# [1]H∞ Fault Detection and Isolation for Descriptor Systems: A Matrix Inequalities Approach

D. Koenig* S. Mammar† and B. Marx*
*Laboratoire d'Automatique de Grenoble
(UMR CNRS-INPG-UJF)
BP 46, 38402 SAINT MARTIN D'HERES CEDEX, FRANCE
† INRETS, 2 av. du Général Malleret-Joinville
BP 34 – 94114 ARCEUIL, FRANCE
Damien.Koenig@esisar.inpg.fr, Mammar@inrets.fr, Benoit.Marx@esisar.inpg.fr

**Abstract:** In this paper, a linear matrix equality (LMI) - based H∞ filtering formulation is presented for fault detection and isolation (FDI) problems of linear time-invariant descriptor systems. The fixed-order H∞ FDI filter design is characterized in terms of definite LMIs with no equality constraint, which are done recently by [1]. Using these matrix inequalities, we show that the solvability of a set of matrix inequality is necessary and sufficient to the existence of a proper FDI filter that satisfies a prescribed H∞ norm condition as well as stabilizing the closed-loop system, estimating the faults and eliminating all impulsive modes.

**Keywords:** Descriptor system, FDI, LMI and H∞ filtering.

## 1 Introduction

The complexity of today's control systems requires fault tolerance schemes to provide early warning of faulty sensors, actuators or system component. Such schemes need to detect and isolate faults before they lead to catastrophes, so that appropriate actions and control reconfiguration can be accomplished. Consequently, the FDI problem has received considerable attention in the last two decades, like observer schemes, parameter estimation methods, or parity state space approaches. Detailed surveys of different FDI methods can be found in [2]. More recently in standard state space equations $(\dot{x} = Ax + \cdots)$, H∞ optimization for FDI have received increased attention for providing disturbance rejection and robustness properties to the FDI schemes [3], [4] and [5]. Here, using the H∞ norm condition for descriptor systems [1] we extend the result of LMI-based H∞ filtering formulation for standard systems [5], to descriptor continuous linear time-invariant systems.

## 2 H∞ control

To begin with, we review the H∞ control theory and LMI problems for descriptor systems. Consider a generalized plant P

$$P: \begin{cases} E_p\dot{x}_p = A_p x_p + B_u u + B_w w \\ z = C_z x_p + D_{zu} u + D_{zw} w \\ y = C_y x_p + D_{yw} w \end{cases} \qquad (1)$$

where $x_p \in \Re^{n_x}$ is the plant state vector, $u \in \Re^{n_u}$ is the control input, $w \in \Re^q$ is the exogenous input, $z \in \Re^p$ is the controlled output and $y \in \Re^m$ is the measured output. The matrix $E \in \Re^{n_x \times n_x}$ has rank r ($\le n_x$). The other matrices have appropriate sizes. Since E is singular, systems (1) can be rewritten as [6]

$$P: \begin{cases} E\dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x \\ y = C_2 x \end{cases} \qquad (2)$$

where

$$x = \left[x_p^T \ \varsigma^T \ \xi^T\right]^T \in \Re^{n = n_x + p + m}, \ B_1 = \left[B_w^T \ D_{zw}^T \ D_{yw}^T\right]^T, \ C_1 = \left[C_z \ I \ 0\right],$$

$$B_2 = \left[B_u^T \ D_{zu}^T \ 0\right]^T, \ E = \begin{bmatrix} E_p & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ A = \begin{bmatrix} A_p & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix}, \ C_2 = \left[C_y \ 0 \ I\right].$$

From now on we assume that $D_{ij} = 0$ (i=z,y; j=u,w) and that augmentation (2) does not change the finite modes or impulsive modes of the original descriptor system [7]. Though such an augmentation of a descriptor system contributes additional components to the descriptor variable and it does not increase the computational complexity of the LMI-based synthesis method of this paper.

