H_∞ Filtering and state feedback control for discrete-time switched descriptor systems

D. Koenig and B. Marx

Abstract

In this paper, the problem of H_∞ filtering for a class of discrete-time switched systems with unknown inputs is investigated. By using switched Lyapunov functionals, sufficient conditions for the solution of this problem are obtained in terms of linear matrix inequalities (LMIs). Filtering is envisaged both with proportional and proportional integral observers. In addition, the results obtained in observer design are transposed to the controller design for switched descriptor systems. The control of switched uncertain descriptor system is also treated. A numerical example is given to illustrate the presented results.

Index Terms

Switched descriptor systems, H_∞-observer, state feedback control, poly-quadratic stability.

I. Introduction

Many natural and artificial systems and processes encompass several modes of operation with a different dynamical behavior in each mode. In practice this phenomenon may occurs when applying gain-scheduled controllers (where different controllers are designed for each operating point and controllers are switched when the operating conditions change), or with power converter systems (where the switching signal is determined by pulse with PWM modulation), or in fault diagnosis (the healthy and faulty behaviors are modeled by different subsystems, then the fault detection consists in determining the switching time [13]). In order to model different behaviors and the switching from one to another, the switched systems were introduced. Switched systems are defined by a collection of dynamical (linear and/or nonlinear) subsystems together with a switching rule that specifies the switching between these subsystems. A survey on basic problems in switched system stability and design is available in [14] (and the references therein).

Recently, the controller and observer synthesis for switched system has been extensively investigated: in [10] stability for arbitrary switching sequences and construction of stabilizing switching sequences are studied. Piecewise quadratic Lyapunov functions were used in [7] to study the stability of continuous-time hybrid systems or in [11]

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for stability characterization and disturbance attenuation. In [2], stability and static output feedback for discrete-time switched systems were treated with switched Lyapunov functions, [5] studied the output feedback for nonlinear switched systems, and [15] addressed the problems of stability and control of switched systems with varying delays. More recently, the problem of H$_\infty$ filtering for a class of discrete-time switched systems with state delays was treated in [3]. In addition, an extension of the results presented in [2] was proposed for switched uncertain systems with polytopic uncertainty [6]. However, to the best of the authors' knowledge, few results exist for the class of switched descriptor systems. Only, [9], has proposed an extension of the Luenberger Observer for switched descriptor systems.

In this note, an H$_\infty$ filtering and state feedback control are developed by using switched Lyapunov functional approach for discrete-time switched descriptor systems with unknown inputs. Contrary to [9] no base changement is needed. The controller and observer design is reduced to compute each gain by an LMI optimization. It should be pointed that, unlike [2], [4], no additional matrix variables are necessary to solve the linear matrix inequalities (LMI). It results in a simple design procedure for both proportional (P), proportional integral (PI) observers and state feedback controller. Moreover, the state feedback control is extended to the design of a robust state feedback controller for switched uncertain systems with polytopic uncertainty. The ease of use of this method should lead to future works dedicated to the control and estimation for a more general class systems like time-delay switched descriptor systems, nonlinear switched descriptor systems, etc.

This note is organized as follows. Section II presents the problem statement, three different problems are addressed: P observer design, PI observer design and full-state feedback control. The state estimation and H$_\infty$-control is envisaged in section III. Before concluding, the PI observer performance is illustrated through a numerical example in section IV.

Notation 1: $(.)^T$ stands for the transpose matrix and $(*)$ is used for the blocks induced by symmetry, $(.) > 0$ denotes a symmetric positive definite matrix, I is used to denote an identity matrix with appropriate dimension, the notation $\ell_2\,[0,\infty)$ refers to the space of square summable infinite vector sequences with the usual norm $||.||_2$.

II. Preliminaries and problem formulation

Consider the following discrete-time switched descriptor system

$$\begin{aligned} \sum_{j=1}^{N}\alpha_j(k+1)E_jx_{k+1} &= \sum_{i=1}^{N}\alpha_i(k)\left(A_ix_k+B_iu_k+W_iw_k\right) \\ y_k &= \sum_{i=1}^{N}\alpha_i(k)C_ix_k \\ z_k &= \sum_{i=1}^{N}\alpha_i(k)T_ix_k \end{aligned} \tag{1}$$

where E_j, $A_i \in \mathbb{R}^{p\times n}$, $B_i \in \mathbb{R}^{p\times t}$, $W_i \in \mathbb{R}^{p\times q}$, are in the general form and may be rectangular, $C_i \in \mathbb{R}^{m\times n}$, $T_i \in \mathbb{R}^{r\times n}$. The signal $x \in \mathbb{R}^n$ denotes the descriptor vector, $u \in \mathbb{R}^t$ is the control input, $y \in \mathbb{R}^m$ is the measurement output, $z \in \mathbb{R}^r$ is the vector to be estimated or controlled and r satisfies $r \leq n$, $w_k \in \mathbb{R}^q$ is the disturbance input which is assumed to belong to $\ell_2[0,\infty)$. $\alpha_i(k)$ is the switching signal

$$\alpha_i : \mathbb{Z}^+ \to \{0,1\}, \quad \sum_{i=1}^{N} \alpha_i(k) = 1, \quad k \in \mathbb{Z}^+ = \{0,1,\cdots\}$$

which specifies which subsystem will be activated at each time k. For example, if $\alpha_i(k) = 1$, $\alpha_{v\neq i}(k) = 0$, $\alpha_j(k+1) = 1$ and $\alpha_{v\neq j}(k+1) = 0$, then it means that the matrices $(E_j, A_i, B_i, W_i, C_i)$ are activated. In the remaining, it is assumed that C_i, T_i are of full row rank and W_i are of full column rank.

