On observer design for nonlinear Takagi-Sugeno systems with unmeasurable premise variable

Dalil Ichalal, Benoît Marx, Didier Maquin, José Ragot

***Abstract*— In this paper, we propose a method for state estimation of nonlinear systems represented by Takagi-Sugeno (T-S) models with unmeasurable premise variables. The main result is established using the differential mean value theorem which provides a T-S representation of the differential equation generating the state estimation error. This allows to extend some results obtained in the case of measurable premise variables to the unmeasurable one. Using the Lyapunov theory, stability conditions are obtained and expressed in term of linear matrix inequalities. Furthermore, an extension for observer design with disturbance attenuation performance is proposed. Finally, this approach is illustrated on a DC series motor and compared to the existing approaches.**

***Index Terms*— Takagi-Sugeno systems, state estimation, differential mean value theorem, Lyapunov stability analysis, linear matrix inequality.**

I. Introduction and problem statement

The control and diagnosis of physical systems often require the knowledge of internal variables of the system (state variables). These last are often not accessible to measurement due to economic or technical reasons. In this situation, state observers are used to provide an estimation of these variables from input-output data and a mathematical model describing the behavior of the system. Therefore, the estimation quality necessarily depends on the precision of the model of nonlinear behaviors, leading to complex nonlinear models.

Early work on the state estimation of nonlinear systems dates back to the work of Thau [24] when he proposed an extension of the Luenberger observer [13] to systems with Lipschitz nonlinearities. Sufficient conditions are then obtained for the convergence of the state estimation error toward zero. Thereafter, in [19], an iterative approach is proposed for observer gain design. In [20], Rajamani obtained necessary and sufficient conditions on the observer matrix that ensure asymptotic stability of the observer and proposed a design procedure, grounded on the use of a gradient based optimization method. In [20] is discussed the equivalence between the stability condition and the H_∞ minimization in the standard form, and pointed out that this design method was not solvable since the regularity assumptions are not satisfied. Based on the result of Rajamani [20], [18] proposed a dynamic observer. The problem of regularity assumptions pointed out in [20] is solved by modifying the H_∞ problem. Other classes of nonlinear systems are also studied in the literature to design observers for nonlinear systems, for Linear Parameter Varying systems (LPV) [3] and for bilinear systems.

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In [22], a new structure for nonlinear representation has been introduced. It is based on the decomposition of the operating space of the system in several zones. To each zone is associated a linear model. Thanks to nonlinear functions satisfying the convex sum property, the overall behavior of the nonlinear system can be represented by the following so-called T-S model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector and $y(t) \in \mathbb{R}^p$ represents the output vector. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ are known matrices. The functions $\mu_i(\xi(t))$ are the weighting functions depending on the variables $\xi(t)$ which can be measurable (as the input or the output of the system) or unmeasurable variables (as the state of the system). These functions verify the following properties

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1, \; 0 \le \mu_i(\xi(t)) \le 1 \quad \forall i \in \{1, ..., r\} \quad (2)$$

Obtaining a T-S model can be performed from different methods such as linearization of an available nonlinear model around some operating points and using adequate weighting functions. It can also be obtained by black-box approaches allowing to identify the parameters of the model from input-output data [7]. Finally, an interesting approach to obtain a T-S model is the well-known nonlinear sector transformations [23], [15]. Indeed, this transformation allows to obtain an exact T-S representation of a general nonlinear model with no information loss, in a compact state space.

Takagi-Sugeno model has proved its effectiveness in the study of nonlinear systems. Indeed, it gives a simpler formulation from the mathematical point of view to represent the behavior of nonlinear systems [22]. Thanks to the convex sum property of the weighing functions, it is possible to generalize some tools developed in the linear domain to the nonlinear systems. This representation is very interesting in the sense that it simplifies the stability study of nonlinear systems and the controller/observer design. In [8], [10], [11], the stability and stabilization tools inspired from the study of linear systems are proposed. In [2], [14], the authors worked on the problem of state estimation and diagnosis of T-S systems. The proposed approaches in these last papers rely on the generalization of the classical observers (Luenberger Observer [13] and Unknown Input Observer (UIO) [9]) to the nonlinear domain. Recently, some works are dedicated to the relaxation of the conservatism of the stability condition. For example, in [21], the Polya's theorem is used in order to reduce the conservatism related to the negativity of a sum matrices inequalities. In [12], the authors proposed a new approach for discrete time T-S systems, it is based on the evaluation of the variation of the Lyapunov function between two samples taken at times k and $k+m$ with $m>1$.

