Robust Fault Diagnosis for Linear Descriptor Systems using Proportional Integral Observers

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Abstract – This paper presents the design of a proportional-integral observer for descriptor systems subject to faults and unknown inputs. The observer is synthesized to minimize the influence of unknown inputs on the estimation. Weighting transfer is introduced to shape the sensitivity of the estimation to the unknown inputs. Particular attention is paid to fault diagnosis objective. The proposed method is based on the solution of LMI and guarantees the estimation of the states and faults to be robust face to unknown inputs. A numerical example is included.

I. INTRODUCTION

Since dynamical systems are becoming more and more complex, control engineering requires safety and reliability. In other words, the plant modelling should include disturbance (or unknown inputs) and possible component failures or malfunctions. Thus diagnosis, and especially model-based fault diagnosis [1], [14], had become a key point in modern control. One of the most popular technique is to generate residual signals which highlight the appearance of an abnormal behaviour of the plant (actuator failure, sensor failure, varying parameters, …). The so-called residual generation problem can be addressed with different approaches such as parity spaces, factorisation approaches, eigenstructure assignment and observer-based methods. Then the design of observers for dynamical systems has received a considerable amount of attention in the field on robust fault diagnosis, in particular observers for systems with unknown inputs (UI) [3], [10].

In this paper the generic class of linear descriptor systems (i.e. E.dx/dt=Ax+…) is considered. This formulation includes both dynamic and static linear relations. Consequently this formalism is much more general than the usual one and can model physical constraints or impulsive behaviour due to an improper part of the system. Descriptor systems appear in many fields of system design and control such as constrained robots, power systems, hydraulic or electrical networks…[5].

Many control issues have been extended to the descriptor case, in particular the observer design for descriptor systems has been intensively addressed see e.g. [4], a linear fractional transformation parametrization of linear observers is done in [9] and [11] introduces the proportional integral (PI) observer.

Unfortunately fault diagnosis is rarely tackled in the descriptor case. In [6] robust fault detection is performed with the use of generalized unknown input observers and in [12] the coprime factorisation approach of robust fault diagnosis is extended to the linear descriptor systems, via strict LMI based solution.

In this paper a simple method is proposed to design a PI observer for descriptor systems subject to failures and disturbances. The proposed approach is proved to be less restrictive since no assumption is made on the matrix distribution of the failures. The presented PI observer gives an asymptotic estimation of both states and failures and bounds the influence of the UI. Moreover a weighting function can be introduced to ensure performance of the estimation in a particular frequency range. For instance the weighting function can take into consideration the power spectrum of the disturbances or the frequency contents of the actuators noise. The design is reduced to the solution of a set of strict LMIs and then is reliably solvable with LMI toolboxes [7], [8]. The PI observer approach is applied to fault diagnosis for descriptor systems. Since the design procedure aims to bound the H_∞-norm of the transfer function from the disturbance to the estimation error, then a threshold for robust failure detection is easily available. Robust fault diagnosis is performed by synthesising a bank of dedicated PI observers. Both full and reduced order observers for UI descriptor systems are studied.

The paper is organized as follows. Section 2 presents the general problem statement the assumptions and the motivation of this note. In section 3 the design of PI observer is studied, weighting functions are included and the reduced order PI observer is treated. Section 4 is dedicated to the use of PI observers for fault diagnosis and section 5 is devoted to a numerical example.

II. PRELIMINARIES

This section recalls some basic knowledge about descriptor systems (taken from [5]) and details the assumptions made and the motivations of this contribution.

A. Backgrounds

We consider a class of linear time invariant (LTI) descriptor systems described by

$$\begin{cases} E^*\dot{x}(t) = A^*x(t) + B^*u(t) + N_1^*f(t) + M^*d(t) \\ y^*(t) = C^*x(t) + D^*u(t) + N_2^*f(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the descriptor variable, $u(t) \in \mathbf{R}^{nu}$ is the control input, $y(t) \in \mathbf{R}^m$ is the measured output, $f(t) \in \mathbf{R}^{nf}$ is the fault vector and $d(t) \in \mathbf{R}^{nd}$ is the unknown input. E^*, A^*, B^*, C^*, D^*, N_1^*, N_2^* and M^* are real known constant matrices with compatible dimensions. E^* is not assumed to be square and may be rank deficient, let note $r = rank(E^*)$.

The system (1) has a unique solution, for any initial conditions, if it is regular (i.e. E^* and A^* square and $det(sE^*-A^*)\neq 0$, for all s). The finite modes of the system correspond to the finite eigenvalues of (E^*, A^*).