Definition 1: A regular descriptor system is said to be

1) *impulse free* if it exhibits no impulse behavior;
2) *stable* if all finite roots of $\det(zE_i - A_i)$ are inside the unit circle;
3) *finite dynamics detectable* if there exists L_i such that $(E_i, A_i - L_iC_i)$ is regular and stable;
4) *impulse observable* if there exists L_i such that $(E_i, A_i - L_iC_i)$ is regular and impulse free;
5) *finite dynamics stabilizable* if there exists K_i such that $(E_i, A_i + B_iK_i)$ is regular and stable;
6) *impulse controllable* if there exists K_i such that $(E_i, A_i + B_iK_i)$ is regular and impulse free;

In the remaining, the three following problems will be addressed.

Problem 1. Consider the following proportional H_∞-observer for switched descriptor system (1):

$$\begin{cases} \sum_{j=1}^{N} \alpha_j(k+1) E_j \hat{x}_{k+1} = \sum_{i=1}^{N} \alpha_i(k) \left(A_i \hat{x}_k + B_i u_k + K_i^p (y_k - C_i \hat{x}_k)\right) \\ \hat{z}_k = \sum_{i=1}^{N} \alpha_i(k) T_i \hat{x}_k \end{cases} \tag{2}$$

where $K_i^p \in \mathbb{R}^{p\times m}$ are the gains of the observer, $\hat{x} \in \mathbb{R}^n$ is the estimate of x and $\hat{z} \in \mathbb{R}^r$ is the estimate of z. The gains K_i^p are determined such that the following specifications are guaranteed:

S_1) the state estimation error ($e_k = x_k - \hat{x}_k$) is globally asymptotically stable and impulse free, when $w_k = 0$;

S_2) the closed loop transfert function $G_{\tilde{z}w}(z) = T_i \left(zE_j - (A_i - K_i^p C_i)\right)^{-1} W_i$ from the unknown input w_k to the estimation error $\tilde{z}_k = z_k - \hat{z}_k$ guarantees the H_∞-norm constraint $\|G_{\tilde{z}w}(z)\|_\infty < \gamma$ for some prescribed positive scalar γ.

In other words, the observer provides an asymptotical estimation of the state variables in the disturbance free case, and a prescribed precision for the estimation of a linear combination of the state variables in the presence of disturbances.

Problem 2. Consider the following proportional integral observer for switched descriptor system (1):

$$\begin{cases} \sum_{j=1}^{N} \alpha_j(k+1) E_j \hat{x}_{k+1} = \sum_{i=1}^{N} \alpha_i(k) \left(A_i \hat{x}_k + B_i u_k + K_i^p (y_k - C_i \hat{x}_k) + W_i \hat{w}_k\right) \\ \hat{w}_{k+1} = \hat{w}_k - \sum_{i=1}^{N} \alpha_i(k) K_i^I (y_k - C_i \hat{x}_k) \\ \hat{z}_k = \sum_{i=1}^{N} \alpha_i(k) T_i \hat{x}_k \end{cases} \tag{3}$$

where the proportional gains K_i^p and the integral gains K_i^I are determined such that S_1, S_2 and the following specifications are guaranteed:

S_3) the estimation error ($e_k = x_k - \hat{x}_k$) and $w_k - \hat{w}_k$ are globally asymptotically stable when w_k is constant ;

S_4) $\hat{w}_k$ represents the mean value of the unknown input w when it is not constant.

In (1), w can be considered as an unknown parameter or a fault affecting the system. In this case, the augmentation of the observer dimension allows to estimate this parameter if it is constant, or at least its mean value.

Problem 3. Consider the following H_∞ state feedback controller for switched descriptor system (1)

$$u_k = -\sum_{i=1}^{N} \alpha_i (k) K_i x_k \tag{4}$$

where the gains $K_i \in \mathbb{R}^{p\times m}$ are determined such that the following specifications are guaranteed:

S_5) the closed loop system $E_i x_{k+1} = (A_i - B_i K_i) x_k$ is regular, globally asymptotically stable and impulse free, when $w_k = 0$;

S_6) the closed loop transfert function $G_{zw}(z) = T_i (zE_i - (A_i - B_i K_i))^{-1} W_i$ from w_k to the controlled output z_k guarantees the $H\infty$ constraint $\|G_{zw}(z)\|_\infty < \gamma$ for some prescribed positive scalar γ.

The design of a robust H_∞ state feedback controller is also envisaged for uncertain switched descriptor system.

III. State estimation, H_∞-filtering and H_∞-state feedback control

In this section, the main results of this note concerning estimation and control are presented. On the one hand, estimation is performed in the H_∞ framework, in order to minimize the influence of the unknown input on the estimates of the state variables (or a linear combination of the state variables). On the other hand, the designed control laws are state feedback ensuring a bounded H_∞-norm of the transfer from the unknown input to the controlled output. Moreover, controller design is envisaged also for uncertain systems described by polytopic matrices.