In this work the considered premise variable $\xi(t)$ depends on the state of the system which is not totally measurable. The problems of state estimation and diagnosis of nonlinear systems using T-S model approach have been addressed with different methods, but most of the published works have considered T-S models with measurable premise variables [1], [17], [14], [2]. It is clear that the choice of measurable premise variables offers a good simplicity to generalize the methods already developed for linear systems. Contrarily, the problem becomes harder when the premise variables are not measurable. However, this formalism is very important in both the exact representation of nonlinear behaviors by T-S model and in observer based diagnosis for sensor/actuator fault detection and isolation. Indeed in this case, the use of measurable premise variables requires to develop two different models. The first one uses the input $u(t)$ as a premise variable and allows to detect and isolate sensor faults. The second one, using the output $y(t)$ of the system as a premise variable, is dedicated to the detection and isolation of actuator faults. Diagnosis based on a single T-S model with unmeasurable premise variables allows to detect and isolate both actuator and sensor faults using observer banks. Furthermore, the T-S models with unmeasurable premise variables may represent a larger class of nonlinear systems compared to the T-S model with measurable premise variables [25]. Only a few works are devoted to the case of unmeasurable premise variables, nevertheless, we can cite [6], [16], [5] where the authors proposed the fuzzy Thau-Luenberger observer which is an extension of the classical Luenberger observer and, in [25], a filter estimating the state and minimizing the effect of disturbances is proposed.

In this paper a new method is proposed for state estimation of nonlinear systems. It is based on the use of the Takagi-Sugeno model representing the behavior of the nonlinear system. The contribution of this work concerns T-S model with unmeasurable premise variables (e.g. the state of the system), such a model is commonly encountered when using the sector nonlinearity approach [23]. The main results on observer design are given in sections II and III. The first result is devoted to the problem of state estimation and the second one concerns the observer design with disturbance attenuation by minimizing the $\mathcal{L}_2$-gain from energy bounded unknown exogenous disturbances to the state estimation error. Finally, in section IV, an application for state estimation of DC series motor is proposed to illustrate the performances of the proposed methodology.

II. Observer design

Let us consider the T-S system with unmeasurable premise variable, given by

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t))\left(A_i x(t) + B_i u(t)\right) \tag{3}$$

$$y(t) = Cx(t) \tag{4}$$

The following observer is proposed

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} \mu_i(\hat{x}(t))\left(A_i \hat{x}(t) + B_i u(t) + L(y(t) - \hat{y}(t))\right) \tag{5}$$

$$\hat{y}(t) = C\hat{x}(t) \tag{6}$$

Let us remark that the comparison between the state $x(t)$ of the system and the state $\hat{x}(t)$ of the observer seems to be difficult. In order to cope with this difficulty, let us introduce the following matrices

$$A_0 = \frac{1}{r}\sum_{i=1}^{r} A_i, \qquad B_0 = \frac{1}{r}\sum_{i=1}^{r} B_i \tag{7}$$

$$\bar{A}_i = A_i - A_0, \qquad \bar{B}_i = B_i - B_0 \tag{8}$$

Then, it is easy to rewrite the system (3) in the following form

$$\begin{aligned}\dot{x}(t) = {} & A_0 x(t) + B_0 u(t) \\ & + \sum_{i=1}^{r} \mu_i(x(t))\left(\bar{A}_i x(t) + \bar{B}_i u(t)\right)\end{aligned} \tag{9}$$

$$y(t) = Cx(t) \tag{10}$$

where it appears that the matrices A_0 and B_0 play the role of nominal values of the system and $\bar{A}_i$ and $\bar{B}_i$ are variations around these values.

The state equation of the observer (5) can also be presented in the following form

$$\begin{aligned}\dot{\hat{x}}(t) = {} & A_0\hat{x}(t) + B_0 u(t) + L(y(t) - \hat{y}(t)) \\ & + \sum_{i=1}^{r} \mu_i(\hat{x})\left(\bar{A}_i \hat{x}(t) + \bar{B}_i u(t)\right)\end{aligned} \tag{11}$$

$$\hat{y}(t) = C\hat{x}(t) \tag{12}$$

which allows a simpler comparison with the state equation (9)-(10). For that purpose, let us define the state estimation error $e(t)$ by $e(t) = x(t) - \hat{x}(t)$. Using (9)-(10) and (11)-(12), the dynamic of the state estimation error is obtained as follows

$$\dot{e}(t) = (A_0 - LC)e(t) + f(x(t), u(t)) - f(\hat{x}(t), u(t)) \tag{13}$$

where $f(x(t), u(t))$ is denoted $f(z)$ with $z = [x^T(t)\ u^T(t)]^T$ and defined by

$$f(z) = \sum_{i=1}^{r} \mu_i(x(t))(\bar{A}_i x(t) + \bar{B}_i u(t)) \tag{14}$$

Note that the stability analysis of (13) cannot be directly achieved with the help of the tools developed for T-S systems with measurable premise variables. Indeed, the fact that the premise variable is the state of the system leads to a more complex form of the state estimation error (see equation (13)). The key point of the proposed observer design is to obtain a suitable form of the state estimation error in order to re-use the tools proposed for stability and relaxed stability analysis of T-S systems with measurable premise variables. In conclusion, the objective is to find the gain L of the observer (11)-(12) that stabilize (13).

