Observer Design with H_∞ Performance for Delay Descriptor Systems

D. Koenig and B. Marx

***Abstract*— This note deals with the problem of full-order observer design for linear, descriptor systems with unknown input (UI) and time-varying delays. The resulting filter is of Luenberger observer type, and it guarantees that the H_∞-norm of the system, relating the exogenous signals to the estimation error, is less than a prescribed level. Four slack matrix variables are introduced in the derivative of the Lyapunov functional in order to reduce conservatism in existing delay dependent stability conditions. Both delay-dependent and delay-independent criteria are obtained. The applicability of the proposed approach is shown through a numerical example.**

KEYWORDS

H_∞observer design, time delay system, time-varying delay, descriptor systems.

I. INTRODUCTION

On the one hand, observer design for standard systems with unknown inputs (UI) has received considerable attention in the last two decades [2], [3] since, in many practical situations, disturbances or some inputs cannot be measured. On the other hand, singular systems have been extensively studied in the past years due to the fact that singular systems provide a more natural description of dynamical systems than the non-singular representation [17], [20]. Specific applications of singular systems are, for instance, constrained mechanical systems [12], electrical circuits [23], robotics [22], social, biological and economic systems [20]. Since, singular systems are very sensitive to slight input changes [5], the presence of UI is very detrimental to the observer design. This fact motivates the design of observers for descriptor systems in presence of UI [3], [1], [15] and [16]. In addition, the existence of delay in practical systems may induce instability, oscillations and poor performances. Moreover, descriptor systems may be destabilized by small delay in the feedback [19]. Therefore, current efforts on this topic have been carried out. Contributions on this topic can be classified into two main categories, namely delay-independent criteria [21], [4] and delay-dependent criteria [24], [25], [9], [14], [28], [13], [4]. It is well known that the delay-independent criteria is more conservative than the delay-dependent conditions especially for small delays. To the best of our knowledge, only in [4], the functional observer design for time-delay systems has been studied. Specifically, Darouach [4] has proposed a delay dependent stability criterion for constant time-delay systems (not in descriptor form), by using a bilinear matrix inequality (BMI) formulation. Unfortunately, only few papers deal with descriptor systems with delay [7], [11], [15].

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In this note, a H_∞ filtering design is proposed for linear, continuous time-invariant systems with time varying-delay and UI. The proposed observer is a full-order observer, which is not in descriptor form, in order to facilitate its implementation. Both delay-dependent and delay-independent stability of the observer are studied.

In section II, the studied systems are defined, they are described by a delay differential-algebraic equation with a state-delay. This model is called singular, implicit or descriptor system with delay. Our main contribution, detailed in section III, is to propose for descriptor time-delay systems, a new H_∞ observer with delay-dependent conditions using a LMI formulation. The recent stabilization method of [13] is extended to the descriptor estimation problem and new stability conditions are given in terms of LMIs. An example is given in section IV.

Notation 1: $(.)^T$ is the transpose matrix and $(*)$ is used for the blocks induced by symmetry. $(.) > 0$ denotes symmetric positive definite matrices. $(.)^+$ denotes any generalized inverse of the matrix $(.)$, where $(.)^+(.)(.)^+ = (.)^+$ and $(.)(.)^+(.) = (.)$. $Sym\left\{X\right\} = \left\{X + X^T\right\}$ for any matrix X. The space of functions in $\mathbb{R}^q$ that are square integrable over $[0,\infty)$ is denoted by $\ell_2^q\left[0,\infty\right)$ with norm $\left\|.\right\|_{L_2}$.

II. PROBLEM FORMULATION

Consider linear continuous time-delay systems of the form

$$\begin{aligned} E\dot{x}(t) &= A_0x(t)+A_1x(t-\tau(t))+Bu(t)+Ww(t)\\ x(t) &= \phi(t), \quad t\in[-\tau_m,0]\\ y\left(t\right) &= Cx\left(t\right)\\ z\left(t\right) &= Lx\left(t\right) \end{aligned} \tag{1}$$

where $\tau\left(t\right)$ is a known positive number denoting the delay, $\tau\left(t\right) \le \tau_m$. E may be rectangular. $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $w \in \mathbb{R}^q$ is the UI, $y \in \mathbb{R}^p$ is the measurement and $z \in \mathbb{R}^r$ is the vector to be estimated. $E \in \mathbb{R}^{k\times n}$, $A_i \in \mathbb{R}^{k\times n}$, $B_i \in \mathbb{R}^{k\times m}$, $C \in \mathbb{R}^{p\times n}$, $W \in \mathbb{R}^{n\times q}$ and $L \in \mathbb{R}^{r\times n}$. Without loss of generality it is assumed that $rankC = p$, $rankL = r$ and $rankW = q$.

For the sake of simplicity, only one delay is considered but the method can be easily extended to multiple delays.