A descriptor system is said to be stable if all the finite eigenvalues of *(E*, A*)* lie in the left half complex plane. Even for a regular input, a descriptor may have impulsive behaviour, due to the non causal part of its transfer function. A matrix pencil *(E*, A*)* has no impulsive mode and is said to be impulse-free if and only if the following equality holds

$$deg(det(sE^*-A^*)=rank(E^*), \text{ for all } s \text{ complex} \quad (2)$$

A descriptor system *(E*, A*, C*)* is impulse observable (resp. *R*-observable) if and only if (3) (resp. (4)) holds

$$rank\begin{bmatrix} E^{*T} & 0 & 0 \\ A^{*T} & E^{*T} & C^{*T} \end{bmatrix} = n + rank(E^*) \quad (3)$$

$$rank\begin{bmatrix} sE^* - A^* \\ C^* \end{bmatrix} = n, \text{ for all } s \text{ complex} \quad (4)$$

Impulse observability (resp. R-observability) reflects the ability to reconstruct the non causal (resp. causal) part of the system. If (3) is satisfied, there exists a matrix L such that *(E*, A*-LC*)* is impulse free. If (4) is satisfied there exists a matrix L such that the finite pole of *(E*, A*-LC*)* are set to prescribed values.

B. Assumptions and problem statement

The matrix pencil *(E*, A*)* is not assumed to be square and the regularity of the matrix pencil is not needed.

In the remaining the following assumptions are made

$$rank\begin{bmatrix} E^* & M^* \end{bmatrix} = rank E^* \quad (A1)$$

$$(E^*, A^*, C^*) \text{ is impulse observable} \quad (A2)$$

$$\dot{f}(t) = 0 \quad (A3)$$

(A1) means that the disturbance d, affect the dynamic relations of the system. (A3) means that the bounded nonlinear failures are approximated by step functions. In the case of PI observer this assumption is reasonable since the approximate error can be minimised by increasing the observer bandwidth (see discussion in [11]). Moreover the compromise between bandwidth increasing and noise sensibility pointed in [11] need not to be treated *a posteriori* but is integrated in the design procedure since the transfer from the UI to the estimate errors is bounded while a decay ration of the estimate error can be imposed. The assumption (A3) is needed for theoretical proofs but our approach remains effective in practical cases where (A3) is not satisfied, as one can see in the example, in section 5. Note that if *[E*/ C*]* is full column rank, (A1) and (A2) are satisfied.

It is of interest to notice that since N_2^* is non null, the approach is more generic than for instance in [11]. Augmenting the state vector with the failure vector to apply [11] to the augmented system will give rise to more restrictive conditions to verify the impulse observability condition.

An analogous argument can be opposed to the common idea that including UI in descriptor systems is redundant since it suffices to augment the state with the UI.

Performing a singular value decomposition of E^* and partitioning A^*, C^*, $N_1^*$and M^* according to it, we obtain the following matrices

$$PE^*Q = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}, PA^*Q = \begin{pmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{pmatrix}, PN_1^* = \begin{pmatrix} \underline{N}_{11} \\ \underline{N}_{12} \end{pmatrix}, PM^* = \begin{pmatrix} \underline{M}_{11} \\ \underline{M}_{12} \end{pmatrix}$$
$$C^*Q = \begin{pmatrix} \underline{C}_1 & \underline{C}_2 \end{pmatrix}$$

On the one hand, the impulse observability of the system augmented with the failure (resp. the UI) is verified if and only if P_f (resp, P_{UI}) is full column rank with

$$P_f = \begin{pmatrix} \underline{A}_{22} & \underline{N}_{12} \\ \underline{C}_2 & N_2^* \end{pmatrix}, P_{UI} = \begin{pmatrix} \underline{A}_{22} & \underline{M}_{12} \\ \underline{C}_2 & 0 \end{pmatrix} \quad (5)$$

On the other hand, the impulse observability of (1) is verified if and only if $[\underline{A}_{22}^T \ \underline{C}_2^T]$ is full row rank, which is less restrictive than the rank condition on P_f (or P_{UI}). The example in section 5 provides an illustration of the above discussion.

