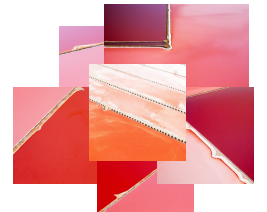




Prognostics aware control design for extended remaining useful life: Application to Liquid Propellant Reusable Rocket Engine

HAC Meeting 22th November 2023

Julien Thuillier, Mayank Shekhar Jha,
Sebastien Le Martelot, and Didier Theilliol



Project supported by CNES 2021-2023 supervised by Marco Galeotta (until November 2022) and by Sebastien Le Martelot



Ferrara, Italy

4-7 June, 2024

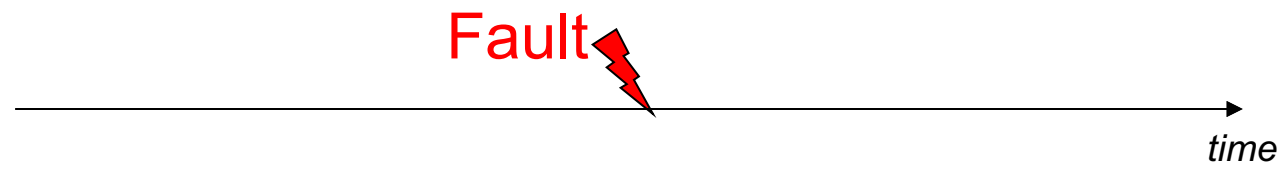


Important dates

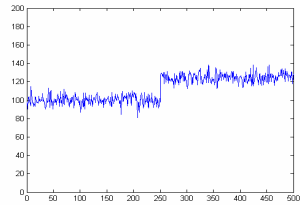
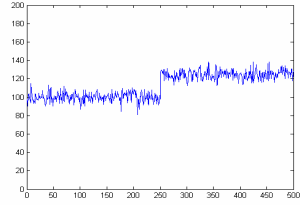
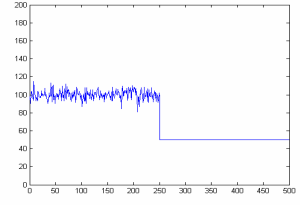
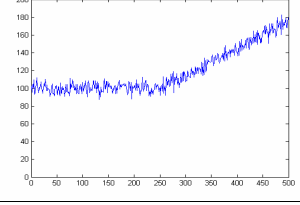
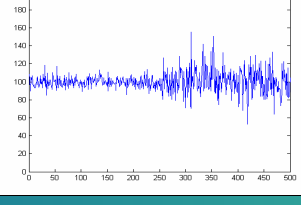
- Invited session proposals: November 15, 2023
- Submission of contributions: November 28, 2023
- Tutorial session proposals: January 8, 2024
- Roundtable proposals: January 8, 2024
- Notification of acceptance: March 1, 2024
- Final paper submission: April 1, 2024
- Early registration: April 1, 2024

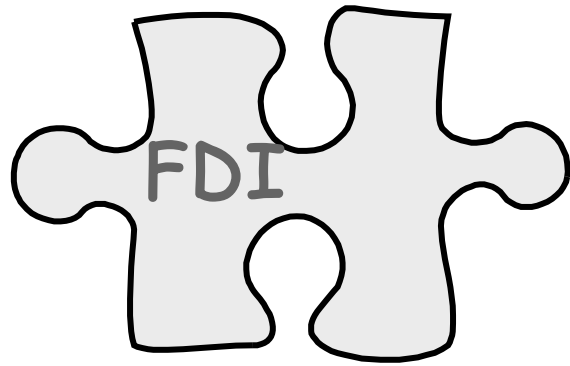
- **Introduction / Context**
- **Problem Statement**
- **Our Solution**
- **Illustrative Examples**
- **Conclusions / Perspectives**

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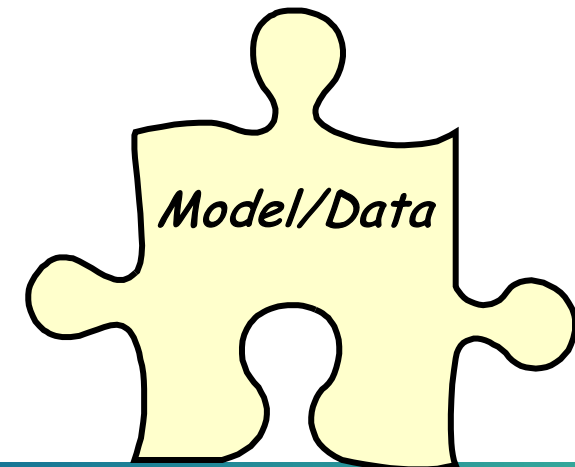
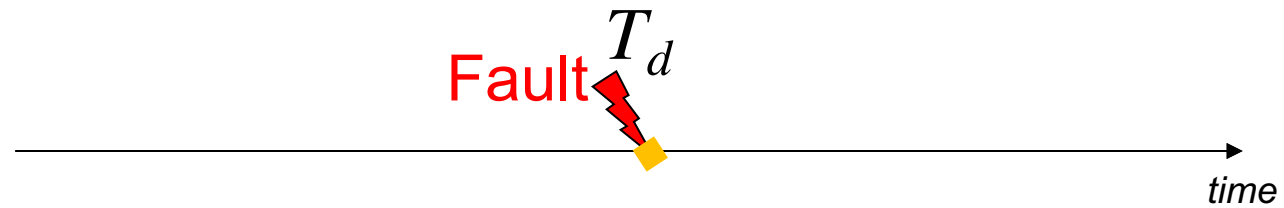