The n different entries of the nonlinear vector function $f(z) : \mathbb{R}^{n+m} \to \mathbb{R}^n$ are denoted $f_i(z)$. It follows

$$f(z) = \begin{bmatrix} f_1(z) & \dots & f_n(z) \end{bmatrix}^T \tag{15}$$

where $f_i(z) : \mathbb{R}^{n+m} \to \mathbb{R}$, $i = 1, ..., n+m$.

Let us denote $e_s(i)$ the vector of $\mathbb{R}^s$ with all entries being null, except the i^{th} being equal to 1 as given below

$$e_s(i) = \begin{pmatrix} \underset{1}{0} & \cdots & \underset{i-1}{0} & \underset{i}{1} & \underset{i+1}{0} & \cdots & \underset{s}{0} \end{pmatrix}^T \tag{16}$$

The function $f(z)$ can be written as follows

$$f(z) = \sum_{i=1}^{n} e_n(i) f_i(z) \tag{17}$$

Theorem 1: Consider $f_i(z) : \mathbb{R}^{n+m} \to \mathbb{R}$. Let $a, b \in \mathbb{R}^{n+m}$. If $f_i(z)$ is differentiable on $[a, b]$ then there exists a constant vector $z^i \in \mathbb{R}^{n+m}$, satisfying $z^i \in]a, b[$ (i.e. $z^i_j \in]a_j, b_j[$, for $j = 1, \dots, n+m$), such that

$$f_i(a) - f_i(b) = \frac{\partial f_i(z^i)}{\partial z}(a - b) \tag{18}$$

Applying the theorem 1 on (17), it is obtained for $a, b \in \mathbb{R}^{n+m}$

$$f(a) - f(b) = \sum_{i=1}^{n} \sum_{j=1}^{n+m} e_n(i) e_{n+m}^T(j) \frac{\partial f_i(z^i)}{\partial z_j}(a - b) \tag{19}$$

Using (19), the dynamic of the state estimation error (13) can be then transformed into

$$\dot{e}(t) = \sum_{i=1}^{n} \sum_{j=1}^{n+m} e_n(i) e_{n+m}^T(j) \frac{\partial f_i(z^i)}{\partial z_j} \left(\begin{pmatrix} x(t) \\ u(t) \end{pmatrix} - \begin{pmatrix} \hat{x}(t) \\ u(t) \end{pmatrix} \right) + (A_0 - LC) e(t) \tag{20}$$

Since $z_j = x_j$, for $1 \le j \le n$, then it follows

$$\dot{e}(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(e_n(i) e_n^T(j) \frac{\partial f_i(z^i)}{\partial x_j} + A_0 - LC \right) e(t) \tag{21}$$

Assumption 1: Assume that $f(z^i)$ is a differentiable function satisfying, for $i = 1, ..., n$ and $j = 1, ..., n$

$$a_{ij} \le \frac{\partial f_i}{\partial x_j}(z^i) \le b_{ij} \tag{22}$$

Each nonlinearity $\frac{\partial f_i}{\partial x_j}(z^i)$ can be represented by

$$\frac{\partial f_i}{\partial x_j}(z^i) = \sum_{l=1}^{2} v_{ij}^l(z^i) \tilde{a}_{ijl} \tag{23}$$

where $\tilde{a}_{ij1} = a_{ij}$ and $\tilde{a}_{ij2} = b_{ij}$ and

$$v_{ij}^1(z^i) = \frac{\frac{\partial f_i}{\partial x_j}(z^i) - a_{ij}}{b_{ij} - a_{ij}} \tag{24}$$

$$v_{ij}^2(z^i) = \frac{b_{ij} - \frac{\partial f_i}{\partial x_j}(z^i)}{b_{ij} - a_{ij}} \tag{25}$$

$$\sum_{l=1}^{2} v_{ij}^l(z^i) = 1, \quad 0 \le v_{ij}^l(z^i) \le 1, \quad l = 1, 2 \tag{26}$$

Using (21) and (23), the dynamic of the state estimation error is represented by

$$\dot{e}(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{l=1}^{2} v_{ij}^l(z^i) (e_n(i) e_n^T(j) \tilde{a}_{ijl} + A_0 - LC) e(t) \tag{27}$$