The aim of this contribution is to design the following PI observer to robustly estimate the state *x(t)* and the failure *f(t)* of (1)

$$\begin{cases} \dot{z} = Fz + (L_1 + L_2)y + Ju + H\hat{f} \\ \dot{\hat{f}} = L_3(y - \hat{y}) \\ \hat{x} = M_1 z + M_2 y + M_3 u \\ \hat{y} = C\hat{x} + Du + K\hat{f} \end{cases} \quad (6)$$

The above observer is termed proportional integral, because of the integral loop due to the second equation of (6). This structure of observer is particularly efficient for failures characterised by low frequency signals. The different matrices in (6) will be determined to ensure the convergence of the estimate errors.

III. DESIGN OF THE PI OBSERVER

In this section the existence condition of the PI observer is established and its design is performed in order to bound the sensitivity of the estimate error to the UI. Full and reduced order PI observer are studied.

A. H_∞ design of the full order PI observer

The design is based on the approach of [4] and [11]. Under assumption (A1) there exists a non singular matrix P such that system (1) is equivalent to the following

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t) + N_1 f(t) + Md(t) \\ y(t) = Cx(t) + Du(t) + N_2 f(t) \end{cases} \quad (7)$$

where E is full row rank and

$$PE^* = \begin{bmatrix} E \\ 0 \end{bmatrix}, PA^* = \begin{bmatrix} A \\ A_1 \end{bmatrix}, PB^* = \begin{bmatrix} B \\ B_1 \end{bmatrix}, PN_1^* = \begin{bmatrix} N_1 \\ N_{12} \end{bmatrix},$$
$$y(t) = \begin{bmatrix} -B_1 u(t) \\ y^*(t) \end{bmatrix}, PM^* = \begin{bmatrix} M \\ 0 \end{bmatrix}, C = \begin{bmatrix} A_1 \\ C^* \end{bmatrix}, N_2 = \begin{bmatrix} N_{12} \\ N_2^* \end{bmatrix}, D = \begin{bmatrix} 0 \\ D^* \end{bmatrix}$$

Proposition 1. Using the P transformation, the three following propositions are equivalent.

i *(E*, A*, C*)* is impulse observable
ii *(E,A,C)* is impulse observable
iii *[E / C]* is full column rank

Proof. *i*⇔ *ii* is obvious since P is non singular and *ii*⇔ *iii* since E is full row rank. ■

Under assumption (A3) and with proposition 1 there exist two matrices T_1 and T_2 such that

$$T_1 E + T_2 C = I_n . \tag{8}$$

Moreover, T_1 and T_2 are given by

$$T_1 = \left(E^T E + C^T C\right)^{-1} E^T$$
$$T_2 = \left(E^T E + C^T C\right)^{-1} C^T$$

and T_1 is full column rank. Let us set

$$\begin{gathered} H = T_1 N_1 - L_2 N_2,\ M_1 = I_n,\ M_2 = T_2,\ M_3 = -T_2 D \\ K = N_2,\ F = T_1 A - L_2 C,\ L_1 = F T_2,\ J = T_1 B - (L_1 + L_2) D \end{gathered} \tag{9}$$

Let e and ef denote the state and fault estimation errors defined by $e = x - \hat{x}$ and $e_f = f - \hat{f}$ respectively. The estimate error $\underline{e}^T = [e^T\ e_f^T]$ is given by

$$\begin{cases} \begin{pmatrix} \dot{e}(t) \\ \dot{e}_f(t) \end{pmatrix} = \left(\underline{A} - \underline{L}\underline{C}\right)\begin{pmatrix} e(t) \\ e_f(t) \end{pmatrix} + \underline{B} d(t) \\ \underline{e}(t) = \underline{D}\begin{pmatrix} e(t) \\ e_f(t) \end{pmatrix} \end{cases} \tag{10}$$

where $\underline{A} = \begin{pmatrix} T_1 A & T_1 N_1 \\ 0 & 0 \end{pmatrix}$, $\underline{B} = \begin{pmatrix} T_1 M_1 \\ 0 \end{pmatrix}$, ,$\underline{L} = \begin{pmatrix} L_2 \\ L_3 \end{pmatrix}$,

$\underline{C} = \begin{pmatrix} C & N_2 \end{pmatrix}$ and $\underline{D} = I_{n+nf}$

Obviously the PI observer exists if and only if there exists a matrix $\underline{L}$ such that the estimate errors asymptotically converges toward zero, in other words if and only if the pair $(\underline{A}, \underline{C})$ is detectable. The following theorem gives the existence condition of the PI observer.