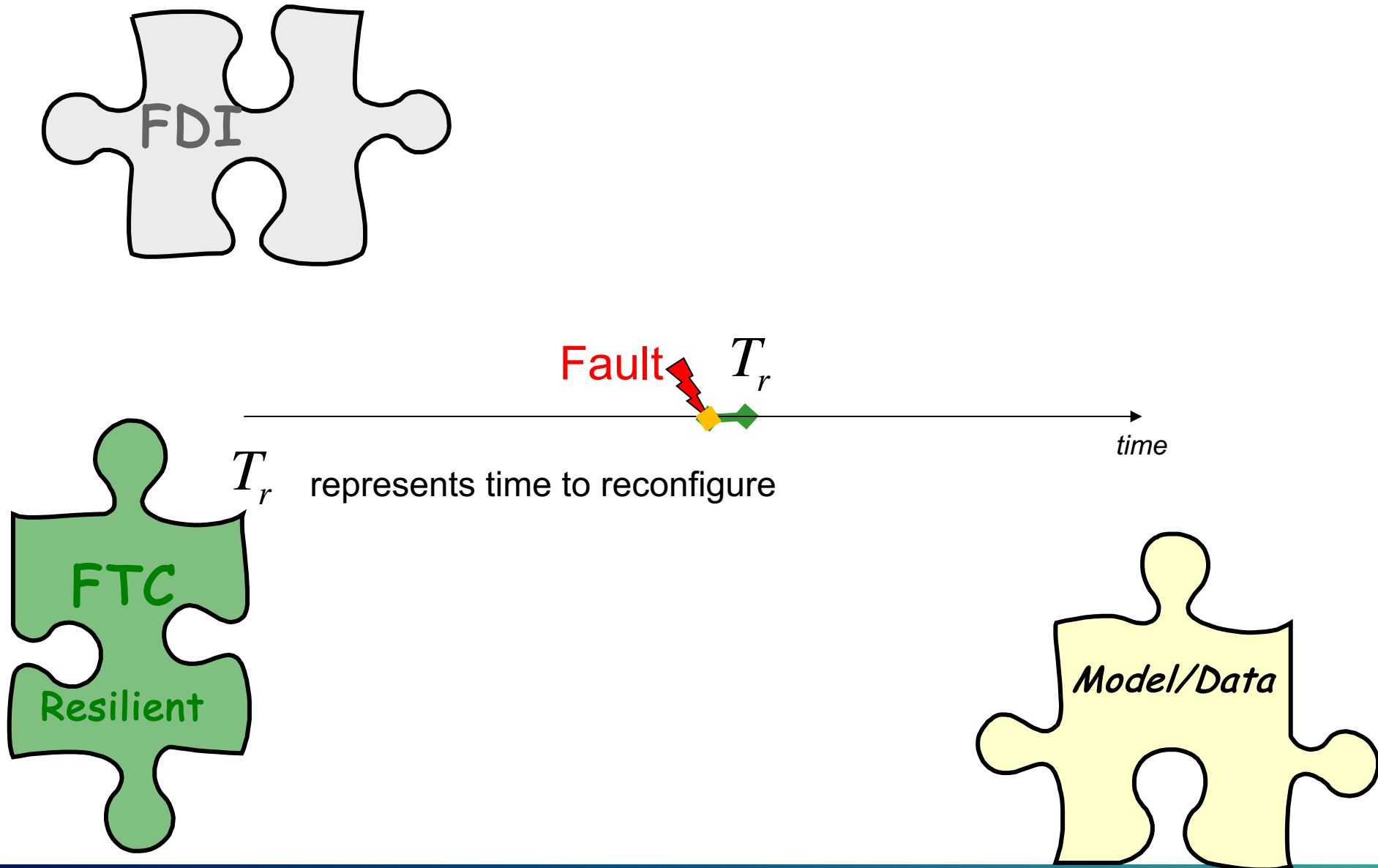
Context

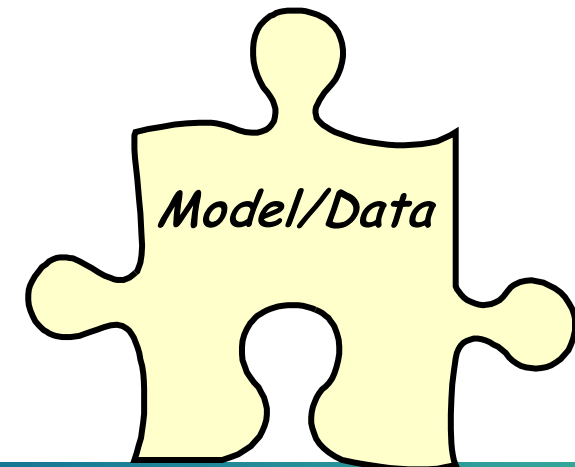
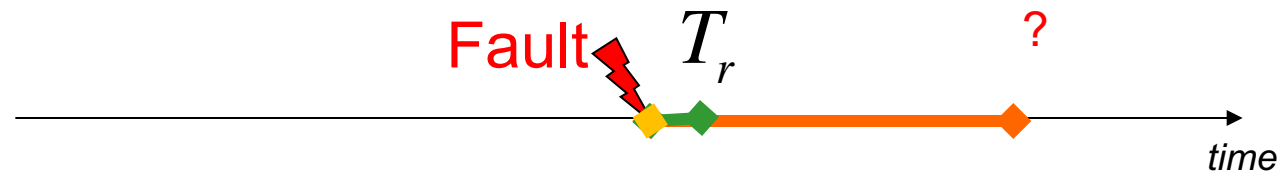
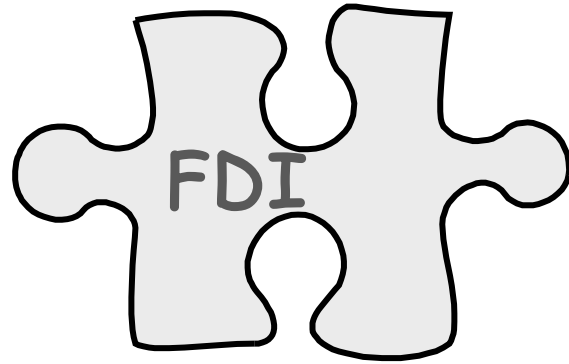
Fault	Equation	Cause	Effect	Graph
Bias	$Y_i(t) = Y_i^*(t) + \varepsilon_i(t) + \alpha \Gamma_\theta$ $\alpha = \text{constant } t \text{ value}$ $\Gamma_\theta = \begin{cases} 0, & t < \theta \\ 1, & t \geq \theta \end{cases}$ <p>the Heavisides function</p>	<ul style="list-style-type: none"> Decalibration 	Jump in the mean value	 <p>The graph shows a noisy signal that starts at a mean value of approximately 100. At t=250, the signal jumps to a new mean value of approximately 120 and continues with the same level of noise.</p>
Gain modification	$Y_i(t) = \alpha \Gamma_\theta Y_i^*(t) + \varepsilon_i(t)$	<ul style="list-style-type: none"> Decalibration 	Jump in the mean value	 <p>The graph shows a noisy signal that starts at a mean value of approximately 100. At t=250, the signal jumps to a new mean value of approximately 120 and continues with the same level of noise.</p>
Total breakdown	$Y_i(t) = (1 - \Gamma_\theta) [Y_i^*(t) + \varepsilon_i(t)] + k$ $k \in \{Y_{min}, 0, Y_{max}\}$	<ul style="list-style-type: none"> Destruction of the sensor Disconnection of an electrical signal 	Signal constant, zero or min/max	 <p>The graph shows a noisy signal that starts at a mean value of approximately 100. At t=250, the signal drops to zero and remains constant at zero for the rest of the time period.</p>
Offset drift Bias drift	$Y_i(t) = Y_i^*(t) + \varepsilon_i(t) + \alpha r_{\theta,t} \Gamma_\theta$ $r_{\theta,t} = \begin{cases} 0 & , t < 0 \\ t - \theta & , t \geq 0 \end{cases}$	<ul style="list-style-type: none"> Ageing Slow destruction of the sensor Temperature drift 	Signal slowly deviates from the true value	 <p>The graph shows a noisy signal that starts at a mean value of approximately 100. At t=250, the signal begins to slowly drift upwards, reaching a mean value of approximately 150 by t=500.</p>
Increased noise	$Y_i(t) = Y_i^*(t) + \varepsilon_i(t) + [\eta_i(t) - \varepsilon_i(t)] \Gamma_\theta$ <p>η_i = increased noise standard deviation</p>	<ul style="list-style-type: none"> Electro-magnetic disturbance Loss of screening Disconnection of the ground signal 	Small signal to noise ratio	 <p>The graph shows a noisy signal that starts at a mean value of approximately 100. At t=250, the signal becomes much noisier, with the noise level increasing significantly, making the signal difficult to discern.</p>