In the present paper our aim is to design an observer for the system (1). The proposed observer (2) is a full-order and non descriptor Luenberger observer.

$$\begin{array}{rl} \dot{\zeta}(t) = & F_0\zeta(t) + F_1\zeta(t-\tau(t)) + TBu(t) \\ & +G_0 y(t) + G_1 y(t-\tau(t)) \\ \hat{x}(t) = & \zeta(t) + Ny(t) \\ \hat{z}(t) = & L\hat{x}(t) \end{array} \tag{2}$$

with the initial state $\zeta(t) = \phi_1(t)\ \forall t \in [-\tau_m, 0]$, and where $\zeta \in \mathbb{R}^n$ is the state observer, $\hat{x} \in \mathbb{R}^n$ is the estimate of x, $\hat{z} \in \mathbb{R}^{r \leq n}$ is the estimate of z. Matrices F_0, F_1, G_0,G_1, T and N are unknown matrices to be designed. The H_∞-observer problem for a performance level $\gamma > 0$ is to find the gains of the observer (2) that stabilizes asymptotically the state estimation error, $e(t) = \hat{x}(t) - x(t)$ and ensures the following performance index.

$$J = \int_0^\infty \left(\tilde{z}^T(\upsilon)\tilde{z}(\upsilon) - \gamma^2 w^T(\upsilon)w(\upsilon)\right) d\upsilon < 0 \tag{3}$$

where $0 \leq \tau(t) \leq \tau_m$, $\dot{\tau}(t) \leq d < 1$, $\tilde{z}(t) = \hat{z}(t) - z(t)$ and where $w(t) \in \ell_2^q$ satisfies $w(t) \neq 0$.

III. Main results

The objective of the observer design is to minimize the H_∞-norm of the transfer from the UI to the estimation of a linear combination of the state variables, namely $z(t) = Lx(t)$.

A. Observer design

The following theorem gives the structure of the state estimation error.

Theorem 1: Under the condition

$$rank \begin{bmatrix} E \\ C \end{bmatrix} = n \tag{4}$$

there exist matrices T, N, F_0, F_1, $\bar{G}_0$ and $\bar{G}_1$ such that the following conditions hold

1) $TE + NC = I_n$
2) $F_i = TA_i - \bar{G}_i C, \quad i = 0, 1$
3) $\dot{e}(t) = (\chi_0 - K\beta_0)\, e(t) + (\chi_1 - K\beta_1)\, e(t-\tau(t)) - (\chi_2 - K\beta_2)\, Ww(t)$

where $\bar{G}_i = G_i - F_i N$ for $i = \{0, 1\}$ and $e(t) = \hat{x}(t) - x(t)$. Matrices χ_i, β_i, for $i = \{0, 1, 2\}$ are defined in (13).

Proof: From (4), the state estimation error $e(t) = \hat{x}(t) - x(t)$ becomes

$$e(t) = \zeta(t) - TEx(t) \tag{5}$$

and the state estimation error (5) satisfies the differential-delay equation

$$\begin{aligned} \dot{e}(t) =& F_0 e(t) + F_1 e(t-\tau(t)) + \left(F_0 + \bar{G}_0 C - TA_0\right) x(t) \\ & + \left(F_1 + \bar{G}_1 C - TA_1\right) x(t-\tau(t)) - TWw(t) \end{aligned} \tag{6}$$

Now, if the condition 2 of theorem 1 holds, then (6) becomes

$$\dot{e}(t) = F_0 e(t) + F_1 e(t-\tau(t)) - TWw(t) \tag{7}$$

In the sequel the observer design reduces to find matrices $T, N, F_0, \bar{G}_0, F_1, \bar{G}_1$ such that conditions 1) and 2) of theorem 1 are satisfied. Rewriting conditions 1), 2) of theorem 1 as

$$\begin{bmatrix} T & N & F_0 & \bar{G}_0 & F_1 & \bar{G}_1 \end{bmatrix} \Theta = \Psi \tag{8}$$

where Θ and Ψ are given by

$$\Theta = \begin{bmatrix} E & A_0 & A_1 \\ C & 0 & 0 \\ 0 & -I_n & 0 \\ 0 & -C & 0 \\ 0 & 0 & -I_n \\ 0 & 0 & -C \end{bmatrix} \in \mathbb{R}^{(k+2n+3p)\times 3n}$$

$$\Psi = \begin{bmatrix} I_n & 0 & 0 \end{bmatrix} \in \mathbb{R}^{n \times 3n}$$

The solution of (8) depends on the rank of matrix Θ. A solution exists if and only if [26]

$$rank \begin{bmatrix} \Theta \\ \Psi \end{bmatrix} = rank \Theta \tag{9}$$

which is obviously equivalent to (4).

