ON PARAMETRIC AND NONPARAMETRIC FAULT DETECTION IN LINEAR CLOSED-LOOP SYSTEMS

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Abstract: This paper deals with the problem of fault detection and isolation (FDI) in linear closed loop control systems. The presented approach uses model-based techniques applied to linear systems. The residual generator proposed in the following is derived from transfer function representation of both open and closed loop system, and it is designed to be sensitive to parametric faults (e.g. actuator, process and sensor faults) and nonparametric faults (e.g. unknown inputs) which can occur in the plant.

Keywords: Fault detection and isolation, linear systems, closed loops, sensitivity analysis, frequency domain.

1. INTRODUCTION

A traditional approach to fault diagnosis in wider application context is based on hardware or physical redundancy methods to use multiple sensors, actuators, components to measure a particular variable. Typically, a voting technique is applied to the hardware redundant system to decide if a fault has occurred or not and to deduce its location among all the redundant system components. The major problems encountered with hardware redundancy are the extra equipments and maintenance cost (Isermann and Ballé, 1997). In view of the conflict between the reliability and the cost of adding more hardware, it is preferable to combine measures of different state variables at different times, rather than replicating hardware individually in order to detect or isolate dysfunctions. This is the meaning of analytical or functional redundancy. It exploits redundant relationships among various variables of the monitored process (Frank, 1990) (Patton *et al.*, 1989). In the analytical redundancy scheme, the resulting difference generated from the comparison of different variables is called a *residual*. The residual is equal to zero when the system is in normal operation and is different from zero when a fault occurs.

Many methods are available in literature for residual generation. Most of them are model-based and designed in both continuous and discrete-time domain:

- fault detection via parameter estimation (Isermann, 1997), (Patton *et al.*, 2000)
- observer-based approaches (Frank, 1990)
- parity relations methods (Chow and Willsky, 1984), (Gertler and Singer, 1990)

However, these methods deal with systems in open loop scheme. In most of industrial applications, the system operates under feedback framework.

Thus, by continuous monitoring of the control loop, it will be possible to give an early warning of a component deterioration, avoiding breakdown failures and enable maintenance actions. In closed loop fault detection context, various faults can produce the same effect, and, moreover, the authors in (Jacobson, 1991), (Wang and Wu, 1993) claim that there will always be a conflict between the aims of fault detection and good feedback controller design. This is certainly true in the sense that the controller at frequencies of high gain effectively hide any plant variation (Jenssen and Zarrop, 1994). Thus generally, it is difficult for the operator who supervise the process to determine which part of the plant is defective. It is therefore required to implement a diagnostic system based on redundancy relationships in order to assist the operator to isolate the fault. The residual signal which will be generated must contain information related to the *health* of the plant and must be sensitive to incipient faults. Based on the sensitivity analysis, the aim of this paper is to design a residual generator which can be applicable in classical control loop configuration.

The paper is organized as follows: in the section 2, we formulate the problem in the case of nonparametric faults. Section 3 is dedicated to the presentation of the residual generator and the analysis of the residual signal properties in the frequency domain in the case where all or a part of the signals are measured. In the section 4, the parametric fault case is considered and in section 5 the sensitivity analysis of the residuals is presented. Simulation results are presented in section 6 and a conclusion is given in section 7.

2. NONPARAMETRIC FAULTS

Commonly designed by additive faults in the literature, the nonparametric faults are unknown inputs acting on the plant. While healthy operating, these signals are zero mean. The occurrence of a fault is modeled by a nonzero signal and causes a change in the plant outputs independent of the known inputs. The figure 1 shows a standard control loop scheme which contains the following elements: the controller $C(s)$, the actuator $G_a(s)$, the plant $G_p(s)$ and the sensor $G_s(s)$. These filters are introduced for example to take into account the dynamics of the actuator and of the sensor. The influence of the actuator fault $F_a(s)$, plant fault $F_p(s)$ and sensor fault $F_s(s)$ are modeled by the transfer functions $H_a(s)$, $H_p(s)$ and $H_s(s)$ respectively. Let $U_p(s) = X_a(s)$ and $U_s(s) = X_p(s)$. Then, the description of each element in the continuous time domain is given by the following relationships:

$$X_\ell(s) = G_\ell(s)U_\ell(s) + H_\ell(s)F_\ell(s) \qquad (1)$$

where the subscript ℓ can be a, p or s respectively standing for actuator, plant and sensor and where