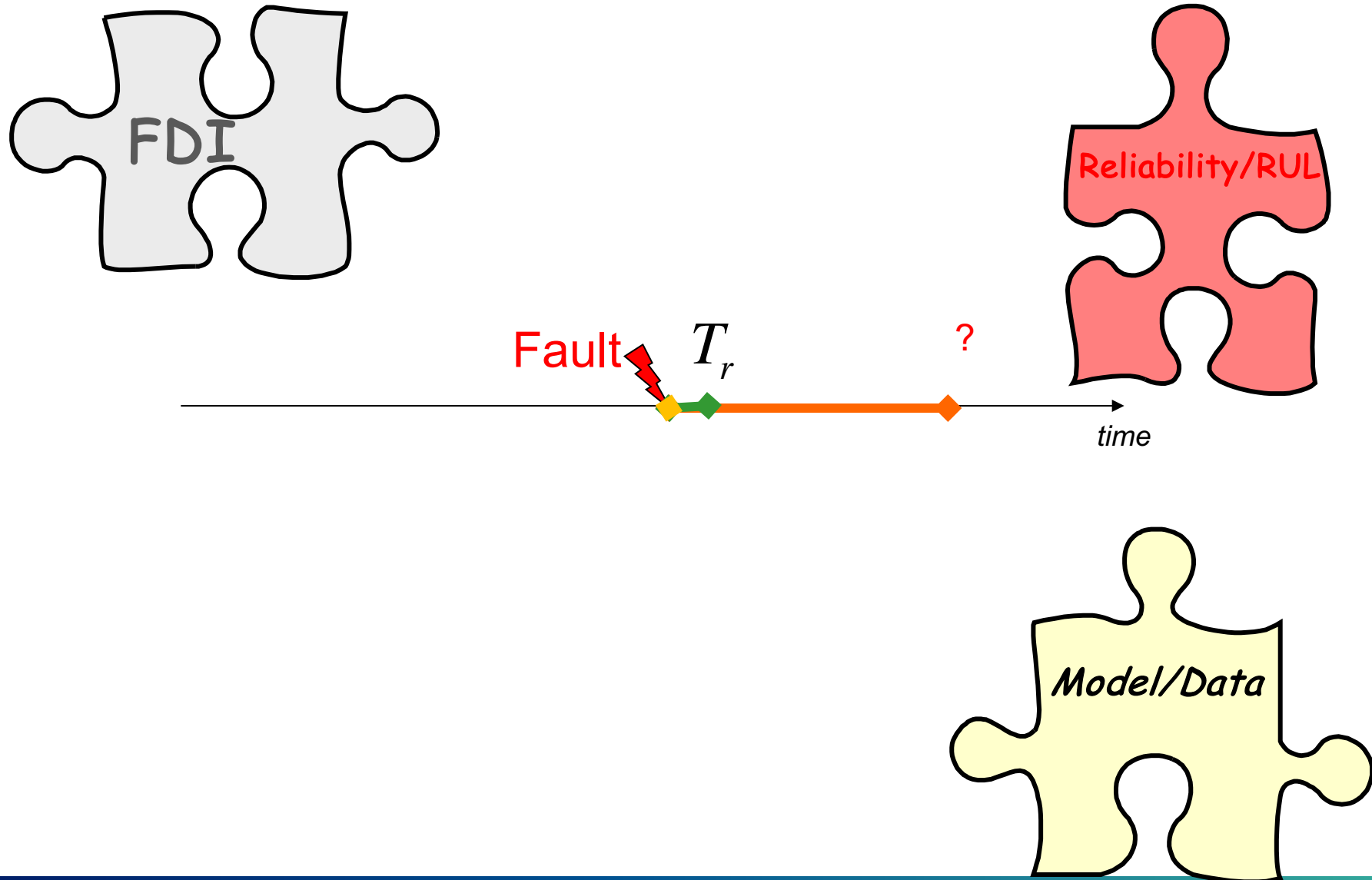
T_d represents time to detect, isolate and to estimate