A. *Problem 1: Proportional H_∞ observer design*

From (1) and (2), the following equation, ruling the estimation error, is obtained

$$\begin{aligned} \textstyle\sum_{j=1}^{N} \alpha_j (k+1) E_j e_{k+1} &= \textstyle\sum_{i=1}^{N} \alpha_i (k) (A_i - K_i^p C_i) e_k + \sum_{i=1}^{N} \alpha_i (k) W_i w_k \\ \tilde{z}_k &= \textstyle\sum_{i=1}^{N} \alpha_i (k) T_i e_k \end{aligned} \tag{5}$$

Theorem 1: The switched proportional observer (2) for the switched descriptor system (1) guaranteeing S_1 and S_2 exists if the triplets (E_i, A_i, C_i), $i \in \varepsilon = \{1, 2, \cdots N\}$ are both *finite dynamics detectable* and *impulse observable* and if there exist symmetric positive definite matrices $P_1, P_2, ..P_N$ and matrices $U_1, U_2, ..U_N$ satisfying the following LMIs

$$E_i^T P_i E_i \geq 0 \quad \forall i \in \varepsilon = \{1, 2, \cdots N\} \tag{6}$$

$$\begin{bmatrix} P_j - 2P_i & P_i A_i - U_i C_i & P_i W_i \\ * & T_i^T T_i - E_i^T P_i E_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \quad \forall i, j \in \varepsilon \tag{7}$$

The gains of the observers are given by $K_i^p = P_i^{-1} U_i$.

Proof: In order to establish sufficient conditions for the existence of (2) satisfying the specifications S_1 and S_2, the following inequality should be verified [1]

$$H(e, \tilde{z}, w, k) = V_{k+1} - V_k + \tilde{z}_k^T \tilde{z}_k - \gamma^2 w_k^T w_k < 0 \tag{8}$$

where $V_k = e_k^T \left(\sum_{i=1}^N \alpha_i(k) E_i^T P_i E_i\right) e_k$ is a switched parameter dependent Lyapunov function and $P_i > 0$ are positive definite matrices. Computing the difference $V_{k+1} - V_k$, along the solution of (5), the relation (8) becomes

$$H(e,\tilde{z},w,k) = e_{k+1}^T \left(\sum_{j=1}^N \alpha_j(k+1) E_j^T P_j E_j\right) e_{k+1} - e_k^T \left(\sum_{i=1}^N \alpha_i(k) \left(E_i^T P_i E_i - T_i^T T_i\right)\right) e_k - \gamma^2 w_k^T w_k \quad (9)$$

In order to take into account all possible switching laws we consider the case $\alpha_i(k) = 1,\ \alpha_{u\neq i}(k) = 0$ and $\alpha_j(k+1) = 1,\ \alpha_{v\neq j}(k+1) = 0$ where $i,j \in \varepsilon = \{1,2,\cdots N\}$. Therefore (9) and (1) are respectively equivalent to

$$H(e,\tilde{z},w,k) = e_{k+1}^T E_j^T P_j E_j e_{k+1} - e_k^T \left(E_i^T P_i E_i - T_i^T T_i\right) e_k - \gamma^2 w_k^T w_k \quad (10)$$

and

$$\begin{aligned} E_j x_{k+1} &= A_i x_k + B_i u_k + W_i w_k \\ y_k &= C_i x_k \\ z_k &= T_i x_k \end{aligned} \quad (11)$$

With (10) and (11), it follows

$$\begin{aligned} H(e,\tilde{z},w,k) =& e_{k+1}^T E_j^T P_j E_j e_{k+1} - e_k^T \left(E_i^T P_i E_i - T_i^T T_i\right) e_k - \gamma^2 w_k^T w_k \\ =& \left[e_k^T \left(A_i^T - C_i^T K_i^{pT}\right) + w_k^T W_i^T\right] P_j \left[(A_i - K_i^p C_i) e_k + W_i w_k\right] \\ &+ e_k^T \left(T_i^T T_i - E_i^T P_i E_i\right) e_k - \gamma^2 w_k^T w_k \\ =& e_k^T \left[\left(A_i^T - C_i^T K_i^{pT}\right) P_j (A_i - K_i^p C_i) + T_i^T T_i - E_i^T P_i E_i\right] e_k \\ &+ e_k^T \left(A_i^T - C_i^T K_i^{pT}\right) P_j W_i w_k + w_k^T W_i^T P_j (A_i - K_i^p C_i) e_k \\ &+ w_k^T \left[W_i^T P_j W_i - \gamma^2 I\right] w_k < 0 \end{aligned}$$

which can be rewritten as

$$\begin{bmatrix} e_k \\ w_k \end{bmatrix}^T \begin{bmatrix} \beta & \left(A_i^T - C_i^T K_i^{pT}\right) P_j W_i \\ * & -\gamma^2 I + W_i^T P_j W_i \end{bmatrix} \begin{bmatrix} e_k \\ w_k \end{bmatrix} < 0 \quad (12)$$

where $\beta = \left(A_i^T - C_i^T K_i^{pT}\right) P_j (A_i - K_i^p C_i) + T_i^T T_i - E_i^T P_i E_i$. It follows that $H(e,\tilde{z},w,k) < 0$ for any nonzero vector $\begin{bmatrix} e_k^T & w_k^T \end{bmatrix}^T$ if

$$\begin{bmatrix} \beta & \left(A_i^T - C_i^T K_i^{pT}\right) P_j W_i \\ * & -\gamma^2 I + W_i^T P_j W_i \end{bmatrix} < 0 \quad (13)$$

and using the Schur complement formula, (13) becomes

$$\begin{bmatrix} -P_j^{-1} & (A_i - K_i^p C_i) & W_i \\ * & T_i^T T_i - E_i^T P_i E_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0$$

which is equivalent to

$$\begin{bmatrix} -P_i P_j^{-1} P_i & P_i \left(A_i - K_i^p C_i\right) & P_i W_i \\ * & T_i^T T_i - E_i^T P_i E_i - \eta I & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{14}$$