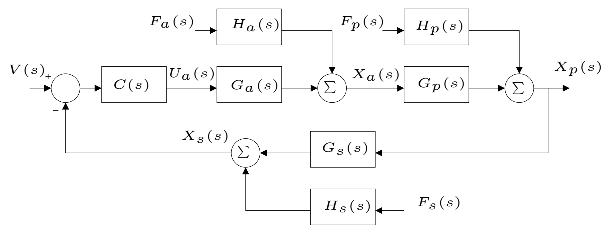

Fig. 1. Closed loop control with nonparametric faults

the signals U_ℓ and X_ℓ are respectively the input and output of the subsystems (controller, actuator, plant and sensor) $G_\ell(s)$. $F_\ell(s)$ designates the unknown fault input which affects the subsystem $G_\ell(s)$ through the transfer function $H_\ell(s)$. The unknown inputs $F_\ell(s)$ are null in free fault case (normal operating condition). In the SISO framework, the transfer functions of the subsystems $G_\ell(s)$ can be written as:

$$G_\ell(s) = \frac{\sum_{j=0}^{m_\ell} b_{\ell_j} s^j}{\sum_{i=0}^{n_\ell} a_{\ell_i} s^i}, \ \ell = a, p, s \text{ and } n_\ell \geq m_\ell \qquad (2)$$

2.1 Assumptions

In the sequel, the following assumptions hold:

(A1) all the signals X_ℓ and U_ℓ are available to measurement.

(A2) the transfer functions $G_\ell(s)$ are perfectly known.

(A3) the system is single-input single-output.

3. RESIDUAL GENERATION

The residual generator studied hereafter is based on a classical model-based methodology using the parity space approach. The commonly desired properties for the residual signal $r(t)$ are:

- $r(t) = 0$ as $f(t) = 0$
- $r(t) \neq 0$ as $f(t) \neq 0$ for fault detection
- $r_\ell(t) \neq 0$ and $r_j(t) = 0$ for $j \neq \ell$ as $f_\ell(t) \neq 0$ for fault isolation
- $\lim_{t\to\infty} (f(t) - r(t)) = 0$ for fault identification

According to the description of the system given by the figure 1 and equation (1) it is possible to derive the following relationships:

$$X_a(s) = G_a(s)U_a(s) + H_a(s)F_a(s) \qquad (3)$$

$$X_p(s) = G_p(s)X_a(s) + H_p(s)F_p(s) \qquad (4)$$

$$X_s(s) = G_s(s)X_p(s) + H_s(s)F_s(s) \qquad (5)$$

$$U_a(s) = C(s)(V(s) - X_s(s)) \qquad (6)$$

Note that (6) is an analytical redundancy relation, i.e. implies only known and measurable variables. Then, following the assumption (A1), the residuals can be generated as follows:

$$R_1(s) = X_a(s) - G_a(s)U_a(s) = H_a(s)F_a(s) \qquad (7)$$

$$R_2(s) = X_p(s) - G_p(s)X_a(s) = H_p(s)F_p(s) \qquad (8)$$

$$R_3(s) = X_s(s) - G_s(s)X_p(s) = H_s(s)F_s(s) \qquad (9)$$

which can be expressed matricially, on the one hand:

$$\underline{R}(s) = \mathcal{G}(s)\underline{X}(s) \tag{10}$$

and on the other hand:

$$\underline{R}(s) = \mathcal{H}(s)\underline{F}(s) \tag{11}$$

where

$$\underline{R}(s) = [R_1(s) \;\; R_2(s) \;\; R_3(s)]^T \tag{12}$$

$$\underline{X}(s) = [U_a(s) \;\; X_a(s) \;\; X_p(s) \;\; X_s(s)]^T \tag{13}$$

$$\underline{F}(s) = [F_a(s) \;\; F_p(s) \;\; F_s(s)]^T \tag{14}$$

$$\mathcal{H}(s) = diag(H_a(s) \;\; H_p(s) \;\; H_s(s)) \tag{15}$$

$$\mathcal{G}(s) = \begin{bmatrix} -G_a(s) & 1 & 0 & 0 \\ 0 & -G_p(s) & 1 & 0 \\ 0 & 0 & -G_s(s) & 1 \end{bmatrix} \tag{16}$$