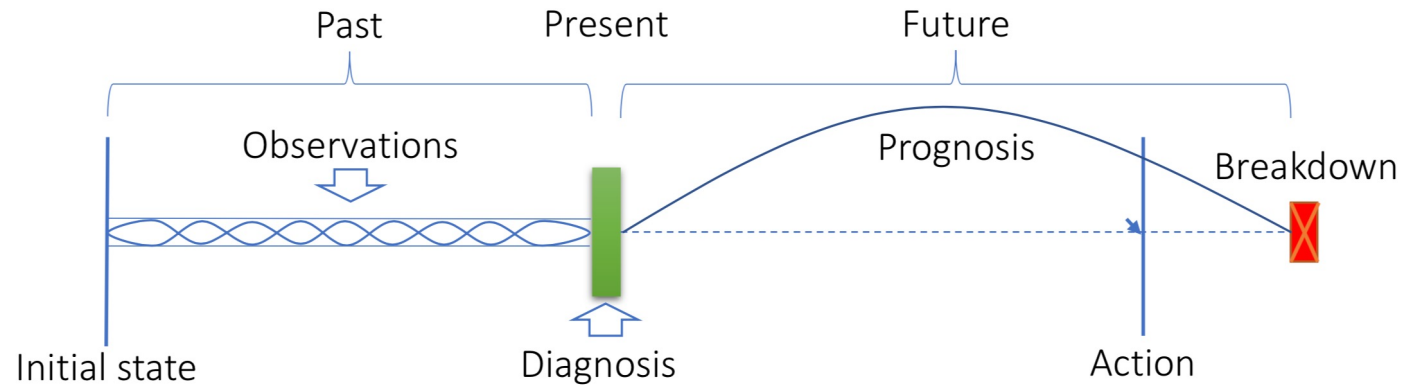




Context

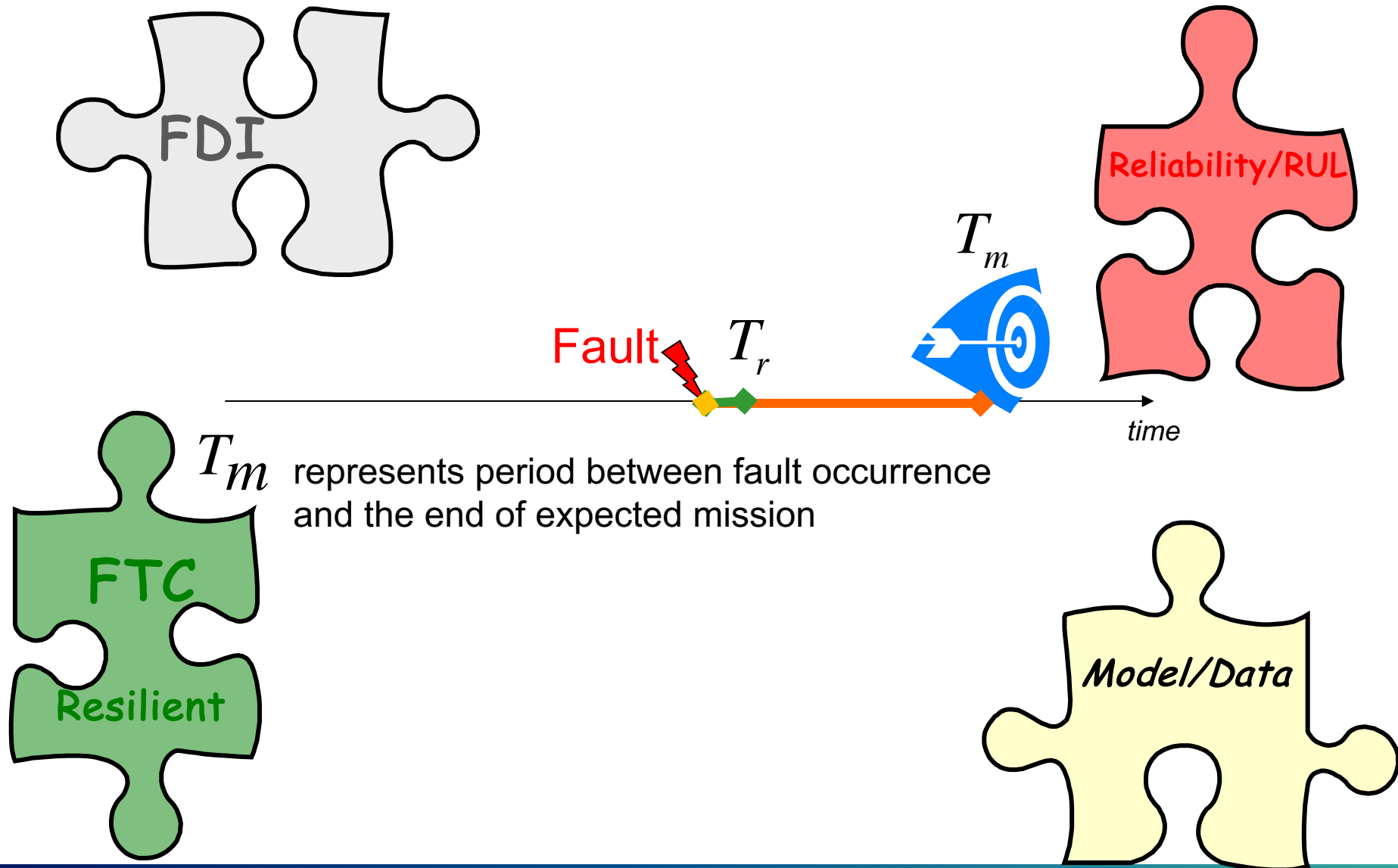


DIAGNOSIS & PROGNOSIS



- Initial state: State distribution at the beginning.
- Observations: Extracted data from sensors on the complex system.
- Diagnosis: Identifying the current hidden health state of the system based on observations.
- Prognosis: prediction of the future health evolution of the system considering operating conditions.
- Action: Plan maintenance.
- Breakdown: System failure.

Heath Aware Control Design



Consider the linear system as follows:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + w(k) \\ z(k) = Cx(k) + v(k) \end{cases}$$

with measured output and tracking control loop:

$$\begin{cases} y(k) = Dx(k) \\ u(k) = -K_1x(k) + K_2(y_{ref}(k) - y(k)) \end{cases}$$

Consider the degraded linear system as follows:

$$x(k+1) = Ax(k) + B(u(k) - d(k)) + w(k)$$

with Unknown degradation (dimension 1)

$$d(k) = \phi(x(k), u(k), \theta(k))$$

where $\phi(\cdot)$ is a nonlinear function

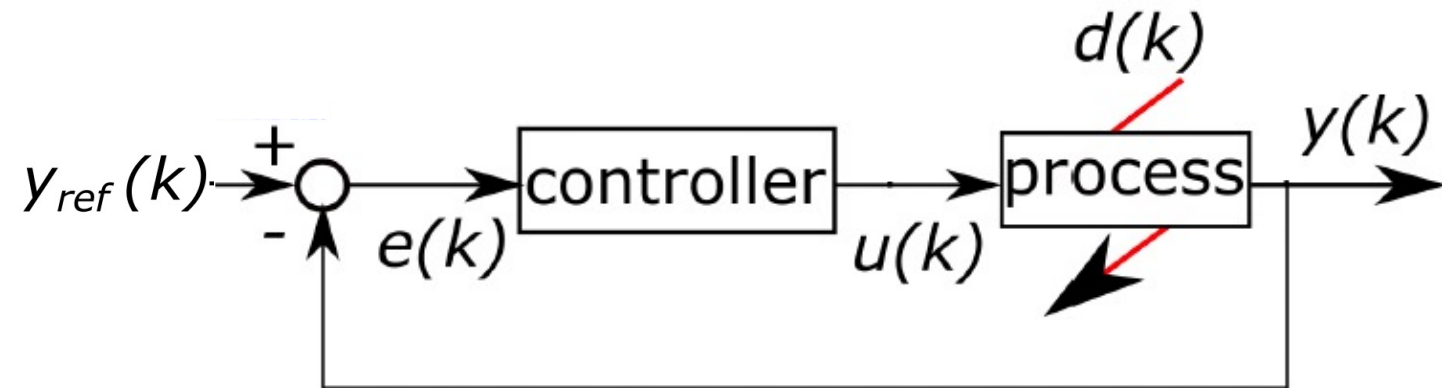
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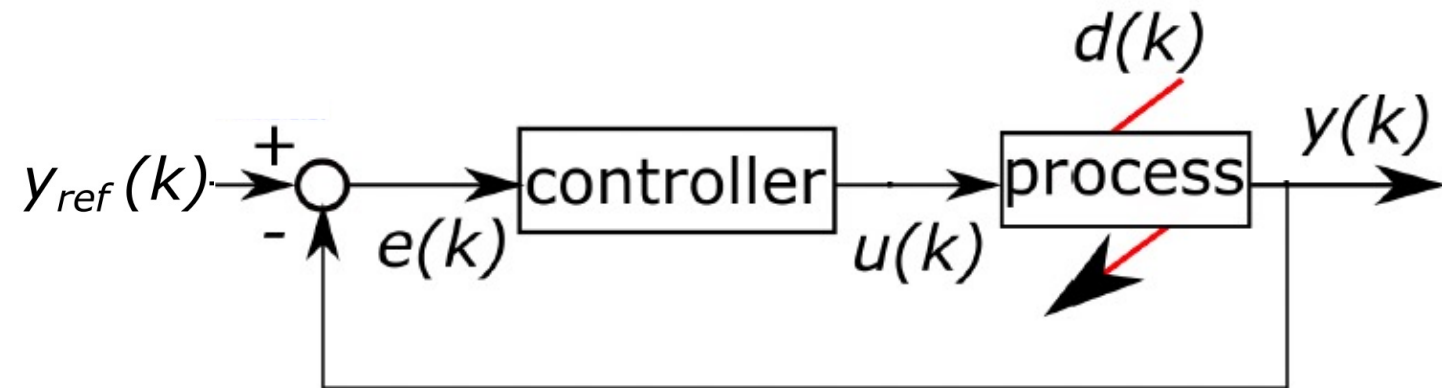
$$x(k+1) = Ax(k) + B(u(k) - d(k)) + w(k)$$

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Remaining Useful Life
Dynamic Behaviour
Etc ...