Using the sector nonlinear transformation method proposed in ([23], chap. 2), it follows

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{l=1}^{2} v_{ij}^l(z^i(t)) e_n(i) e_n^T(j) \tilde{a}_{ijl} = \sum_{i=1}^{q} h_i(\tilde{z}(t)) \mathcal{A}_i \tag{28}$$

where $\tilde{z}(t)$ depends on the $z^i(t)$, $\mathcal{A}_i$ depends on the $\tilde{a}_{ijl}$ and where $q = 2^{n^2}$. Then, the dynamic of the state estimation error is written as

$$\dot{e}(t) = \sum_{i=1}^{q} h_i(\tilde{z}(t)) \Psi_i e(t) \tag{29}$$

with $\Psi_i = \mathcal{A}_i + A_0 - LC$.

The stability of this kind of models is largely studied in the literature. Hence, interesting results exist such as the quadratic stability established by using a quadratic Lyapunov function candidate. Relaxed stability conditions are proposed by using the well-known fuzzy Lyapunov functions in the continuous time case and the Lyapunov function proposed in [11] for the discrete time case.

In this paper, the stability analysis of the system (29) is studied in order to find the gain L. This analysis is performed by using the Lyapunov theorem and a quadratic Lyapunov function, defined by

$$V(e(t)) = e(t)^T P e(t), \quad P = P^T > 0 \tag{30}$$

Theorem 2: The state estimation error asymptotically converges toward zero if there exist a symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$ and a matrix $M \in \mathbb{R}^{n \times n_y}$ such that the following linear matrix inequalities hold $\forall i = 1, ..., q$

$$A_0^T P + P A_0 + \mathcal{A}_i^T P + P \mathcal{A}_i - MC - C^T M^T < 0 \tag{31}$$

The gain of the observer is derived from

$$L = P^{-1} M \tag{32}$$

Proof: Considering Lyapunov function candidate (30) and definition (29), it is straightforward to obtain

$$\dot{V}(e(t)) = \sum_{i=1}^{q} h_i(\tilde{z}(t)) e^T(t) \left(\Psi_i^T P + P \Psi_i \right) e(t) < 0 \tag{33}$$

Then, using the definition of Ψ_i, with properties (2), the proof of the theorem is obvious (see [23] for more details). ■

III. EXTENSION TO $\mathcal{H}_\infty$ PERFORMANCES

The system (3) is now affected by a disturbance input $\omega(t)$

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t)) \left(A_i x(t) + B_i u(t) + E_i \omega(t)\right) \\ y(t) = Cx(t) + W\omega(t) \end{cases} \quad (34)$$

where E_i and W are the incidence matrices defining the influences of $\omega(t)$ on the dynamics and the output of the system. The disturbance vector is assumed to be energy bounded, i.e. $\omega(t) \in \mathcal{L}_2$. As previously, the introduction of the matrices A_0, B_0, $\bar{A}_i$ and $\bar{B}_i$, allows to write the state estimation error as

$$\begin{aligned} \dot{e}(t) = (A_0 - LC)e(t) + \sum_{i=1}^{r} \mu_i(x(t))(E_i - LW)\omega(t) \\ + \sum_{i=1}^{r} \left(\mu_i(x(t)) - \mu_i(\hat{x}(t))\right) \left(\bar{A}_i x(t) + \bar{B}_i u(t)\right) \end{aligned} \quad (35)$$

The application of the previously described method leads to write the state estimation error as a T-S system

$$\dot{e}(t) = \sum_{i=1}^{q} \sum_{j=1}^{r} h_i(z(t)) \mu_j(\hat{x}(t)) \left(\Psi_i e(t) + (E_j - LW)\omega(t)\right) \quad (36)$$

where

$$\Psi_i = A_0 + \mathcal{A}_i - LC \quad (37)$$

Given the system (36), the problem of designing a robust observer (5) is to find the gain L such that

$$\lim_{t \to \infty} e(t) = 0 \quad if \quad \omega(t) = 0 \quad (38)$$

$$\|e(t)\|_2 < \gamma \|\omega(t)\|_2 \quad if \quad \omega(t) \neq 0 \quad (39)$$

where $\gamma > 0$ is a positive scalar representing the attenuation level of the disturbance. To satisfy the constraints (38) and (39), it is sufficient to find a Lyapunov function $V(e(t))$ such that

$$\dot{V}(e(t)) + e(t)^T e(t) - \gamma^2 \omega(t)^T \omega(t) < 0 \quad (40)$$

The following theorem provides sufficient conditions under LMI formulation, for the synthesis of an observer robust to the disturbance $\omega(t)$.