The equations (10) and (11) are respectively the external and internal forms of the residuals. One can note that if the assumption (A1) is valid, then, it is clear that it is possible to achieve the fault detection and isolation procedure without difficulty. In this case, the residual sensitivity with respect to the faults is given by:

$$\frac{\partial \underline{R}(s)}{\partial \underline{F}(s)} = \mathcal{H}(s) \tag{17}$$

Thus, it is obvious that the residual sensitivity depends on the bandwidth of the transfer functions $H_\ell(s), \;\ell = a, p, s$.

Now, investigate the case where the assumption (A1) is not valid, i.e. some signals of the set $\underline{X}$ are not available to measurement. The idea developed in the sequel consists in inspecting the columns of the matrix $\mathcal{G}(s)$ and combine the nonnull elements with an aim to form a new residual which does not depend on the unmeasured signal. By a linear combination of the rows of the matrix $\mathcal{G}(s)$ in (16), one can form new residuals such that (10) and (11) can be written as follows:

$$\tilde{\underline{R}}(s) = \tilde{\mathcal{G}}(s)\underline{X}(s) \tag{18}$$

$$\tilde{\underline{R}}(s) = \tilde{\mathcal{H}}(s)\underline{F}(s) \tag{19}$$

where

$$\tilde{\underline{R}}(s) = [R_1(s) \;\; R_2(s) \;\; R_3(s) \;\; R_4(s) \;\; R_5(s)]^T \tag{20}$$

$$\tilde{\mathcal{G}}(s) = \begin{bmatrix} -G_a(s) & 1 & 0 & 0 \\ 0 & -G_p(s) & 1 & 0 \\ 0 & 0 & -G_s(s) & 1 \\ -G_a(s)G_p(s) & 0 & 1 & 0 \\ 0 & -G_p(s)G_s(s) & 0 & 1 \end{bmatrix} \tag{21}$$

and

$$\tilde{\mathcal{H}}(s) = \begin{bmatrix} H_a(s) & 0 & 0 \\ 0 & H_p(s) & 0 \\ 0 & 0 & H_s(s) \\ G_p(s)H_a(s) & H_p(s) & 0 \\ 0 & G_s(s)H_p(s) & H_s(s) \end{bmatrix} \tag{22}$$

For instance, if one cannot measure the output signal of the plant, $X_p(s)$, it will not be possible to generate the residuals $R_2(s)$ and $R_3(s)$ depending on $X_p(s)$ which is not available. Hence, in that case, regarding the structure of the matrix $\tilde{\mathcal{G}}(s)$, one can generate only two residuals: $R_1(s)$ and $R_5(s)$. The relation which links these residuals to the faults is given by:

$$\begin{bmatrix} R_1(s) \\ R_5(s) \end{bmatrix} = \begin{bmatrix} H_a(s) & 0 & 0 \\ 0 & G_s(s)H_p(s) & H_s(s) \end{bmatrix} \underline{F}(s) \tag{23}$$

In this situation, because of the structure of the matrix $\tilde{\mathcal{H}}(s)$, one can detect all faults but isolate only the actuator one. The following algorithm resumes the method.

3.1 Procedure

`step 1.` Write the residual signals in their internal and external forms in such a way to have:

$$\underline{\mathbf{R}} = [\mathbf{G}]\underline{\mathbf{X}} = [\mathbf{H}]\underline{\mathbf{F}}$$

`step 2.` Let $(N_{\mathbf{G}} \times M_{\mathbf{G}})$ be the dimensions of $[\mathbf{G}]$. For every couple of rows with nonnull j^{th} component, make a linear combination in order to obtain a new row with a null j^{th} component. If α new rows are formed (i.e. α new residuals) the sizes of $[\mathbf{G}]$ will be $(N_{\mathbf{G}} + \alpha \times M_{\mathbf{G}})$.