Consider the degraded linear system as follows:

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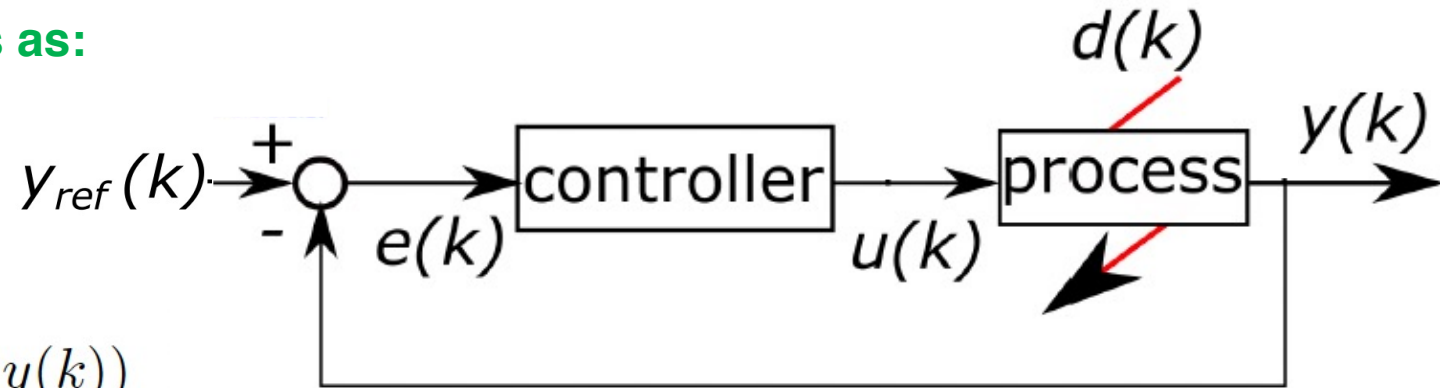
Remaining Useful Life
Dynamic Behaviour
Etc ...

Remaining Useful Life dynamics as:

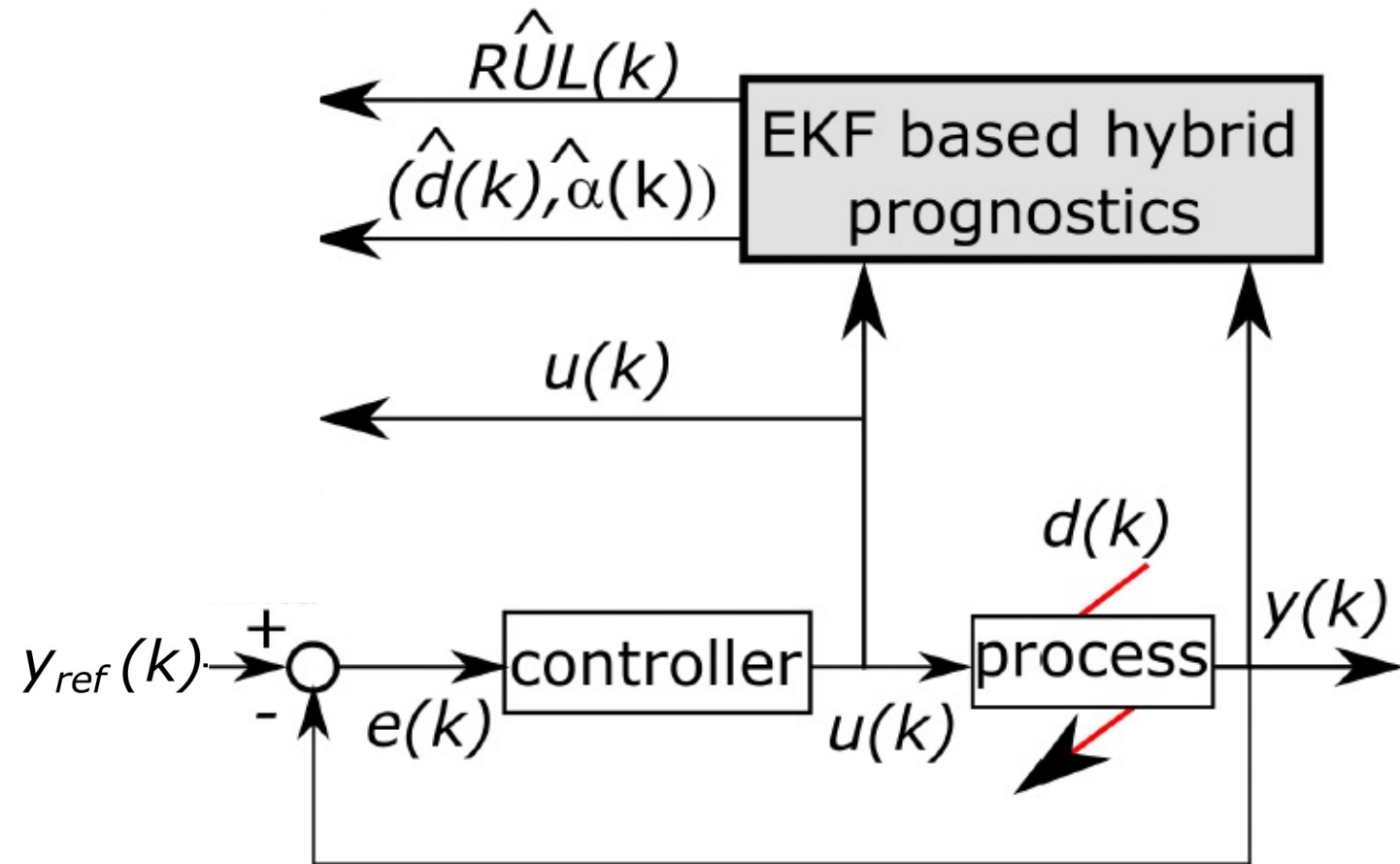
$$\widehat{RUL}(k) = \Phi(\hat{d}(k), u(k))$$

For tracking control as:

$$\widehat{RUL}(k) = \Phi(\hat{d}(k), (y_{ref}(k) - y(k)))$$

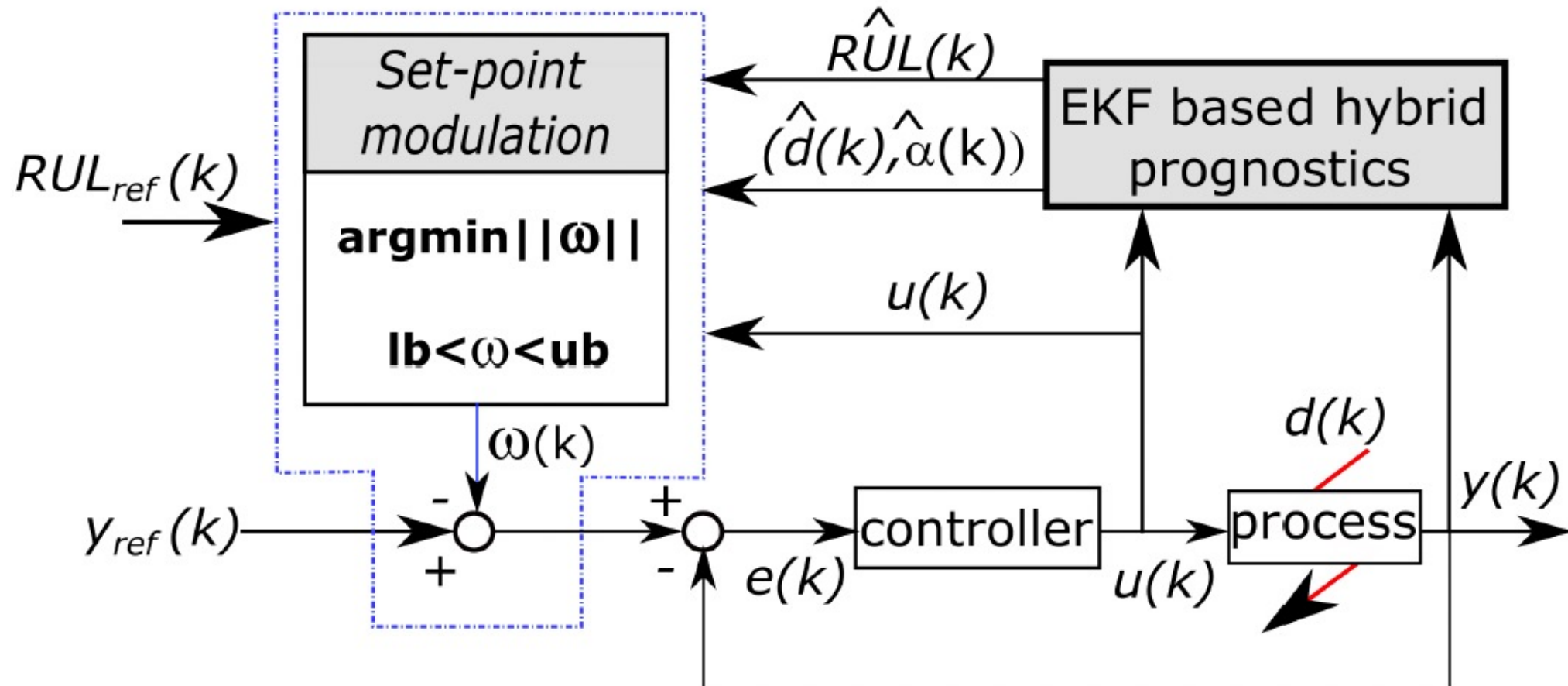


- Introduction / Context
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HYBRID PROGNOSTICS ENABLED SET POINT MODULATION STRATEGY

... cnes ...



Degradation model *sensitive* to system input but unknown $\phi(\cdot)$ so approximated by:

$$d(k) = e^{(\sum_{i=1}^k u(i) \cdot \alpha(i-1) \cdot T_s)} + w_d(k) \quad \text{Time + Cumulative « Energy »}$$

$$\left\{ \begin{array}{l} d(k+1) = e^{(\sum_{i=1}^{k+1} u(i) \cdot \alpha(i-1) \cdot T_s)} + w_d(k+1) \\ \quad = e^{(\sum_{i=1}^k u(i) \cdot \alpha(i-1) \cdot T_s)} \times e^{(u(k+1) \cdot \alpha(k) \cdot T_s)} \\ \quad \quad + w_d(k+1) \\ \quad = d(k) e^{(u(k+1) \cdot \alpha(k) \cdot T_s)} + w_d(k+1) \\ \alpha(k+1) = \alpha(k) + w_\alpha(k+1) \end{array} \right.$$

Recurrent formulation

$$\left\{ \begin{array}{l} d(k+1) = d(k)(1 + u(k+1) \cdot \alpha(k) \cdot T_s) + w_d(k+1) \\ \alpha(k+1) = \alpha(k) + w_\alpha(k+1) \end{array} \right.$$

State space representation included degradation becomes as:

$$\begin{pmatrix} x(k+1) \\ x_d(k+1) \end{pmatrix} = \begin{pmatrix} A & 0_{n,2} \\ 0_{2,n} & A_d(k) \end{pmatrix} \begin{pmatrix} x(k) \\ x_d(k) \end{pmatrix} + Bu(k) + w(k)$$

$$\begin{pmatrix} z(k) \\ z_d(k) \end{pmatrix} = \begin{pmatrix} C \\ C_d \end{pmatrix} \begin{pmatrix} x(k) \\ x_d(k) \end{pmatrix} + v(k)$$

where $x_d(k) = [d(k) \quad \alpha(k)]^T$

with
$$\begin{cases} d(k+1) = d(k)(1 + u(k+1).\alpha(k).T_s) + w_d(k+1) \\ \alpha(k+1) = \alpha(k) + w_\alpha(k+1) \end{cases}$$