From the following inequality

$$\left(P_i - P_j\right) P_j^{-1} \left(P_i - P_j\right) \geq 0$$

it follows

$$-P_i P_j^{-1} P_i \leq P_j - \left(P_i + P_i\right)$$

Using this inequality, and setting $P_i K_i = U_i$, it appears that (14) is satisfied if the following LMI holds

$$\begin{bmatrix} P_j - 2P_i & P_i \left(A_i - K_i^p C_i\right) & P_i W_i \\ * & T_i^T T_i - E_i^T P_i E_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0$$

■

Remark 1: The H_∞-norm bound γ can be considered as an LMI variable to minimize. In this case, it suffices to set $\gamma^2 = \tilde{\gamma}$ in (7), and to find matrices P_i and U_i that minimize $\tilde{\gamma}$ under the constraints (6) and (7).

Remark 2: If no disturbance affect the system (i.e. $W_i = 0$), the asymptotical stability of the state estimation error is sufficient to ensure state estimation, then the LMI (7) becomes

$$\begin{bmatrix} P_j - 2P_i & P_i A_i - U_i C_i \\ * & -P_i \end{bmatrix} < 0 \tag{15}$$

Remark 3: For $N = 1$ (i.e. in the case of a simple linear descriptor system without switches) and $K_i^p = 0$ (i.e. stability analysis by a H_∞-norm bound condition), we have $\left(A_i - K_i^p C_i\right) = A$, $P_j = P_i = P$, $T_i = T$, $W_i = W$, $E_i = E$ and the inequalities (6) and (12) become

$$\begin{aligned} E^T P E &\geq 0 \\ A^T P A + T^T T - E^T P E - A^T P W \left(\gamma^2 I - W^T P W\right) W^T P A &< 0 \end{aligned}$$

which are equivalent to the LMI's (5a) and (5b) in [16].

B. Problem 2: Proportional Integral H_∞ observer design

Here, the H_∞-filtering is extended to PI observer, in order to estimate the unknown input w. From (1) and (3), the state estimation error is described by the following equation

$$\begin{aligned} \textstyle\sum_{j=1}^{N} \alpha_j \left(k+1\right) E_j^a e_{k+1}^a &= \textstyle\sum_{i=1}^{N} \alpha_i \left(k\right) \left(A_i^a - K_i^a C_i^a\right) e_k^a + \sum_{i=1}^{N} \alpha_i \left(k\right) W_i^a w_k \\ \tilde{z}_k &= \textstyle\sum_{i=1}^{N} \alpha_i \left(k\right) T_i^a e_k^a \end{aligned} \tag{16}$$

where $E_j^a = \begin{bmatrix} E_j & 0 \\ 0 & I \end{bmatrix}$, $A_i^a = \begin{bmatrix} A_i & -W_i \\ 0 & I \end{bmatrix}$, $K_i^a = \begin{bmatrix} K_i^p \\ K_i^I \end{bmatrix}$, $C_i^a = \begin{bmatrix} C_i & 0 \end{bmatrix}$, $W_i^a = \begin{bmatrix} W_i \\ 0 \end{bmatrix}$, $T_i^a = \begin{bmatrix} T_i & 0 \end{bmatrix}$ and $e_k^a = \begin{bmatrix} e_k \\ \hat{w}_k \end{bmatrix}$. For a discussion of the advantages of the PI observer, see [8].

Corollary 1: The switched proportional integral observer (3) for the switched descriptor system (1) guaranteeing S_1, S_2, S_3 and S_4 exists if and only if the triplets $\{E_i^a, A_i^a, C_i^a\}$, $(i \in \varepsilon = \{1, 2, \ldots, N\})$ are both *finite dynamics detectable* and *impulse observable* and if there exist symmetric positive definite matrices $P_1, P_2, \ldots, P_N$ and matrices $U_1, U_2, \ldots, U_N$ satisfying LMI's (6) and (7) where E_i, A_i, C_i, W_i, T_i are replaced by $E_i^a, A_i^a, C_i^a, W_i^a, T_i^a$ respectively.

Proof: The proof is similar to the one of Theorem 1, except that the augmented matrices are used. ■

C. Problem 3: $H\infty$ state feedback controller design

The state feedback controller is envisaged in two different cases: firstly, it is assumed that the model of the switched descriptor system is exactly known, and secondly, the control of uncertain switched descriptor systems is addressed. The uncertainty affecting the different matrices is taken into account by a polytopic representation of the matrices where the weighting parameters are unknown.