Under (4), the general solution of (8) is given by

$$\begin{aligned} & \begin{bmatrix} T & N & F_0 & \bar{G}_0 & F_1 & \bar{G}_1 \end{bmatrix} \\ & = \Psi\Theta^+ - K\left(I_{k+2n+3p} - \Theta\Theta^+\right) \end{aligned} \tag{10}$$

where K is an arbitrary matrix of appropriate dimensions and Θ^+ is the generalized inverse matrix of Θ. Let us define φ_i for $i \in \{1, 2, 3\}$ by

$$\begin{aligned} \varphi_0^T &= \begin{bmatrix} A_0^T & 0 & 0 & -C^T & 0 & 0 \end{bmatrix} \\ \varphi_1^T &= \begin{bmatrix} A_1^T & 0 & 0 & 0 & 0 & -C^T \end{bmatrix} \\ \varphi_2^T &= \begin{bmatrix} I_k & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

in this case the matrix T and the conditions 2) of theorem 1 can be rewritten as

$$\begin{aligned} & \begin{bmatrix} T & F_0 & F_1 \end{bmatrix} \\ & = \begin{bmatrix} T & N & F_0 & \bar{G}_0 & F_1 & \bar{G}_1 \end{bmatrix} \begin{bmatrix} \varphi_2 & \varphi_0 & \varphi_1 \end{bmatrix} \end{aligned} \tag{11}$$

Substituting (10) into (11), we obtain

$$\begin{aligned} F_0 &= \chi_0 - K\beta_0 \\ F_1 &= \chi_1 - K\beta_1 \\ T &= \chi_2 - K\beta_2 \end{aligned} \tag{12}$$

where

$$\begin{aligned} \chi_0 &= \Psi\Theta^+\varphi_0 \\ \chi_1 &= \Psi\Theta^+\varphi_1 \\ \chi_2 &= \Psi\Theta^+\varphi_2 \\ \beta_0 &= \left(I - \Theta\Theta^+\right)\varphi_0 \\ \beta_1 &= \left(I - \Theta\Theta^+\right)\varphi_1 \\ \beta_2 &= \left(I - \Theta\Theta^+\right)\varphi_2 \end{aligned} \tag{13}$$

Substituting (12) in (7), relation 3) of theorem 1 follows. ■

In other words, under condition (4), the state estimation error given by (2) is governed by

$$\begin{aligned} \dot{e}(t) =& \left(\chi_0 - K\beta_0\right) e(t) + \left(\chi_1 - K\beta_1\right) e(t-\tau(t)) \\ & - \left(\chi_2 - K\beta_2\right) Ww(t) \\ \tilde{z}(t) =& Le(t) \end{aligned} \tag{14}$$

where the matrices χ_i and β_i are known (13). Then, the observer design reduces to find the matrix gain K such that (14) is asymptotically stable and such that (3) is satisfied.

From [4], we can distinguish two different stability criteria, the first one is delay-dependent and the second one is delay-independent. These criteria are related to the stability of matrices $F = F_0 + F_1$ and F_0. Before giving the stability conditions of the obtained observer, we give the necessary and sufficient condition for the existence of a matrix K such that F and F_0 are Hurwitz.

Lemma 1: There exists a parameter matrix K such that $F = \chi - K\beta$ is Hurwitz if and only if

$$\text{the pair } (\chi,\ \beta) \text{ is detectable} \tag{15}$$

or equivalently

$$rank \begin{bmatrix} \lambda E - A_0 - A_1 \\ C \end{bmatrix} = n, \quad \forall \lambda \in C, \Re(\lambda) \geq 0 \tag{16}$$

where

$$\begin{aligned} \chi &= \chi_0 + \chi_1 = \Psi\Theta^{+}\varphi \\ \beta &= \beta_0 + \beta_1 = \left(I - \Theta\Theta^{+}\right)\varphi \\ \varphi^T &= \begin{bmatrix} A_0^T + A_1^T & 0 & 0 & -C^T & 0 & -C^T \end{bmatrix} \end{aligned}$$

Proof: The proof of (15)⇔(16) is done in appendix. ■

Lemma 2: There exists a matrix K such that $F_0 = \chi_0 - K\beta_0$ is Hurwitz if and only if

$$\text{the pair } (\chi_0,\ \beta_0) \text{ is detectable} \tag{17}$$

or equivalently

$$rank \begin{bmatrix} \lambda E - A_0 \\ C \end{bmatrix} = n, \quad \forall \lambda \in C, \Re(\lambda) \geq 0 \tag{18}$$

Proof: It can be easily derived from the proof of Lemma 1, hence it is omitted. ■

Remark 1: The computation of T is included in the design procedure, contrary to [15], [16]. More precisely, the matrix K involved in T (12) plays the role of a parametrization. Hence, T cannot involve a loss of detectability (for more details see [2]).