Theorem 1. Under assumptions (A1-A3), the PI observer (6) for descriptor system (1) converges asymptotically if and only if the following condition holds

$$rank\begin{bmatrix} sE^* - A^* & -N_1^* \\ 0 & sI_{nf} \\ C^* & N_2^* \end{bmatrix} = n + nf \tag{11}$$
$$\text{for all } s \text{ complex with } Re(s) \geq 0$$

Proof. Since P is non singular, the following equality holds

$$rank\left(\begin{bmatrix} P & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} sE^* - A^* & -N_1^* \\ 0 & sI_{nf} \\ C^* & N_2^* \end{bmatrix}\right) = rank\begin{bmatrix} sE - A & -N_1 \\ 0 & sI_{nf} \\ C & N_2 \end{bmatrix}$$

since T_1 is full column rank, for all s complex we have

$$rank\begin{bmatrix} sE - A & -N_1 \\ 0 & sI_{nf} \\ C^* & N_2 \end{bmatrix} = rank\left(\begin{bmatrix} T_1 & 0 & T_2 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & -sI \\ 0 & 0 & 0 & I \end{bmatrix}\begin{bmatrix} sE - A & -N_1 \\ 0 & sI_{nf} \\ sC & sN_2 \\ C & N_2 \end{bmatrix}\right)$$

$$= rank\begin{bmatrix} sI_n - T_1 A & sT_2 N_2 - T_1 N_1 \\ 0 & sI_{nf} \\ C & N_2 \end{bmatrix}$$

$$= rank\begin{bmatrix} sI_n - T_1 A & -T_1 N_1 \\ 0 & sI_{nf} \\ C & N_2 \end{bmatrix}$$

which is equivalent to the detectability of the pair $(\underline{A}, \underline{C})$. The last equality is obtained by separately considering the cases $s=0$ and $s \neq 0$. ∎

The design parameter $\underline{L}$ is chosen to minimize the H_∞-norm of the transfer $T_{\underline{e}d}(s)$ from d to $\underline{e}$. the following theorem gives a computation of $\underline{L}$ to achieve the objective

$$\| T_{\underline{e}d}(s) \|_\infty < \gamma \tag{12}$$

for a prescribed real positive γ made as small as possible.

Theorem 2. The optimally robust PI observer (6) for descriptor system (1) which satisfies (12), is determined by minimizing γ under the following LMI constraints in the variables $X \in \mathbf{R}^{n\times n}$, $Y \in \mathbf{R}^{n\times m}$ and γ (real positive number)

$$\begin{pmatrix} \underline{A}^T X + X\underline{A} - Y\underline{C} - \underline{C}^T Y^T & X\underline{B} & \underline{D}^T \\ \underline{B}^T X & -\gamma I_{nd} & 0 \\ \underline{D} & 0 & -\gamma I \end{pmatrix} < 0 \tag{13}$$
$$X > 0$$

and $\underline{L}$ is given by $\underline{L} = X^{-1} Y$

where ">0" (resp. "<0") stands for symmetric positive (resp. negative) definite

Proof. Apply the well-known bounded real lemma (BRL) [16] to the system (10), there exists a matrix $\underline{L}$ such that (12) is verified if and only if there exists a symmetric positive definite X solution of

$$\begin{pmatrix} \left(\underline{A} - \underline{L}\underline{C}\right)^T X + X\left(\underline{A} - \underline{L}\underline{C}\right) & X\underline{B} & \underline{D}^T \\ \underline{B}^T X & -\gamma I & 0 \\ \underline{D} & 0 & -\gamma I \end{pmatrix} < 0$$
$$X > 0$$

Let us set $Y = X\underline{L}$, (13) follows. ∎

Remark 1. The minimisation of γ may result in slow dynamics of the estimation error (10). The LMI constraint (14) can be added to (13) to impose a minimal decay ratio λ by shifting the spectrum of $\underline{A}$-$\underline{LC}$ (i.e. the poles of (10) lie in the left half complex plane defined by $\{z \mid Re(z) < -\lambda\}$)

$$X\left(\underline{A} + \lambda I\right) + \left(\underline{A} + \lambda I\right)^T X - Y\underline{C} - \underline{C}^T Y^T < 0 \tag{14}$$

(13) and (14) guaranty a robust and effective estimation of the state and failures. Indeed it can be seen as an LMI formulation of a basic pole placement. More precise pole clustering could be perform in the LMI formulation [2] but would result in conservatism in the robustness objective.

B. H_∞ design of the PI observer using weighting function

The previous design method secures an unique norm bound of the sensitivity of the estimation to all the UIs and on the whole frequency range. This may cause some conservatism, as pointed in [15]. Introducing a stable weighting function on the UI allows to reflect the expected frequency content of d, for example a pass band filter will give a relative importance to a prescribed frequency range in which d is likely to be significant.