1) Switched descriptor systems without uncertainty: The objective is to determine the gains of the control law (4) to be applied to the system (1) in order that the closed-loop system defined by

$$\begin{aligned} \sum_{j=1}^{N} \alpha_j(k+1) E_j x_{k+1} &= \sum_{i=1}^{N} \alpha_i(k) (A_i - B_i K_i) x_k + \sum_{i=1}^{N} \alpha_i(k) W_i w_k \\ z_k &= \sum_{i=1}^{N} \alpha_i(k) T_i x_k \end{aligned} \tag{17}$$

is stable, impulse free, and H_∞-norm bounded.

Theorem 2: The control law (4) for the switched descriptor system (1) guaranteeing S_5 and S_6 exists if and only if the triplets $\{E_i, A_i, B_i\}$, $(i \in \varepsilon = \{1, 2, \cdots N\})$ are both *finite dynamics stabilizable* and *impulse controllable* and if there exist symmetric positive definite matrices $Q_1, Q_2, \ldots, Q_N$ and matrices $U_1, U_2, \ldots, U_N$ satisfying the following LMIs

$$2E_i - Q_i \geq 0 \quad \forall i \in \varepsilon \tag{18}$$

$$\begin{bmatrix} -Q_j & (A_i Q_i - B_i U_i) & W_i & 0 \\ * & -Q_i E_i^T - E_i Q_i + Q_i & 0 & Q_i T_i^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad \forall i, j \in \varepsilon \tag{19}$$

The gains of the controller are given by $K_i = U_i Q_i^{-1}$.

Proof: In order to establish sufficient conditions for the existence of (4) such that the closed-loop system satisfies the specifications S_5 and S_6. A sufficient condition is the existence of a function V_k satisfying the following inequality

$$H(x, z, w, k) = V_{k+1} - V_k + z_k^T z_k - \gamma^2 w_k^T w_k < 0 \tag{20}$$

where $V_k = x_k^T \left(\sum_{i=1}^{N} \alpha_i(k) E_i^T P_i E_i\right) x_k$ and $P_i > 0$. Computing the difference $V_{k+1} - V_k$, along the solution of the closed-loop system (17), the relation (20) becomes

$$\begin{bmatrix} x_k \\ w_k \end{bmatrix}^T \begin{bmatrix} \beta & \left(A_i^T - K_i^T B_i^T\right) P_j W_i \\ * & -\gamma^2 I + W_i^T P_j W_i \end{bmatrix} \begin{bmatrix} x_k \\ w_k \end{bmatrix} < 0 \tag{21}$$

where $\beta = \left(A_i^T - K_i^T B_i^T\right) P_j \left(A_i - B_i K_i\right) + T_i^T T_i - E_i^T P_i E_i$ and $\alpha_i(k) = 1,\ \alpha_{v\neq i}(k) = 0,\ \alpha_j(k+1) = 1,$ $\alpha_{v\neq j}(k+1) = 0,\ i,\ j,\ v \in \varepsilon = \{1, 2, \cdots N\}$.

It follows that $H(x, z, w, k) < 0$ for any nonzero vector $\begin{bmatrix} x_k^T & w_k^T \end{bmatrix}^T$ if

$$\begin{bmatrix} -P_j^{-1} & (A_i - B_i K_i) & W_i \\ * & T_i^T T_i - E_i^T P_i E_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{22}$$

Defining the following nonsingular matrices $X_i \begin{bmatrix} I & 0 & 0 \\ 0 & P_i^{-1} & 0 \\ 0 & 0 & I \end{bmatrix}$, the LMI (22) is equivalent to

$$X_i^T \begin{bmatrix} -P_j^{-1} & (A_i - B_i K_i) & W_i \\ * & T_i^T T_i - E_i^T P_i E_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} X_i < 0$$

or equivalently

$$\begin{bmatrix} -P_j^{-1} & (A_i - B_i K_i) P_i^{-1} & W_i \\ * & P_i^{-1} T_i^T T_i P_i^{-1} - P_i^{-1} E_i^T P_i E_i P_i^{-1} & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0$$

defining $U_i = K_i P_i^{-1}$ and applying Schur complement, it follows

$$\begin{bmatrix} -P_j^{-1} & \left(A_i P_i^{-1} - B_i U_i\right) & W_i & 0 \\ * & -P_i^{-1} E_i^T P_i E_i P_i^{-1} & 0 & P_i^{-1} T_i^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{23}$$

Noticing that

$$(P_i^{-1} E_i^T - P_i^{-1}) P_i (E_i P_i^{-1} - P_i^{-1}) \geq 0$$

implies

$$-P_i^{-1} E_i^T P_i E_i P_i^{-1} \leq -P_i^{-1} E_i^T - E_i P_i^{-1} + P_i^{-1} \tag{24}$$

the LMI (23) is satisfied if the following holds

$$\begin{bmatrix} -P_j^{-1} & \left(A_i P_i^{-1} - B_i U_i\right) & W_i & 0 \\ * & -P_i^{-1} E_i^T - E_i P_i^{-1} + P_i^{-1} & 0 & P_i^{-1} T_i^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{25}$$

Setting $Q_i = P_i^{-1}$, it appears that the LMI (25) implies (19), and thus $H(x, z, w, k) < 0$.