Remark 2: The above results were obtained for the case where no delay and UI are encountered in the measurement. If we consider the general system

$$\begin{aligned} E^{\triangledown}\dot{x}(t) &= A_0^{\triangledown}x(t) + A_1^{\triangledown}x(t-\tau(t)) + B_0^{\triangledown}u(t) + W_x^{\triangledown}w(t) \\ y^{\triangledown}(t) &= C^{\triangledown}x(t) + C_1^{\triangledown}x(t-\tau(t)) + W_y^{\triangledown}w(t) \\ z(t) &= Lx(t) \end{aligned} \tag{19}$$

where we have $rank \begin{bmatrix} C^{\triangledown} & C_1^{\triangledown} & W_y^{\triangledown} \end{bmatrix} = \dim y^{\triangledown}$, $rank \begin{bmatrix} W_x^{\triangledown T} & W_y^{\triangledown T} \end{bmatrix} = q \leq \dim y^{\triangledown}$ and $rank E^{\triangledown} = r \leq n$, or if (4) does not hold, then the system (19) is, firstly, transformed to the following equivalent descriptor system

$$\begin{aligned} \bar{E}\dot{x}(t) &= \Phi_0 x(t) + \Phi_1 x(t-\tau(t)) + \bar{B}u(t) + F_{11}y_1 + F_{12}w_2 \\ y_1(t) &= C_{11_0}x(t) + C_{11_1}x(t-\tau(t)) + w_1 \\ y_2(t) &= C_{12_0}x(t) + C_{12_1}x(t-\tau(t)) \end{aligned} \tag{20}$$

where $\bar{E} \in \mathbb{R}^{r\times n}$, $rank\bar{E} = r$, and where $\bar{E}...C_{12_1}, y_1, y_2, w_1$ and w_2 are easily deduced from the results developed in section II in [3]. Secondly, (20) is transformed to the following equivalent system

$$\begin{aligned} E_a\dot{x}_a(t) &= A_0^a x_a(t) + A_1^a x_a(t-\tau(t)) + B_a u(t) \\ &\quad + F_{a1}y_1(t) + F_{a2}w_2(t) \\ y_1(t) &= C_{11_0}x(t) + C_{11_1}x(t-\tau(t)) + w_1 \\ y_2(t) &= C_0^a x_a(t) \end{aligned} \tag{21}$$

where (22) and $1 \ll \rho$ (see eq. (24) in [6]).

$$\begin{aligned} &x_a(t) = \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} \quad E_a = \begin{bmatrix} \bar{E} & 0 \\ 0 & I \end{bmatrix} \quad A_0^a = \begin{bmatrix} \Phi_0 & 0 \\ 0 & -\rho I \end{bmatrix} \\ &B_a = \begin{bmatrix} \bar{B} \\ 0 \end{bmatrix} \quad F_{a1} = \begin{bmatrix} F_{11} \\ 0 \end{bmatrix} \quad A_1^a = \begin{bmatrix} \Phi_1 & 0 \\ \rho C_{12_1} & 0 \end{bmatrix} \\ &F_{a2} = \begin{bmatrix} F_{12} \\ 0 \end{bmatrix} \quad C_0^a = \begin{bmatrix} C_{12_0} & I \end{bmatrix} \end{aligned} \tag{22}$$

Now, the estimation problem becomes an H_∞ observer design problem of the form

$$\begin{aligned} \dot{\vartheta}(t) &= F_0\vartheta(t) + F_1\vartheta(t-\tau(t)) + T\bar{B}u(t) \\ &\quad + \breve{G}y_1(t) + \breve{G}_0 y_2(t) + \breve{G}_1 y_2(t-\tau(t)) \\ \hat{x}_a(t) &= \vartheta(t) + Ny_2(t), \quad \hat{\vartheta}(t) = L\hat{x}_a(t) \end{aligned}$$

where $\breve{G} = TF_{a1}$ and where the next steps correspond to the methodology described previously (i.e. section III.A). Notice that assumption (4) for system (21), i.e. $rank \begin{bmatrix} E_a^T & C_0^{aT} \end{bmatrix} = n + \dim\xi$, is equivalent to $rank \begin{bmatrix} \bar{E}^T & C_{12_0}^T \end{bmatrix} = n$ which is equivalent to the following impulse observability condition

$$rank \begin{bmatrix} \bar{E} & \Phi_0 \\ 0 & \bar{E} \\ 0 & C_{12_0} \end{bmatrix} - rank \begin{bmatrix} \bar{E} \end{bmatrix} = n \tag{23}$$

since $\bar{E}$ is of full row rank. Now, from Lemma 3.1-1) of [3] we deduce that (23) is equivalent to (24) which is less restrictive than (4).

$$rank \begin{bmatrix} E^{\triangledown} & A^{\triangledown} \\ 0 & E^{\triangledown} \\ 0 & C^{\triangledown} \end{bmatrix} - rank \begin{bmatrix} E^{\triangledown} \end{bmatrix} = n \tag{24}$$

B. The case of delay-dependent and rate-dependent stability

Determination of the observer gains is detailed in the following theorem..

Theorem 2: Under conditions (4) and (16), and for a given function $\tau(t)$ satisfying $0 \leq \tau(t) \leq \tau_m$ and $\dot{\tau}(t) \leq d < 1$, the observer (2) for system (1) is asymptotically stable and achieves (3), if for a prescribed $\gamma > 0$ and some prescribed scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3 \in \mathbb{R}$ there exist $P > 0$, $X > 0$, $S \geq 0$, U and matrices H_i for $i \in \{1..4\}$ satisfying the LMI (25).