`step 3.` Let $(N_{\mathbf{H}} \times M_{\mathbf{H}})$ be the sizes of $[\mathbf{H}]$. Do the same combinations as for $[\mathbf{G}]$. The sizes of $[\mathbf{H}]$ will then be $(N_{\mathbf{H}} + \alpha \times M_{\mathbf{H}})$.

`step 4.` If the j^{th} component of the vector $\underline{\mathbf{X}}$ is not available to measurement, the residuals which can be computed are those corresponding to the null elements of the j^{th} column of $[\mathbf{G}]$, i.e. $\underline{\mathbf{R}}_i$ can be computed if $[\mathbf{G}](i,j) = 0$.

3.2 Discussion

The above algorithm allows to calculate residuals taking into account the unavailability of certain measurements. The j^{th} fault can be detected by the residual $\underline{\mathbf{R}}_i$ if $|[\mathbf{H}](i,j)| \neq 0$. The detectability depends on the band-width of the components of the matrix $[\mathbf{H}]$, i.e, larger is the modulus of $|[\mathbf{H}](i,j)|$, better is the sensitivity of $\underline{\mathbf{R}}_i$ to the fault $\underline{\mathbf{F}}_j$ since $\dfrac{\partial \underline{\mathbf{R}}_i}{\partial \underline{\mathbf{F}}_j} = [\mathbf{H}](i,j)$. However, the number of the faults which are isolable is equal to rank $[\mathbf{H}]$.

4. PARAMETRIC FAULTS

The parametric faults are changes (abrupt or gradual) in some plant parameters (component wears for instance). They cause changes in the plant output which depend also on the magnitude of the known inputs. Such faults best describe the deterioration of the plant or of the plant equipment. For the sake of convenience analysis, single-input and single-output control system (as shown in figure 2) is used for the sequel developments. The control loop consists in a controller which is

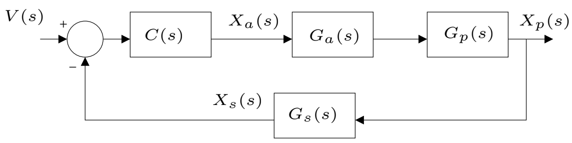

Fig. 2. Control loop with faulty components scheme

represented by the transfer function $C(s)$, the actuator $G_a(s)$, the plant $G_p(s)$ and finally the sensor $G_s(s)$. In free-fault case, all component parameters are given by their nominal values and modeled by $G_{a_0}(s)$, $G_{p_0}(s)$, and $G_{s_0}(s)$ respectively. These latter are gathered in a vector of nominal parameters as $G_0(s) = [G_{a_0}(s)\ G_{p_0}(s)\ G_{s_0}(s)]^T$. Assume that no disturbance is acting on the system and all fault manifestations are due to the component parameter changes.

5. RESIDUAL GENERATION

The residual signals are generated from the difference between the real signals X_ℓ, $\ell = a, p, s$, measured on the actual system and their corresponding signals obtained from the nominal model. Since $X_a = CH_{bf}$, $X_p = CG_aG_pH_{bf}$, $X_s = CG_aG_pG_sH_{bf}$ and $H_{bf} = \frac{1}{1+CG_aG_pG_s}$, we get the following relations:

$$\begin{aligned} R_a &= \left(\frac{C}{1+CG_aG_pG_s} - \frac{C}{1+CG_{a_0}G_{p_0}G_{s_0}}\right)V \\ R_p &= \left(\frac{CG_aG_p}{1+CG_aG_pG_s} - \frac{CG_{a_0}G_{p_0}}{1+CG_{a_0}G_{p_0}G_{s_0}}\right)V \\ R_s &= \left(\frac{CG_aG_pG_s}{1+CG_aG_pG_s} - \frac{CG_{a_0}G_{p_0}G_{s_0}}{1+CG_{a_0}G_{p_0}G_{s_0}}\right)V \end{aligned} \tag{24}$$