The semi-positiveness of the Lyapunov function V_k is ensured if $E_i^T P_i E_i \geq 0$ is satisfied. In order to obtain LMI's in Q_i, the following inequality is used

$$\left(P_i^{-1} - E_i\right)^T P_i \left(P_i^{-1} - E_i\right) \geq 0 \tag{26}$$

which is equivalent to

$$E_i^T P_i E_i \geq 2E_i - P_i^{-1} = 2E_i - Q_i \tag{27}$$

This last inequality proves that (18) implies $E_i^T P_i E_i \geq 0$, and thus the semi-positiveness of V_k, which achieves the proof. ■

If no unknown input affect the system, the closed-loop system is defined by

$$\sum_{j=1}^{N} \alpha_j (k+1) E_j x_{k+1} = \sum_{i=1}^{N} \alpha_i (k) (A_i - B_i K_i) x_k \tag{28}$$

and only asymptotical stability, and admissibility are needed.

Corollary 2: The closed loop system (28) is regular, globally asymptotically stable and impulse free if and only if the the triplets $\{E_i, A_i, B_i\}$, $(i \in \varepsilon)$ are both *finite dynamics stabilizable* and *impulse controllable* and if there exist symmetric positive definite matrices $Q_1, Q_2, \ldots, Q_N$ and matrices $U_1, U_2, \ldots, U_h$ satisfying

$$2E_i - Q_i \geq 0 \quad \forall i \in \varepsilon \tag{29}$$

$$\begin{bmatrix} -Q_i E_i^T - E_i Q_i + Q_i & Q_i A_i^T - U_i^T B_i^T \\ * & -Q_j \end{bmatrix} < 0 \quad \forall i, j \in \varepsilon \tag{30}$$

and the gains of the control law are given by $K_i = U_i Q_i^{-1}$.

Proof: Define $V_k = x_k^T \sum_{i=1}^{N} \alpha_i (k) E_i^T P_i E_i x_k$ and compute the difference $\Delta V_k = V_{k+1} - V_k$ along the trajectory of the closed-loop system (28). The closed-loop system is regular, globally asymptotically stable and impulse free, under arbitrary switching law, if

$$\Delta V_k = x_k^T \left[(A_i - B_i K_i)^T P_j (A_i - B_i K_i) - E_i^T P_i E_i\right] x_k < 0 \quad \forall i, j \in \varepsilon$$

and the difference ΔV_k is negative definite for any $x_k \neq 0$ if

$$(A_i - B_i K_i)^T P_j (A_i - B_i K_i) - E_i^T P_i E_i < 0 \quad \forall i, j \in \varepsilon \tag{31}$$

which is equivalent, to

$$Q_i (A_i - B_i K_i)^T Q_j^{-1} (A_i - B_i K_i) Q_i - Q_i E_i^T Q_i^{-1} E_i Q_i < 0 \quad \forall i, j \in \varepsilon \tag{32}$$

where $Q_i = P_i^{-1}$. Using the Schur complement and setting $U_i = K_i Q_i$, (32) is equivalent to

$$\begin{bmatrix} -Q_i E_i^T Q_i^{-1} E_i Q_i & Q_i \left(A_i^T - K_i^T B_i^T\right) \\ * & -Q_j \end{bmatrix} < 0 \quad \forall i, j \in \varepsilon \tag{33}$$

From $\left(Q_iE_i^T - Q_i\right)Q_i^{-1}\left(E_iQ_i - Q_i\right) \geq 0$ it follows: $-Q_iE_i^T - E_iQ_i + Q_i \geq -Q_iE_i^TQ_i^{-1}E_iQ_i$. Therefore (30) implies (33) and thus $\Delta V_k < 0$. In addition, since it has already been proved that (29) implies $V_k \geq 0$, the proof is achieved. ∎

Remark 4: In the special case of non descriptor system, i.e. with $E_i = I$, the inequality (33) is equivalent to (18) in [2] (with $S_i = Q_i$, $S_j = Q_j$ and $C_i = I$, since state-feedback is envisaged here, whereas output feedback was treated in [2]).

2) Uncertain switched descriptor systems: Here, the previous results are extended to the following uncertain switched descriptor system

$$\begin{array}{l}\sum_{j=1}^{N}\alpha_j(k+1)E_j(k+1)x_{k+1} = \sum_{i=1}^{N}\alpha_i(k)\left(\hat{A}_i(k)x_k + \hat{B}_i(k)u_k + \hat{W}_i(k)w_k\right)\\ z_k = \sum_{i=1}^{N}\alpha_i(k)T_ix_k\end{array} \tag{34}$$

where $\hat{A}_i(k)$, $\hat{B}_i(k)$, $\hat{W}_i(k)$ are polytopic matrices, described by

$$\begin{aligned}\hat{A}_i(k) &= \sum_{l_1=1}^{n_{A_i}}\xi_{il_1}^{A}(k)A_{il_1},\\ \hat{B}_i(k) &= \sum_{l_2=1}^{n_{B_i}}\xi_{il_2}^{B}(k)B_{il_2}, \quad \hat{W}_i(k) = \sum_{l_3=1}^{n_{W_i}}\xi_{il_3}^{W}(k)W_{il_3}\end{aligned}$$

$$\begin{array}{l}\alpha_i : \mathbb{Z}^+ \to \{0,1\}\,, \quad \sum_{i=1}^{N}\alpha_i(k) = 1, \quad k \in \mathbb{Z}^+ = \{0,1,\cdots\}\\ \sum_{l_1=1}^{n_{A_i}}\xi_{il_1}^{A}(k) = \sum_{l_2=1}^{n_{B_i}}\xi_{il_2}^{B}(k) = \sum_{l_3=1}^{n_{W_i}}\xi_{il_3}^{W}(k) = 1\\ \xi_{il_1}^{A}(k), \xi_{il_2}^{B}(k), \xi_{il_3}^{W}(k) > 0\end{array}$$

Each uncertain matrix $\hat{A}_i(k)$, $\hat{B}_i(k)$ and $\hat{W}_i(k)$ is described by a polytope of known vertices. The positive integers n_{A_i}, n_{B_i} and n_{W_i} are the numbers of vertices of the domains in which the uncertain matrices evolve. The positive real numbers $\xi_{il_{(.)}}^{(.)}(k)$ are the time varying, unknown weighting parameters describing the evolution of the uncertain matrices.