Let $W_1(s)$ be a stable weighting function with a state-space realization (A_1, B_1, C_1, D_1) and let denote n_1 its order. The objective is to design a PI observer such that

$$\left\| T_{\underline{e}d}(s) W_1(s) \right\|_\infty < \gamma \tag{15}$$

for a prescribed real positive γ, then it will ensure

$$\left| T_{\underline{e}d}(s) \right| < \gamma \left| W_1(s) \right|^{-1}, \; for\ all\ s \tag{16}$$

Assuming the weighting function to be stable, the existence condition of the PI-observer remains the same. The following theorem establishes the computation of a PI observer satisfying (15).

Theorem 3. Given a stable $W_1(s)$, the optimally robust PI observer (6) for descriptor system (1) is the solution of the minimization of γ under the following LMI constraint in the variables X_1 ((n×n) real matrix), X_2 ((n_1×n_1) real matrix) Y ((n×m) real matrix) and γ (real positive number)

$$\begin{pmatrix} X_1\underline{A} - Y\underline{C} + \left(X_1\underline{A} - Y\underline{C}\right)^T & X_1\underline{B}C_1 & X_1\underline{B}D_1 & \underline{D}^T \\ C_1^T \underline{B}^T X_1 & X_2 A_1 + A_1^T X_2 & X_2 B_1 & 0 \\ D_1^T \underline{B}^T X_1 & B_1^T X_2 & -\gamma I_{nd} & 0 \\ \underline{D} & 0 & 0 & -\gamma I \end{pmatrix} < 0$$

$$X_1 > 0,\; X_2 > 0 \tag{17}$$

and $\underline{L}$ is given by $\underline{L} = X_1^{-1} Y$

Proof. The augmented system is defined by

$$\begin{cases} \begin{pmatrix} \dot{\underline{e}}(t) \\ \dot{x}_1(t) \end{pmatrix} = \underline{\underline{A}} \begin{pmatrix} \underline{e}(t) \\ x_1(t) \end{pmatrix} + \underline{\underline{B}} d(t) \\ \underline{e}(t) = \underline{\underline{C}} \begin{pmatrix} \underline{e}(t) \\ x_1(t) \end{pmatrix} \end{cases}$$

where $\underline{\underline{A}} = \begin{pmatrix} \underline{A} & \underline{B}C_1 \\ 0 & A_1 \end{pmatrix} - \begin{pmatrix} \underline{L} \\ 0 \end{pmatrix} \begin{pmatrix} \underline{C} & 0 \end{pmatrix}, \underline{\underline{B}} = \begin{pmatrix} \underline{B}D_1 \\ B_1 \end{pmatrix}$ and $\underline{\underline{C}} = \begin{pmatrix} \underline{D} & 0 \end{pmatrix}$

applying the BRL to (17), a sufficient condition for (15) is that a symmetric positive definite block diagonal matrix $X=blockdiag(X_1, X_2)$ satisfies

$$\begin{pmatrix} X\underline{\underline{A}} + \underline{\underline{A}}^T X & X\underline{\underline{B}} & \underline{\underline{C}}^T \\ \underline{\underline{B}}^T X & -\gamma I & 0 \\ \underline{\underline{C}} & 0 & -\gamma I \end{pmatrix} < 0 \; and \; X > 0$$

let $Y=X_1\underline{L}$ and the proof is completed. ■

As pointed in the previous section the minimization of γ may result in slow dynamics. The speed of convergence can be forced by adding the LMI (18) to the set (17)

$$X_1\left(\underline{A} + \lambda I\right) + \left(\underline{A} + \lambda I\right)^T X_1 - Y\underline{C} - \underline{C}^T Y^T < 0 \tag{18}$$

C. Reduced order PI observer

For the sake of simplicity of implementation, it may be of interest to reduce the order of the PI observer. Here, following [11], the PI observer of order $r+nf$ is studied. Since E is full row rank, computing a column compression, there exists a non singular matrix $P_1=[E^+ W_E]$, where W_E is an orthonormal basis of the null space of E, such that $EP_1=[I_r\ 0]$. Then system (7) is equivalent to

$$\begin{cases} \dot{x}_1(t) = A_1 x_1(t) + A_2 x_2(t) + Bu(t) + N_1 f(t) + Md(t) \\ y(t) = C_1 x_1(t) + C_2 x_2(t) + Du(t) \end{cases}$$