Then, the H∞ control problem is to find a dynamic output feedback controller K [6], [1]

$$K: \begin{cases} E\dot{x}_K = A_K x_K + B_K y \\ u = C_K x_K \end{cases} \qquad (3)$$

where $x_K \in \Re^{n = n_x + p + m}$ is the descriptor variable, and $A_K$, $B_K$, $C_K$ are coefficient matrices to be determined such that the closed loop system written as

$$\begin{aligned} E_c\dot{x}_c &= A_c x_c + B_c w \\ z &= C_c x_c \end{aligned} \qquad (4)$$

where $x_c = \left[x \ \ x_K\right]$, and $E_c = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}$, $B_c = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$

$$A_c = \begin{bmatrix} A & B_2 C_K \\ B_K C_2 & A_K \end{bmatrix}, \ C_c = \left[C_1 \ 0\right]$$

is admissible and such that $\left\| C_c\left(sE_c - A_c\right)^{-1} B_c \right\|_\infty < \gamma$ for a given scalar $\gamma > 0$.

Definition [6]:

I A pencil sE-A (or a pair (E,A) is *regular* if det(sE-A) is not identically zero.

II For a regular pencil sE-A, the finite eigenvalues of sE-A are said to be the *finite modes* of (E,A). Suppose that $Ev_1=0$. Then the infinite eigenvalues associated with the generalized principal vector vk satisfying $Ev_k=Av_{k-1}$, k=2, 3, 4, … are impulsive modes of (E,A).

III A pair (E,A) is *admissible* if it is regular and has neither impulsive modes nor unstable finite modes.

Together with the H∞ controller K the necessary and sufficient condition for the existence of the controller (3) is given by the following theorem.

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Theorem1 [1]: The pair ($E_c$, $A_c$) is admissible and $\left\| C_c (sE_c - A_c)^{-1} B_c \right\|_\infty < \gamma$ if and only if symmetric matrices P, Q $\in \Re^{n \times n}$ and matrices R, S $\in \Re^{(n-r)\times(n-r)}$, M $\in \Re^{n_u \times n}$, L $\in \Re^{n\times m}$ exist for which the LMIs

$$\begin{bmatrix} \Phi_1 & \Phi_{12} \\ \Phi_{12}^T & -\gamma^2 I \end{bmatrix} < 0 \qquad (5)$$

$$\Phi_1 = A\left(PE^T + VSU^T\right) + \left(PE^T + VSU^T\right)^T A^T + B_2 M + M^T B_2^T + B_1 B_1^T$$

$$\Phi_{12} = \left(PE^T + VSU^T\right)^T C_1^T$$

$$V = \mathrm{null}(E) \in \Re^{n\times(n-r)}, \quad U = \mathrm{null}\left(E^T\right) \in \Re^{n\times(n-r)}$$

$$\mathrm{rank}(V) = \mathrm{rank}(U) = n - r$$

$$\begin{bmatrix} \psi_1 & \psi_{12} \\ \psi_{12}^T & -\gamma^2 I \end{bmatrix} < 0 \qquad (6)$$

$$\psi_1 = A^T\left(QE + URV^T\right) + \left(QE + URV^T\right)^T A + LC_2 + C_2^T L + C_1^T C_1$$

$$\psi_{12} = \left(QE + URV^T\right)^T B_1$$

$$\begin{bmatrix} E_R^T P E_R & \gamma I \\ \gamma I & E_L^T Q E_L \end{bmatrix} > 0 \qquad (7)$$

$$E = E_L E_R^T, \quad E_L = \mathrm{null}\left(U^T\right) \in \Re^{n\times r}$$

$$E_R = \mathrm{null}\left(V^T\right) \in \Re^{n\times r}, \quad \mathrm{rank}(E_L) = \mathrm{rank}(E_R) = r$$

hold.

Then, such the controller (3) is given by

$$A_K = \left\{\left(QE + URV^T\right) - \gamma^2\left(PE^T + VSU^T\right)^{-1}\right\}^{-T} \times \left\{\gamma^2 A^T \left(PE^T + VSU^T\right)^{-1} + \left(QE + URV^T\right)^T A + \left(QE + URV^T\right)^T B_2 M \left(PE^T + VSU^T\right)^{-1} + LC_2 + C_1^T C_1 + \left(QE + URV^T\right)^T B_1 B_1^T \left(PE^T + VSU^T\right)^{-1}\right\}$$

$$B_K = -\left\{\left(QE + URV^T\right) - \gamma^2\left(PE^T + VSU^T\right)^{-1}\right\}^{-T} L$$

$$C_K = M\left(PE^T + VSU^T\right)^{-1}$$

Proof is done in [1].