Where, the Laplace operator s is omitted in order to simplify the residual expressions. To quantify the residual variations with respect to the variations in the components $G_\ell(s)$, we recall the Bode's sensitivity function (Frank, 1978):

5.1 Definition

Let $G(s) = G(s, \alpha)$ and $G_0(s) = G(s, \alpha_0)$ be the actual and nominal transfer functions of the system respectively and let α_0 represent the nominal value of the parameter vector α. Then, the logarithmic derivative

$$S_{\alpha_j}^{G(s)} \triangleq \left(\frac{\partial \ln G(s)}{\partial \ln \alpha_j}\right)_{\alpha_0} = \left(\frac{\partial G(s)}{\partial \alpha_j}\right)_{\alpha_0} \frac{\alpha_{j_0}}{G_0(s)} \tag{25}$$

is called Bode's sensitivity function.

Now, apply the definition above to compute the sensitivity of the three residuals $R_a(s)$, $R_p(s)$ and $R_s(s)$ with respect to the parameter variations in the actuator, plant, and sensor respectively. It reduces to compute the variation of the transfer functions of the input $V(s)$ towards the residuals $R_\ell(s)$. We get the following sensitivity matrix:

$$\Phi(s) = \left[S_{G_\ell(s)}^{R_{\bar{\ell}}(s)}\right]_{\substack{\bar{\ell}=a,p,s \\ \ell=a,p,s}} \tag{26}$$

The matrix $\Phi(s)$ describes the residual sensitivities to changes in the whole subsystem $G_\ell(s)$. However, in order to compute the residual sensitivities with respect to the parameters of the subsystem $G_\ell(s)$, the following proposition is formulated.

Proposition

If $S_{G(s)}^{R(s)}$ is the sensitivity function of the residual $R(s)$ with respect to the component $G(s)$, then the sensitivity function $S_{\alpha_i}^{R(s)}$ of $R(s)$ with respect to the parameter α_i of $G(s)$ is given by:

$$S_{\alpha_i}^{R(s)} = S_{G(s)}^{R(s)} S_{\alpha_i}^{G(s)}$$

Proof: The proof is demonstrated by using the composite function derivative theorem.

After calculation of the various elements of $\Phi(s)$ we obtain the following structured matrix

$$\Phi(s) = \begin{bmatrix} H_1(s) & H_1(s) & H_1(s) \\ H_0(s) & H_0(s) & H_1(s) \\ H_0(s) & H_0(s) & H_0(s) \end{bmatrix} \tag{27}$$

where $H_0(s)$ and $H_1(s)$ are given by:

$$H_0(s) = \frac{1}{1+\Sigma(s)} \tag{28a}$$

$$H_1(s) = \frac{\Sigma(s)}{1+\Sigma(s)} \tag{28b}$$

and $\Sigma(s) = C(s)G_{a_0}(s)G_{p_0}(s)G_{s_0}(s)$. Let define a frequency domain $\mathcal{D}$ such that

$$\mathcal{D} = \{s :\ |\Sigma(s)| \gg 1\} \tag{29}$$

Then, for frequencies in $\mathcal{D}$ defined by (29), $H_1(s)$ can be approximated by 1 and $H_0(s)$ by 0. The approximated sensitivity matrix takes the form:

$$\tilde{\Phi} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tag{30}$$

Note that $H_0(s)$ and $H_1(s)$ are complementary functions. This means that, according to the frequency range, the modulus of $H_0(s)$ decreases when the modulus of $H_1(s)$ increases and conversely. If the $G_\ell(s)$ are proper transfer functions, the modulus of $\Sigma(s)$ depends on the form of the controller $C(s)$. For instance:

- If $C(s)$ is a PI controller, the static gain of $\Sigma(s)$ tends to infinity, so $|H_0(s)| \to 0$ and $|H_1(s)| \to 1$.
- If $C(s)$ is a proportional controller, then $\lim_{s\to 0} \Sigma(s) \ll \infty$. In this case, $|H_0(s)| \to h_0$ and $|H_1(s)| \to h_1$ where $h_0 \in \Re$ and $h_1 \in \Re$.