where $AP_1=[A_1\ A_2]$ and $CP_1=[C_1\ C_2]$ (19)

and it is easy to verify that (A3) implies that C_2 is full column rank, then computing a row compression there exists a non singular matrix $P_2=[W_C / C_2^+]$, where W_C^T is an orthonormal basis of C_2^T, such that $P_2C_2=[0\ I_{n-r}]^T$ and (19) becomes

$$\begin{cases} \dot{x}_1(t) = \left(A_1 - A_2C_2^+C_1\right)x_1(t) + \left(B - A_2C_2^+D\right)u(t) + ... \\ \qquad ...A_2C_2^+ y(t) + Md(t) + \left(N_1 - A_2C_2^+N_1\right)f(t) \\ x_2(t) = C_2^+ y(t) - C_2^+C_1x_1(t) - C_2^+N_2f(t) \\ W_C y(t) = W_C C_1 x_1(t) + W_C Du(t) + W_C N_2 f(t) \end{cases}$$

Let synthesise the PI observer to estimate $x_1(t)$ and $f(t)$ and reconstruct $x_2(t)$ by

$$\begin{cases} \dot{\hat{x}}_1(t) = F\hat{x}_1(t) + L_1 y(t) + Ju(t) + H\hat{f}(t) + L_2P_3\left(y(t) - \hat{y}(t)\right) \\ \dot{\hat{f}}(t) = L_3P_3\left(y(t) - \hat{y}(t)\right) \\ \hat{x}_2(t) = C_2^+ y(t) - C_2^+C_1\hat{x}_1(t) - C_2^+N_2\hat{f}(t) \\ \hat{y}(t) = C_1\hat{x}_1(t) + C_2\hat{x}_2(t) + Du(t) + N_2\hat{f}(t) \end{cases} \tag{20}$$

where

$$\begin{gathered} F = A_1 - A_2C_2^+C_1,\; J = B - A_2C_2^+D \\ L_1 = A_2C_2^+ \; and \; H = N_1 - A_2C_2^+N_1 \end{gathered} \tag{21}$$

Since $P_1^{-1}=[E^T\ W_E]^T$, the estimate errors are given by the following dynamic system, where $e_1(t)$ denotes the estimate error of $x_1(t)$

$$\begin{cases} \begin{pmatrix} \dot{e}_1(t) \\ \dot{e}_f(t) \end{pmatrix} = \left(\underline{A} - \underline{L}\underline{C}\right) \begin{pmatrix} e_1(t) \\ e_f(t) \end{pmatrix} + \underline{B}d(t) \\ \begin{pmatrix} e(t) \\ e_f(t) \end{pmatrix} = \underline{D} \begin{pmatrix} e_1(t) \\ e_f(t) \end{pmatrix} \end{cases} \tag{22}$$

where $\underline{A} = \begin{pmatrix} A_1 - A_2C_2^+C_1 & N_1 - A_2C_2^+N_1 \\ 0 & 0 \end{pmatrix}, \; \underline{B} = \begin{pmatrix} M_1 \\ 0 \end{pmatrix}$

$$\underline{C} = \begin{pmatrix} W_C C_1 & W_C N_2 \end{pmatrix}, \; \underline{D} = \begin{pmatrix} E^+ - W_E C_2^+ C_1 & -W_E C_2^+ N_2 \\ 0 & I \end{pmatrix}$$

Similarly to the full order case, the existence condition of such an observer is the detectability of the pair $(\underline{A}, \underline{C})$. An optimally robust reduced order PI observer satisfying (12) is obtained with theorem 2 and the weighting function can be introduced by applying theorem 3.

IV. ROBUST FAULT DIAGNOSIS

In this section the PI observers are specifically designed for fault diagnosis. In the framework of robust fault detection, we are looking forward to generating alarm signals indicating whether a given fault occurs or not. Alarms are usually generated by a decision making based on residual signals. The estimation of the faults and the value of γ can be interpreted as residual signals and threshold respectively since

$$\left| \hat{f}_i(t) \right| \geq \gamma \Rightarrow f_i(t) \neq 0 \tag{23}$$

where $(*)_i$ denotes the i^{th} component of the vector $(*)$, then a very simple decision making (fixed threshold) generates failure alarms without any false alarm due to the UI.

If the objective is failure diagnosis rather than state estimation, the robustness of the fault estimation can be improved by minimizing the transfer from d to e_f instead of $\underline{e}$ (set $\underline{D}=[0\ I_{nf}]$ in (13) or (17)). A bank of nf dedicated PI observer –an observer is designed for each fault– can be synthesised for more accurate fault estimation. Each PI observer is designed to minimize the transfer from d to e_{fi} .