Theorem 3: Given $\gamma > 0$, the robust observer (5) for the system (34) exists, if there exists a matrix $P = P^T > 0 \in \mathbb{R}^{n \times n}$ and a matrix $M \in \mathbb{R}^{n \times n_y}$ such that the following LMIs hold

$$\begin{pmatrix} \Gamma_i & PE_j - MW \\ E_j^T P - W^T M^T & -\gamma^2 I \end{pmatrix} < 0 \quad (41)$$

$$i = 1, \ldots, q \quad / j = 1, \ldots, r$$

where

$$\Gamma_i = A_0^T P + PA_0 + \mathcal{A}_i^T P + P\mathcal{A}_i - MC - C^T M^T + I \quad (42)$$

The observer gain is given by the equation $L = P^{-1} M$.

Proof: The condition (40) expressed with the Lyapunov function $V(e(t)) = e^T(t)Pe(t)$ (with $P = P^T > 0$) and the convex property of the weighting functions lead to the LMIs (41). ■

Remark 1: Note that the observer minimizing the $\mathcal{L}_2$ gain of the transfer from the disturbances $\omega(t)$ to the state estimation error $e(t)$ is obtained by introducing the LMI variable $\bar{\gamma} = \gamma^2$ and by minimizing $\bar{\gamma}$ under the LMI constraints (41), where $\bar{\gamma}$ is substituted to γ^2 in (41).

IV. SIMULATION EXAMPLE: DC SERIES MOTOR

In this section, the proposed observer design is applied to a series DC motor in order to estimate its current $I(t)$ and its angular velocity $\omega(t)$. This kind of motors is generally used in electrical traction due to their high torque and their power autoregulation. The inductor and the armature of this type of motor are connected in series, as shown in Figure 1, hence the term "DC series motor". The parameteres r and

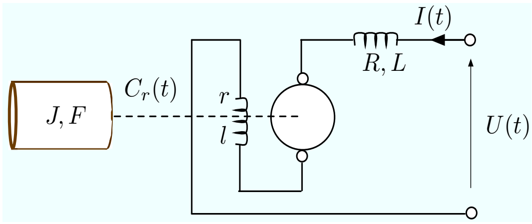

Fig. 1. DC series motor

l respectively represent the resistance and the inductance of the inductor (stator), while R and L respectively represent the armature (rotor) resistance and inductance. The voltage U of the motor must be between 0 and 1000 V and the current I is limited to 1000 A. The nonlinear model is given by the following equations

$$\dot{x}_1(t) = -\frac{F}{J}x_1(t) + K_m \frac{L}{J} x_2(t)^2 - \frac{C_r(t)}{J} \quad (43)$$

$$\dot{x}_2(t) = -\frac{R}{L_t}x_2(t) + K_m \frac{L}{L_t} x_1(t)x_2(t) + \frac{U(t)}{L_t} \quad (44)$$

the state vector is given by $x(t) = [\omega(t) \quad I(t)]^T$ where $\omega(t)$ represents the speed of the motor and $I(t)$ the armature current. We define $R_t = R + r$ and $L_t = L + l$. The motor is powered by a variable voltage which is given by $U(t) = -70\exp(-\frac{t}{35}) + 70$. The resisting torque $C_r(t)$ is usually unknown, but for this example, we consider it as a known input. Both inputs U and C_r are illustrated in Figure 2. The numerical values of the motor parameters are given by $R = 0.001485\Omega$, $r = 0.00989\Omega$, $L = 0.06H$, $K_m = 0.04329$, $J = 30.1N/rad.s^{-1}$, $F = 0.1N/rad.s^{-1}$. The inductance of the rotor is very large compared to the stator one, so we have

$$L >> l \Rightarrow L_t = L = 0.06H \quad (45)$$

The method based on nonlinear sector transformation allows to exactly transform the system (43)-(44) into the T-S model

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i(x)(A_i x(t) + Bu(t)) \quad (46)$$