From the approximated matrix (30), one can observe that the controller output can be used to generate the residual signal for all the system components subject to parameter changes or faults. While from the feedback sensor output, one can hardly get any information about faults. Between these two locations, the output system is only effective in detecting the feedback sensor fault. Following the structure of the approximated sensitivity matrix $\tilde{\Phi}$, a simple isolation logic allows to isolate parametric change in the sensor and to detect parametric change in the actuator and in the plant.

6. SIMULATION RESULTS

6.1 System description

A DC motor scheme is used to illustrate the proposed method. The process is modeled by a second order transfer function:

$$G_p(s,\alpha) = \frac{W(s)}{V(s)} = \frac{K}{(Js+b)(Ls+R)+K^2} \tag{31}$$

where $V(s)$ is the source voltage, $W(s)$ the controlled shaft rotational speed and $\alpha = [J \;\; R \;\; L \;\; K]^T$ is the parameter set:

- moment of inertia of the rotor $J = 0.01$ kg.m^2.s^{-2}
- damping ratio of the mechanical system $b = 0.1$ Nms
- electromotive force constant $K = 0.01$ Nm/A
- electric resistance $R = 1$ ohm
- electric inductance $L = 0.5$ H

The transfer function of the DC motor is then given by:

$$G_{p_0}(s) = \frac{2}{s^2+12s+20.02} \tag{32}$$

The control objectives are:

- settling time less than 2 seconds
- zero overshoot
- zero steady-state error.

All of the design requirements are satisfied with the following PI controller

$$C(s) = \frac{10s+20}{s} \tag{33}$$

The figure 3 shows the control efficiency to

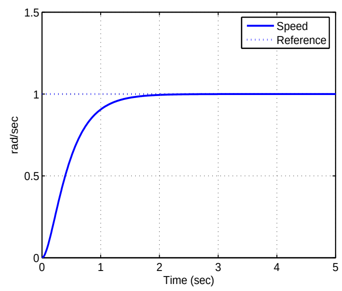

Fig. 3. Closed loop step response

achieve the laid down goals. In the sequel, we are interested in the detection of two types of faults: parametric or nonparametric faults.

6.2 Nonparametric faults

The simulations are performed with $H_\ell(s) = 1$, $\ell = a, p, s$, $G_a(s) = 1$ and $G_s(s) = \frac{1}{0.01s+1}$. The case where all signals are measurable is not studied since, as shown by (10) and (11), all faults are detectable and isolable. We focus on the case when the output of the system $X_p(s)$ is not measurable. Thus, according to (13) and (16), only the residual $R_1(s)$ which can detect actuator and plant faults can be computed. Though, following the previous procedure, one can see that it is possible to compute an additional residual, namely $R_5(s)$, which can detect plant and sensor faults. Now, with these two residuals, one can see that all faults are detectable and, in addition, the actuator fault is isolable. In the temporal residuals simulation, the signals are corrupted by Gaussian noises of variance $\Sigma = 10^{-3}$. The simulated faults are biases. The considered scenario is summarized in the table 1. The figure 4 shows the temporal

	Instant of appearance	duration	magnitude
f_a	t=20 sec	10 sec	0.20
f_p	t=40 sec	10 sec	0.15
f_s	t=60 sec	10 sec	0.10

Table 1. Fault characteristics

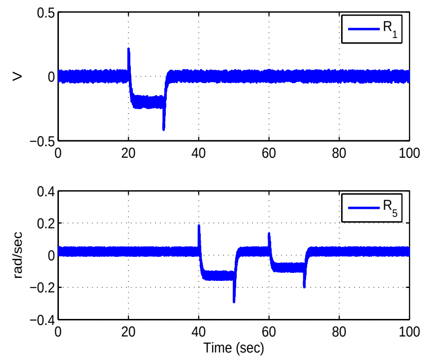

Fig. 4. Residuals R_1 and R_5

evolution of the residuals R_1 and R_5, obtained by following the given algorithm. Thus, one can see that the all faults are detectable. The residual R_1 is only sensitive to the actuator fault which means that it is isolable. The residual R_5 cannot detect the actuator fault but it is sensitive to plant and sensor faults. We can see that the simulation results are in conformity with the theoretical results developed in the section 2.