Algorithm of robust fault diagnosis For each failure f_i synthesise a PI observer (6) by minimizing γ_1 under the LMI constraint in the variables X_i ((n×n) real matrix), Y_i ((n×m) real matrix) and γ_i (real positive number)

$$\begin{pmatrix} \underline{A}^T X_i + X_i \underline{A} - Y_i \underline{C} - \underline{C}^T Y_i^T & X_i \underline{B} & \underline{D}_i^T \\ \underline{B}^T X_i & -\gamma_i I_{nd} & 0 \\ \underline{D}_i & 0 & -\gamma_i \end{pmatrix} < 0 \qquad (24)$$

$$X_i > 0$$

where $\underline{A}$, $\underline{B}$, $\underline{C}$ are defined in (10) and $\underline{D}_i$ is the $(n+i)^{th}$ column of $\underline{D}$ in (10),then $\underline{L}_i$ is given by

$$\underline{L}_i = X_i^{-1} Y_i$$

and an alarm $a_i(t)$ associated to f_i is defined by

$$a_i(t) = \begin{cases} 1, \ if \ \left|\hat{f}_i(t)\right| \geq \gamma_i \\ 0, \ if \ \left|\hat{f}_i(t)\right| < \gamma_i \end{cases}$$

Remark 2. Weighting function and/or reduced order PI observer can obviously be used in the previous algorithm, applying theorem 3 with $\underline{D}i$ and/or using $\underline{A}$, $\underline{B}$, $\underline{C}$, $\underline{D}$ defined in (22).

V. NUMERICAL EXAMPLE

In this section an illustrative example is examined. Let consider the LTI descriptor (1) subject to actuator and sensor failures and disturbances, defined by

$$E^* = \begin{pmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} A^* = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} B^* = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} C^* = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}^T$$

$$N_1^* = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} M^* = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} D^* = \begin{pmatrix} 0 \\ 0 \end{pmatrix} N_2^* = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad (25)$$

First it is easy to check that (25) verifies (A1), (A2) and (A3). According to the discussion of section 2.2 one can see that neither ($[E^*\ 0]$, $[A^*\ N_1^*]$,$[C^*\ N_2^*]$) nor ($[E^*\ 0]$, $[A^*\ M^*]$, $[C^*\ 0]$) are impulse observable which proves the efficiency of the above contribution.

The unknown inputs d are random signals uniformly distributed in $[-O.5\ 0.5]$. $f_1(t)$ is an actuator failure and $f_2(t)$ is a sensor offset defined by (26)

$$f_1(t) = \begin{cases} -u(t), \ for \ \ 3 < t < 6 \\ 0 \qquad , \ else \end{cases}$$
$$f_2(t) = \begin{cases} 1, \ for \ \ 7 < t < 9 \\ 0, \ else \end{cases} \qquad (26)$$

Firstly the design of a PI observer is illustrated, introducing a weighting function and secondly, application to fault diagnosis is illustrated.

A. Design of a PI observer with weighting function

The PI observer is synthesised to satisfy (15) with $W_1(s)=2*(1+0.01s)/(1+0.0002s)$. Solving the LMI (17) the PI observer (6) is designed. Figure 1 displays the minimal and maximal singular values of $T_{\underline{e}d}(s)$ and the magnitude of $W_1(s)$. The design objective (16) is fulfilled and we have $||T_{\underline{e}d}(s)||_\infty=0.23$. The observer provides a correct estimation of the states, as seen on figure 2, and of the failures, see figure 3. On figure 3, one can see that the estimation of f_2 is perturbed by the variation of f_1 ($t=3$ and $t=5$) this can be significantly improved by the use of dedicated observers instead of a global one.

B. Fault diagnosis with dedicated PI observers

In this section fault diagnosis is the principal objective, thus following the algorithm of section 4 we design two dedicated PI observers to minimize the transfer of the perturbation to each estimate error separately. Figure 4 displays the obtained estimation of $f_2(t)$. The H_∞ norms of $T_{ef1d}(s)$ and $T_{ef2d}(s)$ are $\gamma_1=0.14$ and $\gamma_2=0.1$ respectively.