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \tau_m H_1 \\ * & \alpha_{22} & \alpha_{23} & \alpha_{24} & \tau_m H_2 \\ * & * & \alpha_{33} & \alpha_{34} & \tau_m H_3 \\ * & * & * & \alpha_{44} & \tau_m H_4 \\ * & * & * & * & -\tau_m X \end{bmatrix} < 0 \tag{25}$$

where

$$\begin{aligned} \alpha_{11} &= Sym\left\{\varepsilon_1\left(P\chi_0 - U\beta_0\right) + H_1\right\} + S + L^T L \\ \alpha_{12} &= \varepsilon_1\left(P\chi_1 - U\beta_1\right) + \varepsilon_2\left(P\chi_0 - U\beta_0\right)^T - H_1 + H_2^T \\ \alpha_{13} &= \left(I - \varepsilon_1 I\right)P + H_3^T + \varepsilon_3\left(P\chi_0 - U\beta_0\right)^T \\ \alpha_{14} &= -\varepsilon_1 P\chi_2 W + \varepsilon_1 U\beta_2 W + H_4^T \\ \alpha_{22} &= -\left(1-d\right)S + Sym\left\{\varepsilon_2\left(P\chi_1 - U\beta_1\right) - H_2\right\} \\ \alpha_{23} &= -H_3^T + \varepsilon_3\left(P\chi_1 - U\beta_1\right)^T - \varepsilon_2 P \\ \alpha_{24} &= -\varepsilon_2 P\chi_2 W + \varepsilon_2 U\beta_2 W - H_4^T \\ \alpha_{33} &= \tau_m X - 2\varepsilon_3 P \\ \alpha_{34} &= -\varepsilon_3 P\chi_2 W + \varepsilon_3 U\beta_2 W \\ \alpha_{44} &= -\gamma^2 I_q \end{aligned}$$

The parameter matrix K is given by $K = P^{-1}U$.

Proof: Consider the following Lyapunov candidate function

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (26)$$

where $V_1 = e^T(t)Pe^T(t)$, $V_2 = \int_0^{\tau_m}\int_{t-\theta}^{t} \dot{e}^T(s)X\dot{e}(s)dsd\theta$, $V_3 = \int_{t-\tau(t)}^{t} e^T(\tau)Se(\tau)d\tau$, $P > 0$, $X > 0$ and $S \geq 0$. Note that V_1 corresponds to necessary and sufficient conditions for stability of system without delay, V_2 is typical for delay-dependent criteria, while V_3 corresponds to delay-independent stability conditions. In order to establish sufficient conditions of the existence of the observer (2) we apply the Lyapunov-Krasovskii method and requires that $\dot{V}(e,\ t)$ is strictly negative in order to guarantee the asymptotic stability of system (14) and that

$$H(e,w,t) = \dot{V}(e,t) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2 w^T(t)w(t) < 0 \quad (27)$$

is strictly negative in order to satisfy (3).

Since

$$\begin{aligned} &e(t) - e(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{e}(s)ds = 0 \\ &F_0 e(t) + F_1 e(t-\tau(t)) - TWw(t) - \dot{e}(t) = 0 \end{aligned}$$

there exists matrices H_i, $i \in \{1..4\}$, free scalars ε_i, $i \in \{1,2,3\}$ and P such that

$$\left\{ \begin{array}{l} 2\left[e^T(t)H_1 + e^T(t-\tau(t))H_2 + \dot{e}^T(t)H_3 + w^T(t)H_4\right] \\ \times \left[e(t) - e(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot{e}(s)ds\right] = 0 \end{array} \right. \quad (28)$$

$$\left\{ \begin{array}{l} 2\left[\varepsilon_1 e^T(t)P + \varepsilon_2 e^T(t-\tau(t))P + \varepsilon_3 \dot{e}^T(t)P\right] \\ \times \left[F_0 e(t) + F_1 e(t-\tau(t)) - TWw(t) - \dot{e}(t)\right] = 0 \end{array} \right. \quad (29)$$

The time derivative of V_i for $i \in \{1,2,3\}$ is defined by

$$\begin{aligned} \dot{V}_1 &= 2e^T(t)P\dot{e}(t) \\ \dot{V}_2 &= \tau_m \dot{e}^T(t)X\dot{e}(t) - \int_{t-\tau_m}^{t} \dot{e}^T(s)X\dot{e}(s)ds \\ \dot{V}_3 &= e^T(t)Se(t) - (1-\dot{\tau}(t))e^T(t-\tau(t))Se(t-\tau(t)) \end{aligned}$$