defined by

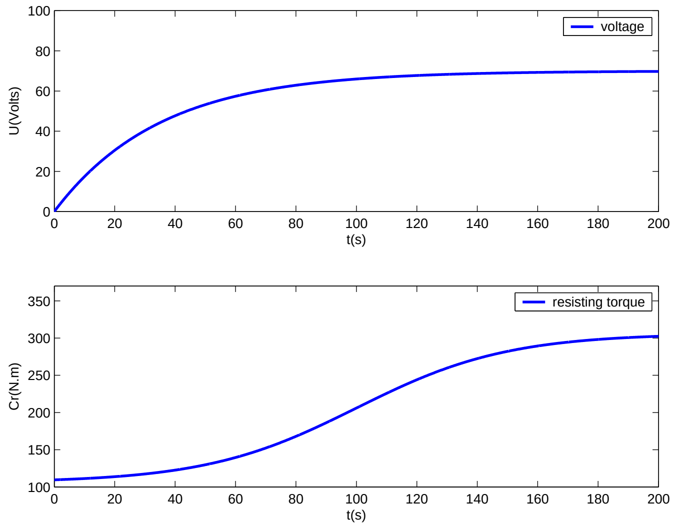

Fig. 2. Inputs of the DC series motor

$$A_1 = \begin{bmatrix} -0.003 & 0.035 \\ -17.317 & -0.412 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.003 & 0 \\ 0 & -0.412 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.033 & 0 \\ 0 & 16.667 \end{bmatrix}$$

The input vector is $u(t) = [C_r(t) \quad U(t)]^T$. The weighting functions are given by the following equations

$$\begin{cases} \mu_1(x_2(t)) = \dfrac{K_m L x_2(t)}{1.039} \\ \mu_2(x_2(t)) = \dfrac{1.039 - K_m L x_2(t)}{1.039} \end{cases} \quad (47)$$

Suppose that only the current I is measured, which gives the output equation

$$y(t) = [0 \quad 1]x(t) \quad (48)$$

The state observer is constructed, applying the proposed method using the differential mean value theorem, and the obtained matrices $\mathcal{A}_i, i = 1, \ldots, 8$ are defined as follows

$$\mathcal{A}_1 = \begin{bmatrix} 0 & -0.016 \\ -8.661 & -3.637 \end{bmatrix} \quad \mathcal{A}_2 = \begin{bmatrix} 0 & 0.052 \\ -8.661 & -3.637 \end{bmatrix}$$

$$\mathcal{A}_3 = \begin{bmatrix} 0 & -0.016 \\ 8.658 & -3.637 \end{bmatrix} \quad \mathcal{A}_4 = \begin{bmatrix} 0 & 0.052 \\ 8.658 & -3.637 \end{bmatrix}$$

$$\mathcal{A}_5 = \begin{bmatrix} 0 & -0.016 \\ -8.661 & 0.476 \end{bmatrix} \quad \mathcal{A}_6 = \begin{bmatrix} 0 & 0.052 \\ -8.661 & 0.476 \end{bmatrix}$$

$$\mathcal{A}_7 = \begin{bmatrix} 0 & -0.016 \\ 8.658 & 0.476 \end{bmatrix} \quad \mathcal{A}_8 = \begin{bmatrix} 0 & 0.052 \\ 8.658 & 0.476 \end{bmatrix}$$

The pairs $(A_0 + \mathcal{A}_i)$ are observable, then the LMIs conditions in the theorem 2 result in the gain L of the observer

$$L = \begin{bmatrix} -0.3891 \\ 51.3786 \end{bmatrix} \quad (49)$$

A first simulation is performed without measurement noises to show the convergence of the observer states to the real states, simulation results are depicted in the figure 3.