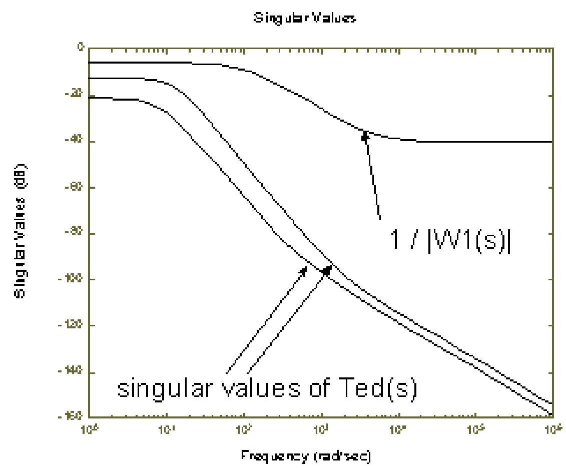

Fig 1. Singular values of $T_{\underline{e}d}(s)$ and $W_1(s)$.

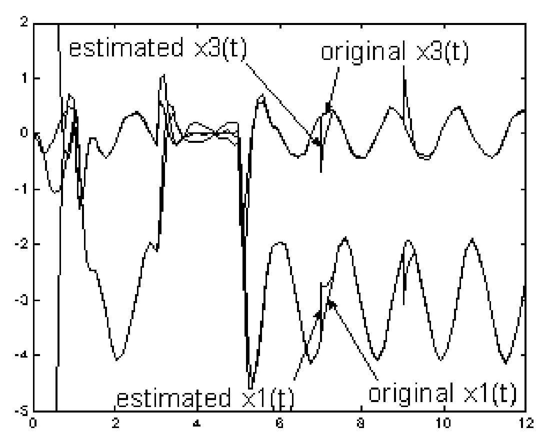

Fig 2. State estimation of x_1 and x_3.

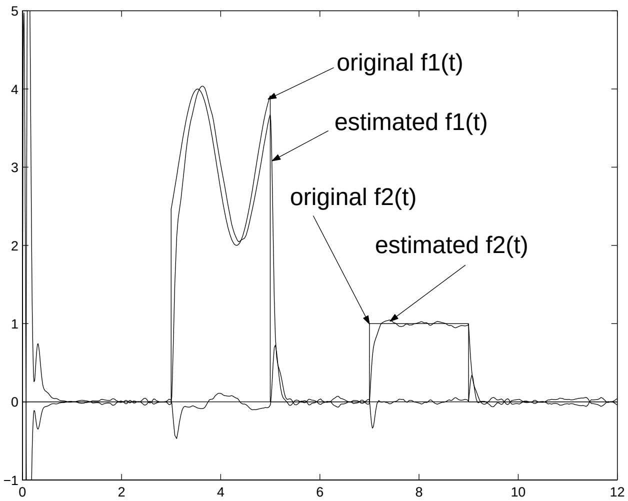

Fig 3. Fault estimation of f_1 and f_2.

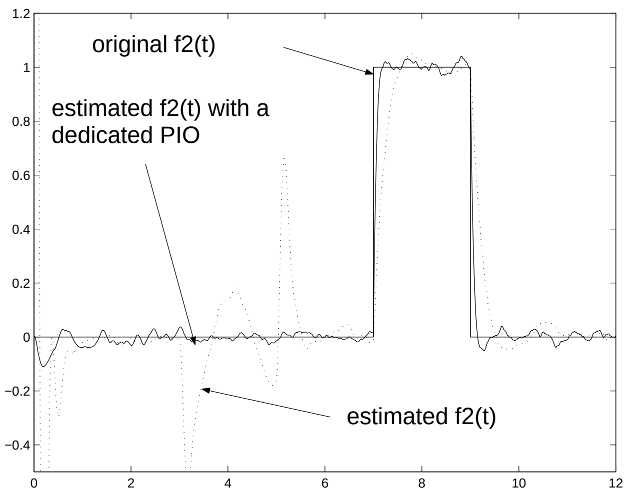

Fig 4. Comparison of fault estimations of f_2 with a dedicated PI observer (solid line) and with a general PI observer (dashed line).

VI. CONCLUSION

The design of robust PI observer for descriptor systems has been studied. The existence condition and an LMI-based computation have been established. PI observers provide the estimation of the state variables and of the failures. They are optimally robust since they are synthesised in order to minimise the sensitivity of the estimate errors to the UI. The introduction of weighting function ensures the performance of the estimation in a prescribed frequency range. The H_∞-norm bound of the sensitivity to perturbations can be interpreted as a threshold for fault diagnosis. An algorithm for robust fault diagnosis is derived. For ease of implementation, reduced order PI observers have also been studied.

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