## 3 Design methodology

In order to formulate the FDI filter problem in a H∞ filtering framework, it will be represented in a linear fraction transformation (LFT) form [5]. Let us consider the following actuator and sensor fault linear time-invariant descriptor system

$$\begin{aligned} E\dot{x} &= Ax + B_{u_1} u_1 + B_d d + B_{u_1} f_a \\ y &= C_y x + D_{yd} d + f_s \end{aligned} \qquad (8)$$

where x is the state vector, $u_1$ is the control input vector, d is the disturbance vector, $f_a$ is the actuators fault vector, , $f_s$ is the sensors fault vector, and E, A, $B_{u1}$, $B_d$, $C_y$, $D_{yd}$, are real matrices of appropriate dimensions.

Define now the vector of actuator and sensor faults $f = \left[f_a^T \; f_s^T\right]^T$ and the extra input $u_2$, which be the estimate $\hat{f}$ of the fault vector f

$$u_2 = \hat{f} \qquad (9)$$

The filter objectives are twofold: stabilizing the closed loop system and achieving the H∞ norm of the closed loop transfer function from $w = \left[d^T \; f^T\right]^T$ to $z = f - u_2$.

Using the defining vector of unknown input $w = \left[d^T \; f^T\right]^T$, of controlled input $u = \left[u_1^T \; u_2^T\right]^T$ and of regulated output $z = f - u_2$, system (8) can thus be put in the general form of H∞ optimization given by system (1) where $B_u = \left[B_{u_1} \; 0\right]$, $B_w = \left[B_d \; B_{u_1} \; 0\right]$, $C_z = [0]$, $D_{zu} = [0 - I]$, $D_{zw} = [0 \, I]$ and $D_{yw} = \left[D_{yd} \; 0 \, I\right]$.

It should be noticed that P includes possible scaling or weighting filters [4] for performance enhancement and frequency range faults. These scaling or weighting functions can be easily included in the representation of the plant (1), it increases the order but without significant difference to the treatment here presented.

Now to complete the design methodology, transform the obtained system (1) as system (2) and find by theorem 1 the dynamic output feedback controller K (3) whose output u is the estimate of the actuator and sensor faults.

## 4 Conclusion

In this paper, H∞ optimization combined with sensor and actuator FDI for descriptor systems has been presented. Using LMIs, we show that the solvability of a set of matrix inequalities is necessary and sufficient to the existence of a proper H∞ FDI filter that satisfies a H∞ norm conditions as well as stabilizing the closed-loop system, estimating the faults and eliminating all impulsive modes.

## Reference


[1] E. Uezato and M. Ikeda, Strict LMI Conditions for Stability, Robust Stabilization, and H∞ Control of Descriptor Systems, Proc. 38th Conference on Decision and Control, Phoenix, Arizona, USA, pp. 4092-4097, 2000.

[2] R.J. Patton, Robust model-based fault diagnosis: the state of the art, IFAC, pp. 1-24, 1994.

[3] J. Chen and R.J. Patton, H∞ formulation and solution for robust fault diagnosis, 14th World Congress of IFAC, pp. 127-132, Beijing, China, 1999.

[4] S. Mammar and D. Koenig, Robust Control and Fault Detection Synthesis with Application to Tractor-Semitrailer Automatic Sterring, IEEE CCA, Anchorage, Alaska, 2000.

[5] E.G. Nobrega, M.O. Abdalla and K.M. Grigoriadis, LMI-Based Filter Design for Fault Detection and Isolation, 14th World Congress of IFAC, pp. 127-132, Beijing, China, 1999

[6] I. Masubuchi, Y. Kamitane, A. Ohara and N. Suda, H∞ Control for Descriptor Systems: A Matrix Inequalities Approach, Automatica, Vol, 33, No4, pp. 669-673, 1997.

[7] G. C. Verghese, B. C. Levy and T. Kailath, A generalized state-space for singular systems, IEEE Trans. Autom. Control. AC-26, 811- 831, 1981.