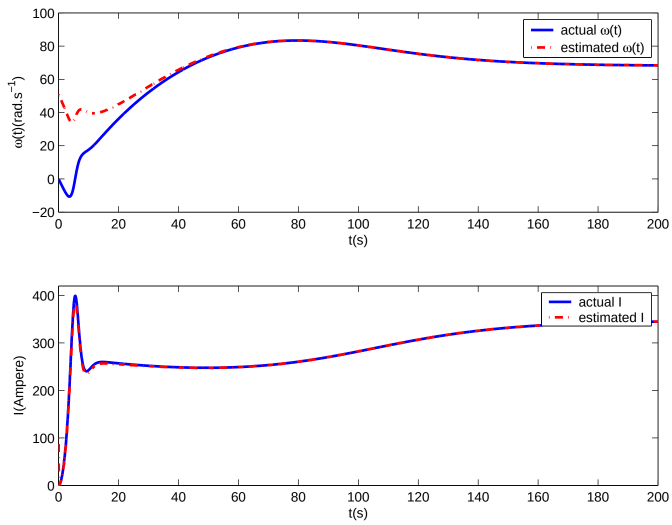

Fig. 3. State of the motor and their estimation

A second simulation is performed by introducing a measurement noise bounded by 20. The goal of this second application is to show that even if the premise variable $z(t) = x_2(t)$ can be measured, using the estimated state in the weighting functions gives better state estimation compared to that obtained when using the noisy measurement $y(t)$ as premise variable. In order to compare the two approaches, two observers are designed. For the first observer design, it is supposed that the weighting functions depend on the measured output $y(t) = x_2(t)$. This observer is constructed using the approach proposed in [17] and the estimation results are displayed on the figure 4. For the design of the second one, it is considered that the weighting functions depend on the state variable $x_2(t)$, then the weighting functions of the observer depend on $\hat{x}_2(t)$. The observer is synthesized by the proposed method using the differential mean value theorem and nonlinear sector transformation and the estimation results are displayed on the figure 5. In figure 4 (resp. 5) the estimated states obtained with the first (resp. second) observer are represented by the red continuous lines while the real growths are depicted by green dashed lines. As a conclusion, the observer using $\hat{x}_2(t)$ as a premise variable gives a better state estimation then the second observer using $y(t)$ as a premise variable. In addition, using the approach developed in [4], the LMIs have no solution because of the high value of the considered Lipschitz constant.

V. Conclusion

In the present article is proposed a new method to design observers for nonlinear systems described by the well-known Takagi-Sugeno systems with unmeasurable premise variables. The method is based on the writing of the system generating the state estimation error in the form of a T-S system. To do that, the differential mean value theorem and

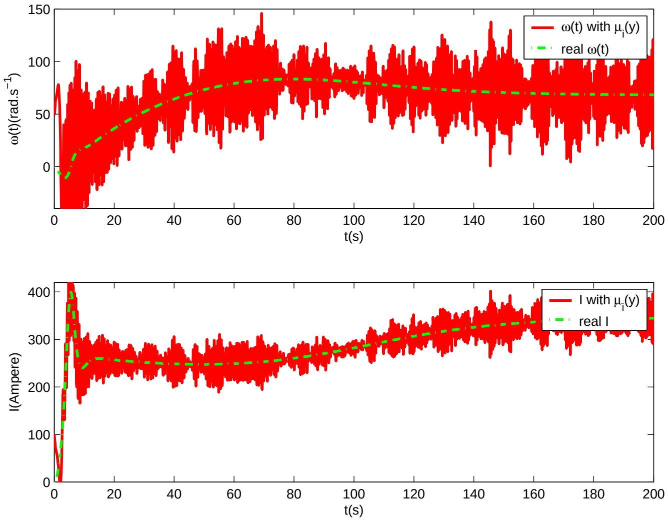

Fig. 4. Observer with measurable premise variable $\xi(t) = y(t)$

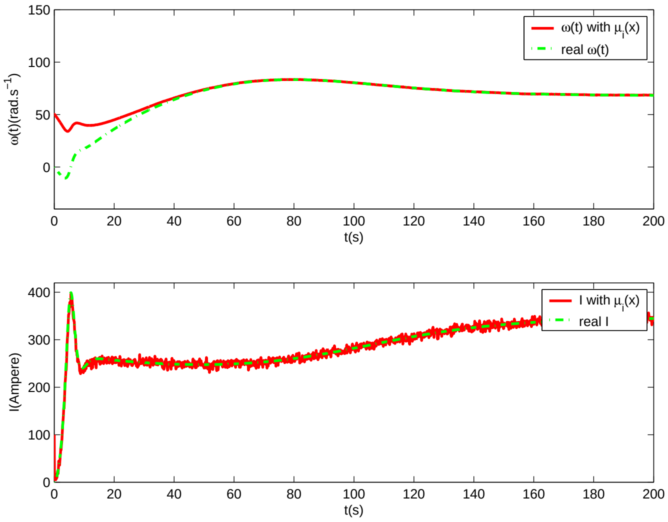

Fig. 5. Observer with estimated premise variable $\xi(t) = \hat{x}_2(t)$

the nonlinear sector transformations are used. After, convergence conditions are obtained by using the Lyapunov theory and a quadratic Lyapunov function. An extension to robust observer design with disturbance attenuation is proposed by minimizing the $\mathcal{L}_2$ gain of the transfer from disturbances to the state estimation error. The stability conditions are expressed in terms of Linear Matrix Inequalities. In order to illustrate the proposed method, an example is devoted to the state estimation of a DC series motor. For future works, we plan to address the problem of relaxed stability condition for observer and controller design for T-S systems with unmeasurable premise variables by using Polya's theorem for example.

REFERENCES

[1] A. Akhenak, M. Chadli, J. Ragot, and D. Maquin. Design of sliding mode unknown input observer for uncertain Takagi-Sugeno model. In *15th Mediterranean Conference on Control and Automation, MED'07*, Athens, Greece, 2007.

[2] A. Akhenak, M. Chadli, J. Ragot, and D. Maquin. Fault detection and isolation using sliding mode observer for uncertain Takagi-Sugeno fuzzy model. In *16th Mediterranean Conference on Control and Automation Congress Centre*, Ajaccio, France, June 25-27 2008.

