, to minimize the influence of ω when $\omega(t) \neq 0$. The $\mathcal{L}_2$ approach is used to obtain the observer gains. The bounded real lemma [31] states that $e(t) \to 0$ when $\omega(t) = 0$ and that the $\mathcal{L}_2$ gain from $\omega(t)$ to $e(t)$ is bounded by γ if there exist a symmetric positive definite matrix X such that the following LMIs* *hold*

$$\begin{bmatrix} N_i^T X + X N_i + I & X T_1 \\ T_1^T X & -\gamma^2 I \end{bmatrix} < 0 \qquad i = 1, ..., r \tag{A.12}$$

Therefore, the observer design reduces to satisfy the equalities (A.6) - (A.10) *and the inequalities* (A.12)*. In order to solve* (A.12)*, let us notice that using* (A.6) *and* (A.8) *the matrices* N_i *are defined by*

$$N_i \;=\; T_1 A_i + (N_i T_2 - L_i) C \tag{A.13}$$

Defining $K_i = N_i T_2 - L_i$ *and gathering the searched matrices in* $\Psi \in \mathbb{R}^{n\times(n+l(r+1))}$

$$\Psi = \begin{bmatrix} T_1 & T_2 & \Big| & K_1 & \dots & K_r \end{bmatrix} \tag{A.14}$$

the conditions (A.6) - (A.10) *become*

$$N_i = \Psi\, Y_i \tag{A.15}$$

$$\Psi\, W = Y \tag{A.16}$$

where the matrices Y_i*,* W *and* Y *were defined in* (15) *and* (18)*. If the assumption* (14) *holds, then the solution of the equation* (A.16) *is given by*

$$\Psi = Y\, W^{+} + Z\, W^{\perp} \tag{A.17}$$

where $Z \in \mathbb{R}^{n\times(n+l(r+1))}$ *is an arbitrary matrix and where the other matrices involved were defined in the theorem statement (equation* (15)*). From* (A.15) *and* (A.17)*, the matrices* N_i *are determined by* Z *according to*

$$N_i = Y\, W^{+}\, Y_i + Z\, W^{\perp}\, Y_i \tag{A.18}$$

and the (1,1) block of (A.12)*, denoted* Φ_i*, becomes*

$$\Phi_i = \mathbb{S}\left(X(Y W^{+} Y_i) + X Z W^{\perp} Y_i\right) + I \tag{A.19}$$

The nonlinear term XZ in (A.19) *is linearized by defining a new LMI variable $\tilde{Z}$ given by $\tilde{Z} = X\,Z$. The nonlinear term XT_1, in* (A.12) *is also linearized since it can be expressed by*

$$
\begin{aligned}
XT_1 &= X\,\Psi\,\Omega \\
&= (X\,Y\,W^{+} + \tilde{Z}\,W^{\perp})\,\Omega
\end{aligned}
\tag{A.20}
$$

where $\Omega = \begin{bmatrix} I_n & 0 & \big| & 0 & \cdots & 0 \end{bmatrix}^T$ *and Ψ is given by* (A.17). *Substituting* (A.19) *and* (A.20) *in* (A.12) *and defining $\bar{\gamma} = \gamma^2$ to be minimized, the LMI* (16) *is obtained, which completes the proof.* □

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