Theorem 3: The robust control law (4) for the uncertain switched descriptor system (34) guaranteeing S_5 and S_6 exists if and only if the triplets $\{E_i, A_{il_1}, B_{il_2}\}\,,\ (i \in \varepsilon = \{1,2,\ldots,N\})$ are both *finite dynamics stabilizable* and *impulse controllable* and if there exist symmetric positive definite matrices $Q_1, Q_2, \ldots, Q_N$ and matrices $U_1, U_2, \ldots, U_N$ satisfying the following LMIs

$$2E_i - Q_i \geq 0 \quad \forall i \in \varepsilon \tag{35}$$

$$\begin{bmatrix} -Q_j & (A_{il_1}Q_i - B_{il_2}U_i) & W_{il_3} & 0\\ * & -Q_iE_i^T - E_iQ_i + Q_i & 0 & Q_iT_i^T\\ * & * & -\gamma^2 I & 0\\ * & * & * & -I\end{bmatrix} < 0 \quad \forall i,j \in \varepsilon \tag{36}$$

$$l_1 = 1,\ldots,n_{A_i}, l_2 = 1,\ldots,n_{B_i}, l_3 = 1,\ldots,n_{W_i}$$

The gains of the controller are given by $K_i = U_iQ_i^{-1}$.

Proof: The closed-loop dynamic system controlled by the switched state feedback (4) is described by the following system

$$\begin{aligned} \sum_{j=1}^{N} \alpha_j(k+1)E_j x_{k+1} &= \sum_{i=1}^{N} \alpha_i(k) \left[\left(\hat{A}_i - \hat{B}_i K_i \right) x_k + \hat{W}_i w_k \right] \\ z_k &= \sum_{i=1}^{N} \alpha_i(k) T_i x_k \end{aligned} \tag{37}$$

or equivalently by the following system

$$\begin{aligned} \sum_{j=1}^{N} \alpha_j(k+1)E_j x_{k+1} &= \sum_{i=1}^{N} \alpha_i(k) \sum_{l_1=1}^{n_{A_i}} \xi_{il_1}(k) \sum_{l_2=1}^{n_{B_i}} \xi_{il_2}(k) \sum_{l_3=1}^{n_{W_i}} \xi_{il_3}(k) \left[\left(A_{il_1} - B_{il_2} K_i \right) x_k + W_{il_3} w_k \right] \\ z_k &= \sum_{i=1}^{N} \alpha_i(k) T_i x_k \end{aligned} \tag{38}$$

Let define the following switched parameter Lyapunov function

$$V_k = x_k^T \sum_{i=1}^{N} \alpha_i(k) E_i^T P_i E_i x_k \tag{39}$$

Now, in order to take into account all possible switching laws we consider the case $\alpha_i(k) = 1$, $\alpha_{v \neq i}(k) = 0$, $\alpha_j(k+1) = 1$ and $\alpha_{v \neq j}(k+1) = 0$, where $i, j, v \in \varepsilon = \{1, 2, \cdots N\}$. Then the system (37), can be written as

$$\begin{aligned} E_j x_{k+1} &= \left(\hat{A}_i - \hat{B}_i K_i \right) x_k + \hat{W}_i w_k \\ z_k &= T_i x_k \end{aligned} \tag{40}$$

Computing the sufficient condition for H_∞-norm bound (20) with the Lyapunov function (39) along the trajectory of (40), the following sufficient condition for S_5 and S_6 is obtained

$$H(x, z, w, k) = \quad x_{k+1}^T E_j^T P_j E_j x_{k+1} - x_k^T E_i^T P_i E_i x_k + x_k^T T_i^T T_i x_k - \gamma^2 w_k^T w_k \tag{41}$$

$$\begin{aligned} = \; & x_{k+1}^T \left(\left(\hat{A}_i - \hat{B}_i K_i \right)^T + w_k^T \hat{W}_i^T \right) P_j \left(\left(\hat{A}_i - \hat{B}_i K_i \right) x_k + \hat{W}_i w_k \right) \\ & - x_k^T E_i^T P_i E_i x_k + x_k^T T_i^T T_i x_k - \gamma^2 w_k^T w_k < 0 \end{aligned} \tag{42}$$

Applying the same computations than in the proof of theorem 2, and setting $U_i = K_i P_i^{-1}$ and $Q_i = P_i^{-1}$, the following sufficient condition for $H(x, z, w, k) < 0$ can be obtained