6.3 Parametric faults

The same nominal transfer function of the plant, controller and sensor as above are used for simulations of the parametric fault case. As shown by the relation (27), the residual sensitivities depend on the frequency range of $H_0(s)$ and $H_1(s)$ given by (28). The magnitude of $H_0(s)$ and $H_1(s)$ are displayed on figure 5. One can see that for frequencies smaller than 0.1 rad/sec, the modulus of $H_0(s)$ tends to zero whereas that of $H_1(s)$ tends to 1 and for frequencies upper than 90 rad/sec, the modulus of $H_0(s)$ tends to unity whereas that

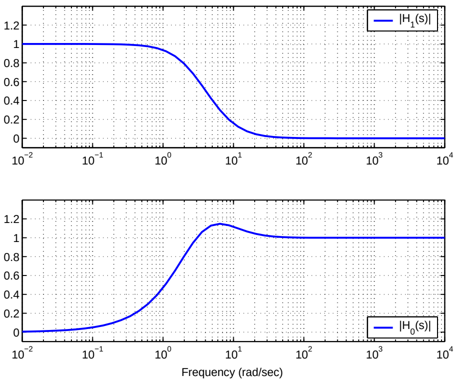

Fig. 5. Residual sensitivities

of $H_1(s)$ tends to zero. For the intermediate frequencies, which are upper than 0.1 rad/sec and lower than 90 rad/sec, the modulus of $H_0(s)$ is included between 0 and 1.15, and that of $H_1(s)$ is included between 0 and 1. If we take into account

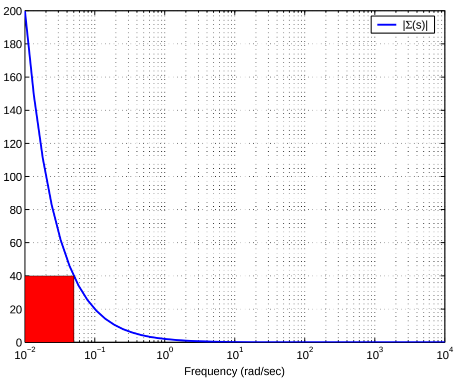

Fig. 6. Validity domain of the condition (29)

the condition (29), one can restrict the analysis to the frequencies belonging to the domain $\mathcal{D}$. The figure 6 shows the variation of the validity domain with respect to the frequency. For instance, if it is considered that 40 is much higher than 1, then, the frequencies of interest are lower than 0.05 rad/sec. The figure 7 shows the residual evolutions

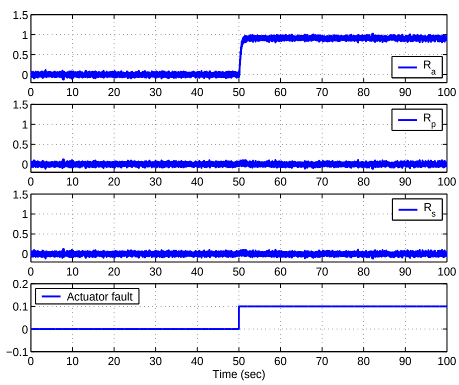

Fig. 7. Residual evolutions for changes in $G_{\tilde{a}}$

when a change in the actuator transfer function is occurred. One can see that only the residual generated from the output of the controller deviates from zero when the fault occurs. These results are in conformity with the analysis of the sensitivity carried out above.

7. CONCLUSION

In this paper, some studies of fault detection in closed loop framework were shown. Both parametric and nonparametric faults are considered. In the case of nonparametric faults, the given algorithm allows to generate residuals both when all signals are available to measurement or when only a set of measurement are available. The sensitivity analysis allows to determine in which frequency range the residuals are able to detect the faults. In the case of parametric faults, the residual sensitivities depend on the frequency of the reference signal. The influence of the type of controller used on the residuals sensitivity is also discussed.

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