Substituting (28) and (29) into (27), we obtain

$$\begin{aligned} H(e,w,t) =& \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + (28) + (29) \\ &+ \tilde{z}^T(t)\tilde{z}(t) - \gamma^2 w^T(t)w(t) \\ \leq& \Gamma_1^T(t)\bar{\Psi}_1\Gamma_1(t) - \int_{t-\tau_m}^{t} \dot{e}^T(s)X\dot{e}(s)ds \\ &- 2g(t)\int_{t-\tau(t)}^{t}\dot{e}(s)ds \end{aligned} \quad (30)$$

where

$$g(t) = e^T(t)H_1 + e^T(t-\tau(t))H_2 + \dot{e}^T(t)H_3 + w^T(t)H_4$$

$$\bar{\Psi}_1 = \begin{bmatrix} \bar{\alpha}_{11} & \bar{\alpha}_{12} & \bar{\alpha}_{13} & \bar{\alpha}_{14} \\ * & \bar{\alpha}_{22} & \bar{\alpha}_{23} & \bar{\alpha}_{24} \\ * & * & \bar{\alpha}_{33} & \bar{\alpha}_{34} \\ * & * & * & \bar{\alpha}_{44} \end{bmatrix} \quad \Gamma_1(t) = \begin{bmatrix} e(t) \\ e(t-\tau(t)) \\ \dot{e}(t) \\ w(t) \end{bmatrix}$$

$$\begin{aligned} \bar{\alpha}_{11} &= Sym\{\varepsilon_1 PF_0 + H_1\} + S + L^T L \\ \bar{\alpha}_{12} &= \varepsilon_1 PF_1 + \varepsilon_2 F_0^T P - H_1 + H_2^T \\ \bar{\alpha}_{13} &= P + H_3^T - \varepsilon_1 P + \varepsilon_3 F_0^T P \\ \bar{\alpha}_{14} &= -\varepsilon_1 PTW + H_4^T \\ \bar{\alpha}_{22} &= -(1-d)S + Sym\{\varepsilon_2 PF_1 - H_2\} \\ \bar{\alpha}_{23} &= -H_3^T - \varepsilon_2 P + \varepsilon_3 F_1^T P \\ \bar{\alpha}_{24} &= -\varepsilon_2 PTW - H_4^T \\ \bar{\alpha}_{33} &= \tau_m X - 2\varepsilon_3 P \\ \bar{\alpha}_{34} &= -\varepsilon_3 PTW \\ \bar{\alpha}_{44} &= -\gamma^2 I_q \end{aligned}$$

Defining $\bar{X}$ by (31), then for any semipositive-definite matrix $\bar{X}$, the inequality (32) is always true.

$$\bar{X} = \begin{bmatrix} \bar{X}_{11} & \bar{X}_{12} & \bar{X}_{13} & \bar{X}_{14} \\ * & \bar{X}_{22} & \bar{X}_{23} & \bar{X}_{24} \\ * & * & \bar{X}_{33} & \bar{X}_{34} \\ * & * & * & \bar{X}_{44} \end{bmatrix} \quad (31)$$

$$\begin{aligned} 0 &\leq \tau_m \Gamma_1^T(t)\bar{X}\Gamma_1(t) - \int_{t-\tau(t)}^{t} \Gamma_1^T(t)\bar{X}\Gamma_1(t)ds \\ &= \tau_m \Gamma_1^T(t)\bar{X}\Gamma_1(t) - \tau(t)\Gamma_1^T(t)\bar{X}\Gamma_1(t) \end{aligned} \quad (32)$$

With (30) and (32) we obtain

$$H(e,w,t) \leq \Gamma_1^T(t)\Psi_1\Gamma_1(t) - \int_{t-\tau(t)}^{t} \Gamma_2^T(t,s)\Psi_2\Gamma_2(t,s)ds \quad (33)$$

where

$$\begin{aligned} \Gamma_2^T(t,s) &= \begin{bmatrix} \Gamma_1^T(t) & \dot{e}^T(s) \end{bmatrix} \\ \Psi_2 &= \begin{bmatrix} \bar{X}_{11} & \bar{X}_{12} & \bar{X}_{13} & \bar{X}_{14} & H_1 \\ * & \bar{X}_{22} & \bar{X}_{23} & \bar{X}_{24} & H_2 \\ * & * & \bar{X}_{33} & \bar{X}_{34} & H_3 \\ * & * & * & \bar{X}_{44} & H_4 \\ * & * & * & * & X \end{bmatrix} \\ \Psi_1 &= \bar{\Psi}_1 + \tau_m \bar{X} \end{aligned}$$