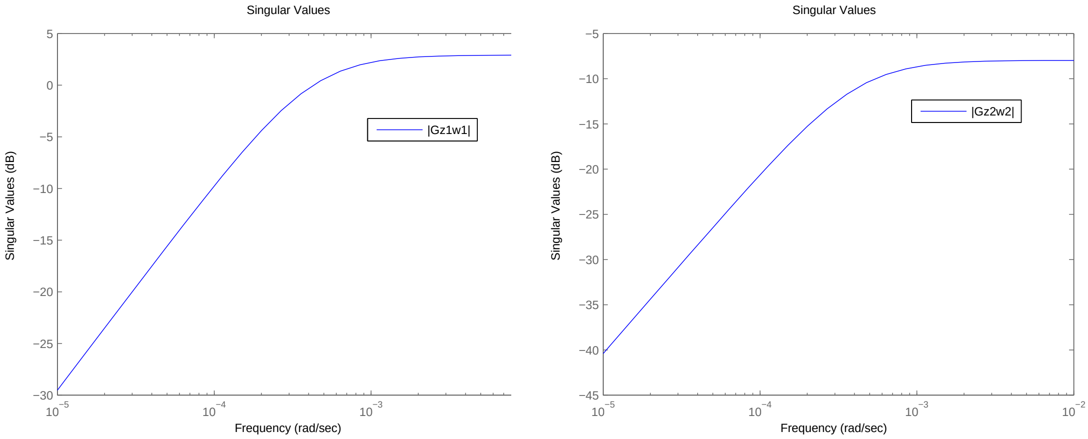
$$\begin{bmatrix} -P_j^{-1} & \left(\hat{A}_i P_i^{-1} - \hat{B}_i U_i \right) & \hat{W}_i & 0 \\ * & -P_i^{-1} E_i^T - E_i P_i^{-1} + P_i^{-1} & 0 & P_i^{-1} T_i^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{43}$$

or equivalently

$$\sum_{l_1=1}^{n_{A_i}} \sum_{l_2=1}^{n_{B_i}} \sum_{l_3=1}^{n_{W_i}} \xi_{il_1}^A(k) \xi_{il_2}^B(k) \xi_{il_3}^W(k) \begin{bmatrix} -Q_j & \left(A_{il_1} Q_i - B_{il_2} U_i \right) & W_{il_3} & 0 \\ * & -Q_i E_i^T - E_i Q_i + Q_i & 0 & Q_i T_i^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{44}$$

Since $\xi_{il_1}^A(k$, $\xi_{il_2}^B(k)$ and $\xi_{il_3}^W(k)$ are positive real numbers, the condition (36) is sufficient to imply (20), and the positiveness of V_k is implied by (35), thus the proof is achieved. ■

Remark 5: For $E_i = I$, Theorem 3 is similar to Theorem 1 in [6].



(a) Transfer function between w_1 to $\tilde{z}_1$

(b) Transfer function between w_2 to $\tilde{z}_2$

Fig. 1. Transfer function between w to $\tilde{z}$

IV. Example

In order to shorten the present note, only the PI observer design is presented. Let consider the system (1) defined by the following matrices

$$\begin{aligned} E_1 &= E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.4 & 0.05 & 0 \\ 0 & -0.7 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.025 & 0 & 0 \\ -0.1 & -0.35 & 0 \\ 0.2 & 0 & 0.1 \end{bmatrix} \\ W_1 &= W_2 = \begin{bmatrix} 1 & 0.4 & 0 \end{bmatrix}^T, \quad C_1 = \begin{bmatrix} 0.29 & 0.15 & 0.2 \\ 0 & 0 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.19 & 0.17 & 0.4 \\ 0 & 0 & 1 \end{bmatrix} \\ T_1 &= \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

The objective is to compute the gains of the PI observer (3) in order to illustrate the performance of the estimation of the state variables of a switched descriptor system. Each triplets $\{E_i^a, A_i^a, C_i^a\}$, $(i \in \varepsilon)$ are both *finite dynamics detectable* and *impulse observable*, therefore we can solve the LMI's defined in Corollary 1. After some iterations, we find $\gamma = 1.4$ and

$$K_1^a = \begin{bmatrix} 1.2 & -0.24 \\ -0.8 & 0.16 \\ 0 & 0.1 \\ 0 & 0 \end{bmatrix}, \quad K_2^a = \begin{bmatrix} -0.204 & 0.082 \\ 1.967 & -0.787 \\ -1.636 & 0.75 \\ 0 & 0 \end{bmatrix}$$

The observer gives a good state estimation. More precisely, the UI attenuation properties can clearly be observed in the bode transfer function between w to $\tilde{z}$ given in Fig. 1a and 1b.

V. Conclusion

The problem of H_∞-filtering and state feedback control for a class of discrete-time switched systems with unknown inputs was investigated. Sufficient conditions for the existence of H_∞ observers and state feedback controller have been derived in terms of linear matrix inequalities. The proposed approaches not only makes the resulting closed-loop system regular, causal and stable, but also guarantees a bounded H_∞-norm of the closed-loop system from the unknown input to a combination of the state estimation error or to the controlled output variable. To our knowledge, the observer and controller designs for switched descriptor systems are new. The determination of the gains of the observers (or controller) is reduced to solve a set of LMI's. Moreover, in order to improve the robustness of the proposed state-feedback controller, the design method is extended to switched uncertain systems with polytopic uncertainty. Future works will be dedicated to extend the approach for non-linear and/or time delay descriptor systems.

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