Algorithm 1 Algorithm for degradation estimation using EKF

Input: $z_{d_k}, P_{k|k}, u_k$

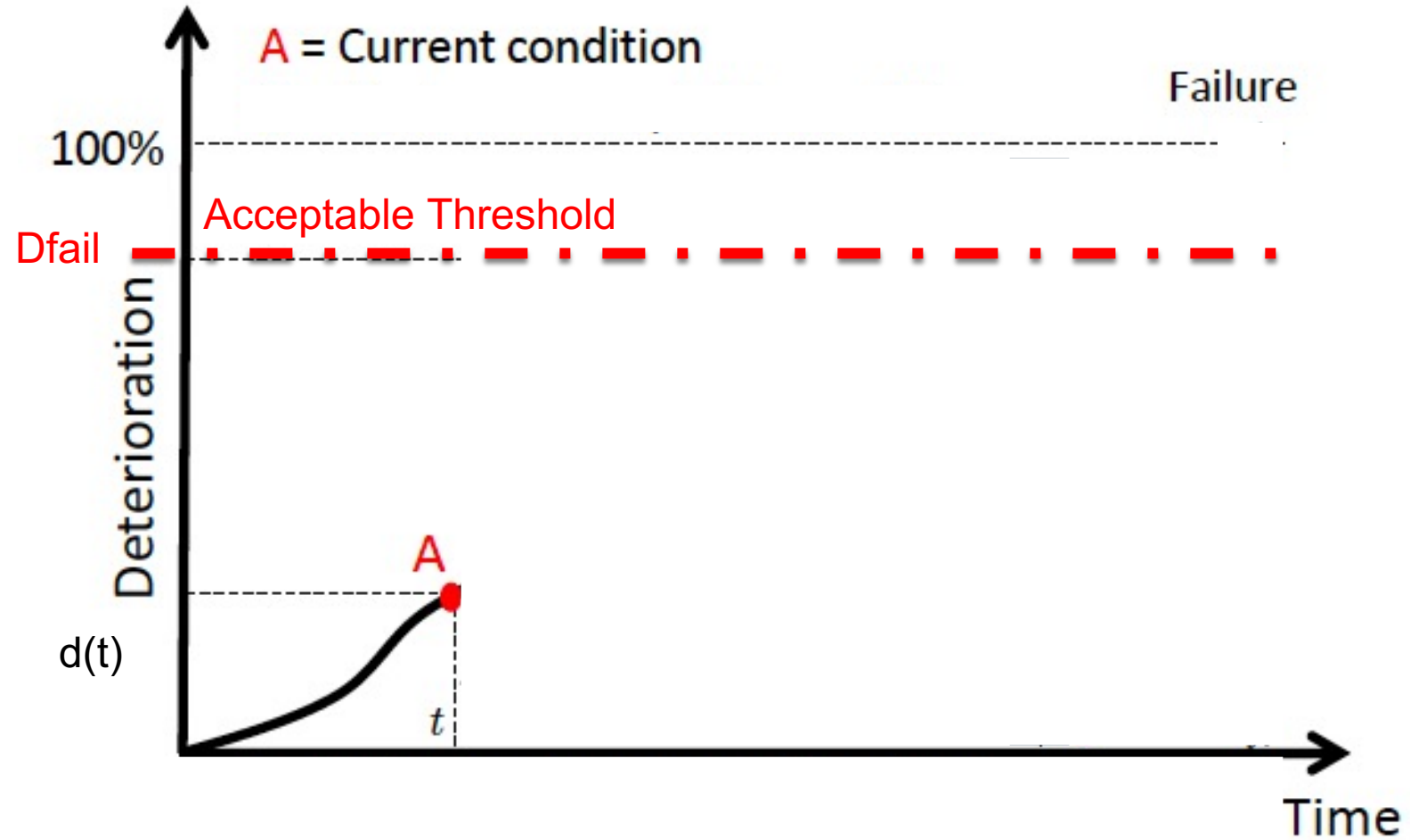
Output: $\hat{x}_{d_{k+1}|k+1}$

Initialisation : $z_{d_0}, \hat{x}_{0|0}, P_{0|0}$

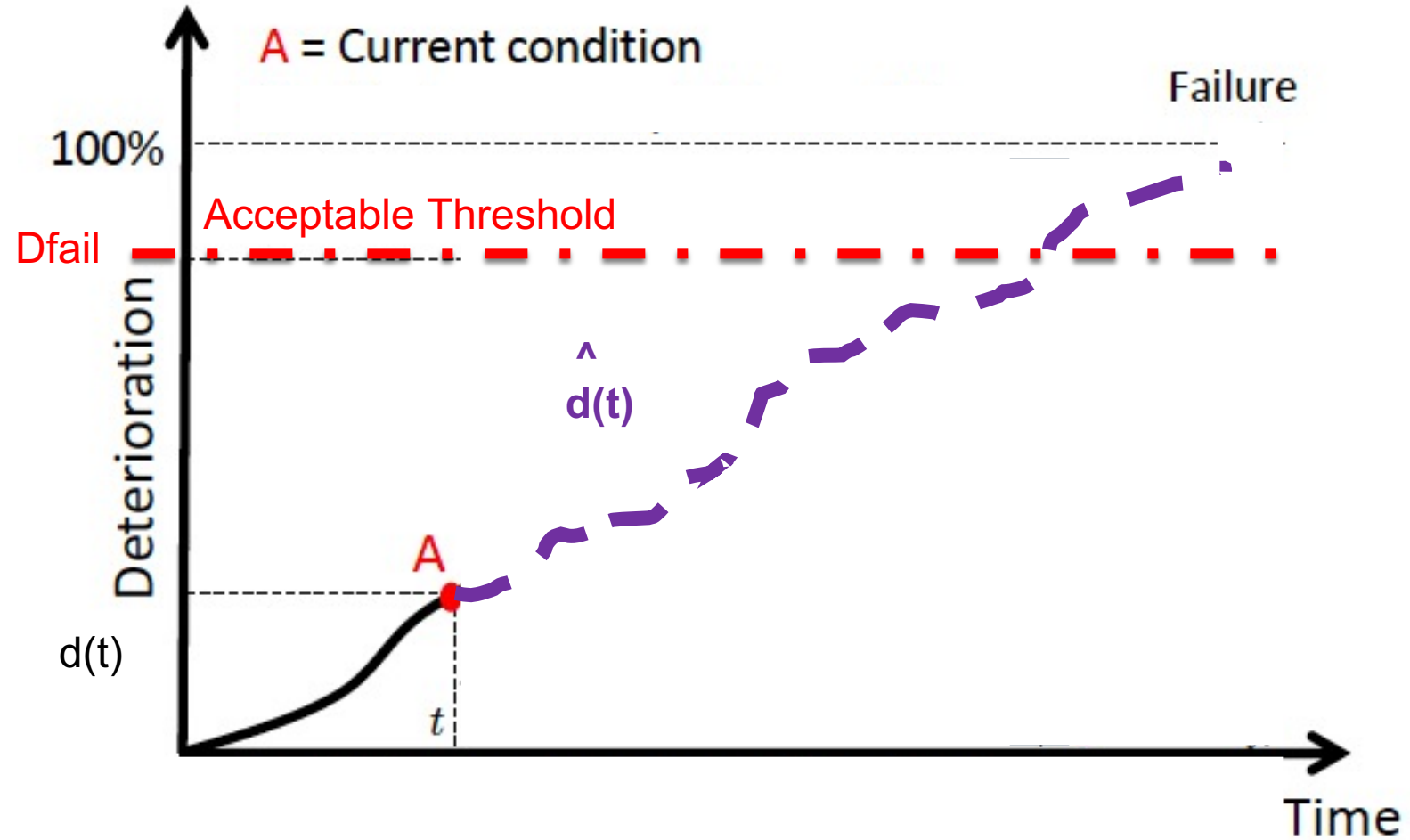
- 1: **if** ($i \neq 0$) **then**
 - 2: $\hat{x}_{d_{k+1}|k} = A_d(k)x_{d_k|k}$
 - 3: $P_{k+1|k} = J(k)P_{k|k}J(k) + Q$
 - 4: $z_{d_{k+1}|k} = C_d x_{d_{k+1}|k}$
 - 5: $S_{k+1} = H_{k+1}P_{k+1|k}H_{k+1}^T + GR_{k+1}G^T$
 - 6: $K_{k+1} = P_{k+1|k}H_{k+1}^T S_{k+1}^{-1}$
 - 7: $\hat{x}_{d_{k+1}|k+1} = \hat{x}_{k+1|k} + K_{k+1|k}(z_{d_{k+1}} - z_{dk+1|k})$
 - 8: $P_{k+1|k+1} = P_{k+1|k} - K_{k+1}S_{k+1}K_{k+1}^T$
 - 9: $z_{d_{k+1}|k+1} = C_d \hat{x}_{d_{k+1}|k+1}$
 - 10: **end if**
 - 11: **return** $\hat{x}_{d_{k+1}|k+1}$
-