If $\Psi_1 < 0$ and $\Psi_2 \geq 0$ then $H(e,w,t) < 0$ and thus the performance index (3) is satisfied. Specifically, if we select a matrix $\bar{X}$, as

$$\bar{X} = \bar{H}X^{-1}\bar{H}^T \quad (34)$$

where $\bar{H}^T = \begin{bmatrix} H_1^T & H_2^T & H_3^T & H_4^T \end{bmatrix}$ and $X > 0$ it follows that $\bar{X} \geq 0$ and $\Psi_2 \geq 0$. In this case, $H(e,\ w,\ t)$ becomes negative definite for any nonzero $\Gamma_1(t)$ if $\Psi_1 < 0$ which is equivalent to

$$\begin{bmatrix} \bar{\alpha}_{11} & \bar{\alpha}_{12} & \bar{\alpha}_{13} & \bar{\alpha}_{14} & \tau_m H_1 \\ * & \bar{\alpha}_{22} & \bar{\alpha}_{23} & \bar{\alpha}_{24} & \tau_m H_2 \\ * & * & \bar{\alpha}_{33} & \bar{\alpha}_{34} & \tau_m H_3 \\ * & * & * & \bar{\alpha}_{44} & \tau_m H_4 \\ * & * & * & * & -\tau_m X \end{bmatrix} < 0 \quad (35)$$

according to the Schur's complement. Substituting (12) into (35), we obtain the LMI (25). ■

Algorithm 1: For a prescribed $\gamma > 0$ and scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3$ solve the LMI (25) on $P > 0$, $X > 0$, $S \geq 0$, U and $H_i, (i = 1..4)$. The matrix gain K is deduced by $K = P^{-1}U$. From (10) and $G_i = \bar{G}_i + F_i N$ deduce $\begin{bmatrix} T & N & F_0 & \bar{G}_0 & F_1 & \bar{G}_1 \end{bmatrix}$ and G_i, $i = 0, 1$ respectively.

Remark 3: The delay-dependent and rate-independent criteria can be derived from Theorem 2 by setting $S = 0$.

C. The case of delay-independent and rate-dependent stability

The delay-independent and rate-dependent criterion can be derived from Theorem 2 by setting $\varepsilon_1 = 1$, $\varepsilon_2 = \varepsilon_3 = 0$, $H_1 = H_2 = H_3 = H_4 = 0$, $X = 0$, then the following corollary is obtained.

Corollary 1: Under conditions (4) and (18), and given a function $\tau(t)$ satisfying $\dot{\tau}(t) \le d < 1$, the observer (2) for system (1) is asymptotically stable and achieves (3), if for a prescribed $\gamma > 0$ there exist $P > 0$, $S > 0$ and U solving the following LMI

$$\begin{bmatrix} x_{11} & P\chi_1 - U\beta_1 & -P\chi_2 W + U\beta_2 W \\ * & -(1-d)S & 0 \\ * & * & -\gamma^2 I_q \end{bmatrix} < 0 \qquad (36)$$

where $x_{11} = Sym\{P\chi_0 - U\beta_0\} + S + L^T L$

Algorithm 2: For a prescribed $\gamma > 0$ solve the LMI (36) on $P > 0$, $S > 0$ and U . The matrix gain K is deduced by $K = P^{-1}U$. From (10) and $G_i = \bar{G}_i + F_i N$ deduce $\begin{bmatrix} T & N & F_0 & \bar{G}_0 & F_1 & \bar{G}_1 \end{bmatrix}$ and G_i, $i = 0, 1$ respectively.

Note that for $d = 0$, (i.e. constant delay) and $L = I_n$, the LMI (36) is equivalent to the LMI (7) defined in [15].

Remark 4: The unknown matrices of the observers (2) is obtained from the general solution K of the LMI (25).

Remark 5: The H_∞ observer design for linear time-delay systems has been addressed in [29], [6], [8] and [10] where the case of the time-varying delay was considered in [8], [10]. The advantages of our results is that the system considered is really a descriptor system. While, the descriptor system (3) described in [6], is equivalent to the system (1) described in [6], if and only if the system (1) is not in a descriptor form. In fact, the descriptor system $E\dot{x} = A_0 x(t) + A_1 x(t-h)$, where h is a constant delay, can be rewritten in the equivalent descriptor form [6], $E\dot{x}(t) = \bar{y}(t)$, $0 = -\bar{y}(t) + (A_0 + A_1)x(t) - A_1 \int_{t-h}^{t} \bar{y}(s)\,ds$ if and only if $E = I$, since $\int_{t-h}^{t} \bar{y}(s)\,ds = Ex(t) - Ex(t-h)$.

D. The case of delay-independent stability

The delay-independent criterion can be derived from Theorem 3 by setting $\varepsilon_1 = 1$, $\varepsilon_2 = 0$, $H_1 = H_2 = H_3 = H_4 = 0$ and $X = 0$.