[3] G. I. Bara, J. Daafouz, F. Kratz, and J. Ragot. Parameter dependent state observer design for affine LPV systems. *International Journal of Control*, 74(16):1601–1611, November 2001.

[4] P. Bergsten and R. Palm. Thau-Luenberger observers for TS fuzzy systems. In *9th IEEE International Conference on Fuzzy Systems, FUZZ IEEE*, San Antonio, TX, USA, 2000.

[5] P. Bergsten, R. Palm, and D. Driankov. Fuzzy observers. In *IEEE International Fuzzy Systems Conference*, Melbourne, Australia, 2001.

[6] P. Bergsten, R. Palm, and D. Driankov. Observers for Takagi-Sugeno fuzzy systems. *IEEE Transactions on Systems, Man, and Cybernetics - Part B: Cybernetics*, 32(1):114–121, 2002.

[7] A. Boukhris, G. Mourot, and J. Ragot. Non-linear dynamic system identification: a multi-model approach. *International Journal of Control*, 72:591–604, 1999.

[8] M. Chadli, D. Maquin, and J. Ragot. Non quadratic stability analysis of Takagi-Sugeno systems. In *IEEE Conference on Decision and Control, CDC'2002*, Las Vegas, Nevada, USA, 2002.

[9] M. Darouach, M. Zasadzinski, and S.J. Xu. Full-order observers for linear systems with unknown inputs. *IEEE Transactions on Automatic Control*, 39(3):606–609, March 1994.

[10] T-M. Guerra, A. Kruszewski, L. Vermeiren, and H. Tirmant. Conditions of output stabilization for nonlinear models in the Takagi-Sugeno's form. *Fuzzy Sets and Systems*, 157(9):1248–1259, May 2006.

[11] A. Kruszewski, R. Wang, and T.M. Guerra. Nonquadratic stabilization conditions for a class of uncertain nonlinear discrete time TS fuzzy models: A new approach. *IEEE Transactions on Automatic Control*, 53(2):606 – 611, March 2008.

[12] A. Kruzewski, R. Wang, and T.M. Guerra. Non-quadratic stabilization conditions for a class of uncertain non linear discrete-time T-S fuzzy models: a new approach. *IEEE Transactions on Automatic Control*, 53(2):606–611, 2008.

[13] D.G. Luenberger. An introduction to observers. *IEEE Transactions on Automatic Control*, 16:596–602, 1971.

[14] B. Marx, D. Koenig, and J. Ragot. Design of observers for takagi sugeno descriptor systems with unknown inputs and application to fault diagnosis. *IET Control Theory and Application*, 1:1487–1495, 2007.

[15] A.M. Nagy, G. Mourot, B. Marx, G. Schutz, and J. Ragot. Model structure simplification of a biological reactor. In *15th IFAC Symposium on System Identification, SYSID'09*, Saint Malo, France, 2009.

[16] R. Palm and P. Bergsten. Sliding mode observers for Takagi-Sugeno fuzzy systems. *9th IEEE International Conference on Fuzzy Systems, FUZZ IEEE, San Antonio, TX, USA*, 2000.

[17] R.J. Patton, J. Chen, and C.J. Lopez-Toribio. Fuzzy observers for non-linear dynamic systems fault diagnosis. In *37th IEEE Conference on Decision and Control*, Tampa, Florida USA, 1998.

[18] A.M. Pertew, H.J. Marquez, and Q. Zhao. H_∞ observer design for lipschitz nonlinear systems. *IEEE Transactions on Automatic Control*, 51:1211–1216, 2006.

[19] S. Raghavan and J. K. Hedrick. Observer design for a class of nonlinear systems. *International Journal of Control*, 59(2):515–528, 1994.

[20] R. Rajamani. Observers for Lipschitz nonlinear systems. *IEEE Transactions on Automatic Control*, 43:397–401, March 1998.

[21] Antonio Sala and Carlos Ario. Asymptotically necessary and sufficient conditions for stability and performance in fuzzy control: Applications of polya's theorem. *Fuzzy Sets and Systems*, 158(24):2671–2686, December 2007.

[22] T. Takagi and M. Sugeno. Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man, and Cybernetics*, 15:116–132, 1985.

[23] K. Tanaka and H.O. Wang. *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. John Wiley and Sons, 2001.

[24] F.E. Thau. Observing the state of non-linear dynamic systems. *International Journal of Control*, 18:471–479, 1973.

[25] J. Yoneyama. H_∞ filtering for fuzzy systems with immeasurable premise variables: an uncertain system approach. *Fuzzy Sets and Systems*, 160(12):1738–1748, 2009.