IV. EXAMPLE

Consider the linear algebra-differential system (1) where

$$L = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \quad B = 0_{3\times 1} \quad E = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_0 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_1 = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$W = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

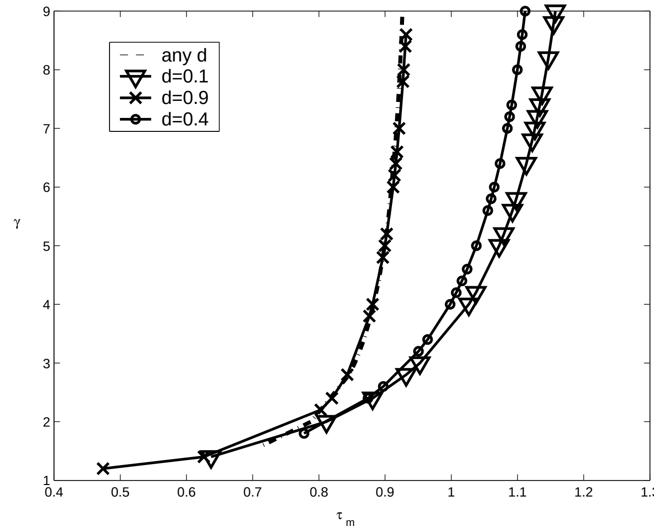

Fig. 1. γ function of τ_m for $d = 0.1$, $d = 0.4$, $d = 0.9$ and any d

One can easily verify that the design techniques presented in [13], [10] and [4] cannot be applied, since the system is in singular form. Only [11] and [15] have presented some results about the observers design for linear singular time-delay systems. More precisely in [11] the delay- independent filtering design for an undisturbed descriptor system was proposed. While in [15] the delay-independent H_∞ filtering design for disturb descriptor system with constant time-delay was proposed. We can note that, for the case of $d = 0$, applying the result of Theorem 2 in [15], no solution has been found.

A procedure of dichotomy were used in order to determine the bound τ_m, γ and the scaling factors ε_i for $i \in \{1, 2, 3\}$. Applying algorithm 1 to the above descriptor system it is found that in the case of time varying delay satisfying $0 \le \tau(t) \le \tau_m$ and $\dot{\tau}(t) \le d < 1$ the state estimation error (7) is asymptotically stable and the specified H_∞ upper bound constraint (3) is simultaneously guaranteed. The figure 1 summarized the results where for any d: $\varepsilon_1 = 1.3, \varepsilon_2 = -0.2, \varepsilon_3 = 1.3$, for $d = 0.1 : \varepsilon_1 = 1.5, \varepsilon_2 = -0.4, \varepsilon_3 = 1.2$, for $d = 0.4 : \varepsilon_1 = 1.5, \varepsilon_2 = -0.1, \varepsilon_3 = 1.3$ and for $d = 0.9 : \varepsilon_1 = 1.6, \varepsilon_2 = -0.3, \varepsilon_3 = 1.3$. For the case of delay-independent criteria (algorithm 2), no solution has been found since the detectability condition (18) is not satisfied.

V. CONCLUSION

This paper have presented the design of observer for time-delay descriptor systems affected by unknown inputs (UI). The objective is to minimize the H_∞-norm of the transfer from the UI to the estimation of a linear combination of the state variables. The convergence of the observer is based on some new stability criteria for time-delay descriptor system. Like [13], the delay-dependent stability criterion is by duality less conservative than [8], [30] and [4]. Sufficient conditions to achieve prescribed disturbance attenuation level are derived in terms of LMIs for the case of descriptor systems with time-varying delay. The delay-independent condition for the case of time-varying delay is derived as a special case of our conditions. The method developed in this paper generalize the results of [13], [15] and [4].

VI. APPENDIX

Proof of Lemma 1. There exists K such that F is Hurwitz if and only if the pair (χ, β) is detectable i.e.,

$$\begin{bmatrix} sI_n - \chi \\ \beta \end{bmatrix} = n, \ \forall s \in \mathbb{C}, \ \Re(s) \geq 0 \Leftrightarrow (15)$$

Let us define the nonsingular matrices V_1, V_2 and the full-column rank matrix V_3 by

$$V_1 = \begin{bmatrix} I_n & 0 \\ -\Theta^+\varphi & I_{3n} \end{bmatrix} \quad V_2 = \begin{bmatrix} -I_n & 0 & 0 \\ sI_n & I_n & 0 \\ 0 & 0 & I_{2n} \end{bmatrix}$$

$$V_3 = \begin{bmatrix} I & -\Psi\Theta^+ \\ 0 & I - \Theta\Theta^+ \\ 0 & \Theta\Theta^+ \end{bmatrix}$$

Since

$$rank \begin{bmatrix} sE - A_0 - A_1 \\ C \end{bmatrix} = rank \begin{bmatrix} sI_n & \Psi \\ \varphi & \Theta \end{bmatrix} V_2 - 3n$$
$$= rank \begin{bmatrix} sI_n & \Psi \\ \varphi & \Theta \end{bmatrix} - 3n = n$$

we obtain

$$rank \begin{bmatrix} sI_n & \Psi \\ \varphi & \Theta \end{bmatrix} = rank V_3 \begin{bmatrix} sI_n & \Psi \\ \varphi & \Theta \end{bmatrix} V_1 = 4n$$

or equivalently

$$rank \begin{bmatrix} sI_n - \chi & \times \\ \beta & \times \\ 0 & \Theta \end{bmatrix} = 4n$$

or equivalently (15) since from (9), $rank\Theta = 3n$.

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