RUL PREDICTION OVER LONG TIME HORIZON

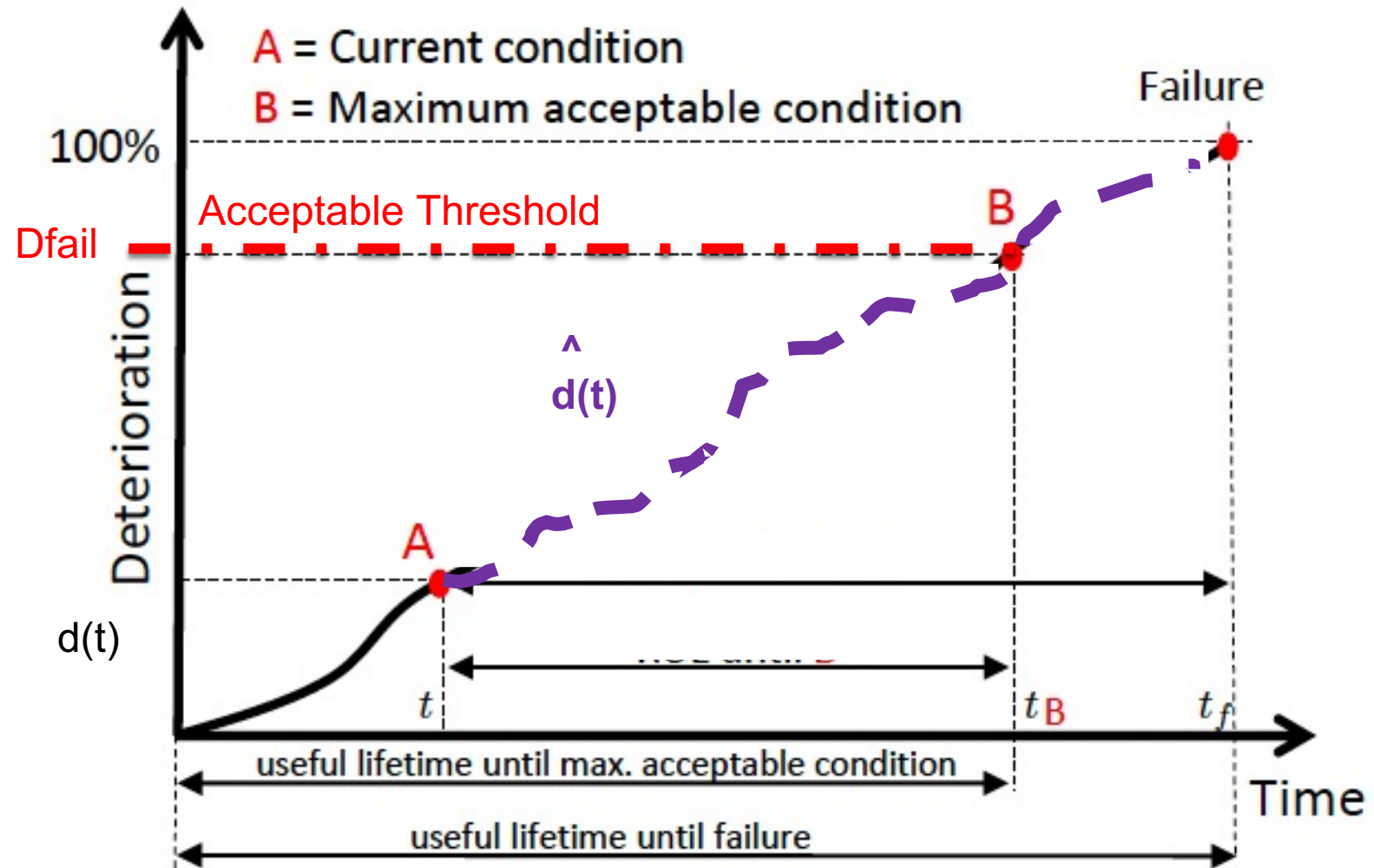
RUL DEFINITION



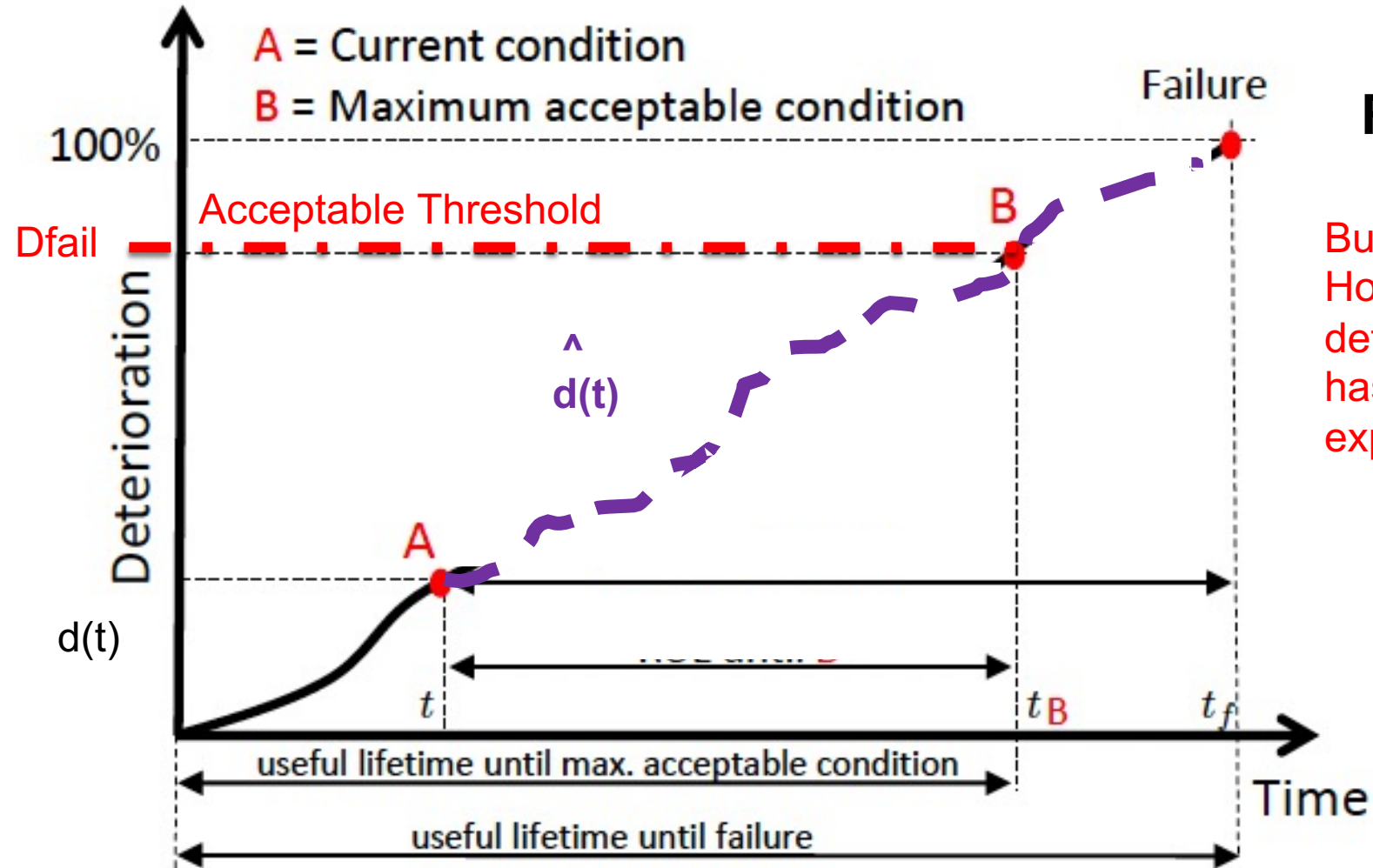
RUL DEFINITION



RUL DEFINITION



RUL DEFINITION



$$RUL(t) = t_B - t$$

But t_b is unknown
However only
deterioration level
has been fixed by
expert

Algorithm 2 L -step ahead RUL prediction

Input: u_k

Output: \hat{RUL}

Initialisation : $L = 0$

- 1: $[\hat{d}(i), \hat{\alpha}(i)] = \text{Algorithm 1}(z_{d_k}, P_{k|k}, u_k)$
 - 2: $d_L(i) = \hat{d}(i)$
 - 3: **while** $d_L(i) \leq D_{fail}$ **do**
 - 4: $d_L(i) = d_L(i) \cdot (1 + \hat{\alpha}(i) \cdot u(i) \cdot T_s)$
 - 5: $L = L + T_s$
 - 6: **end while**
 - 7: $\hat{RUL} = L$
 - 8: **return** \hat{RUL}
-

or based on
formal equations

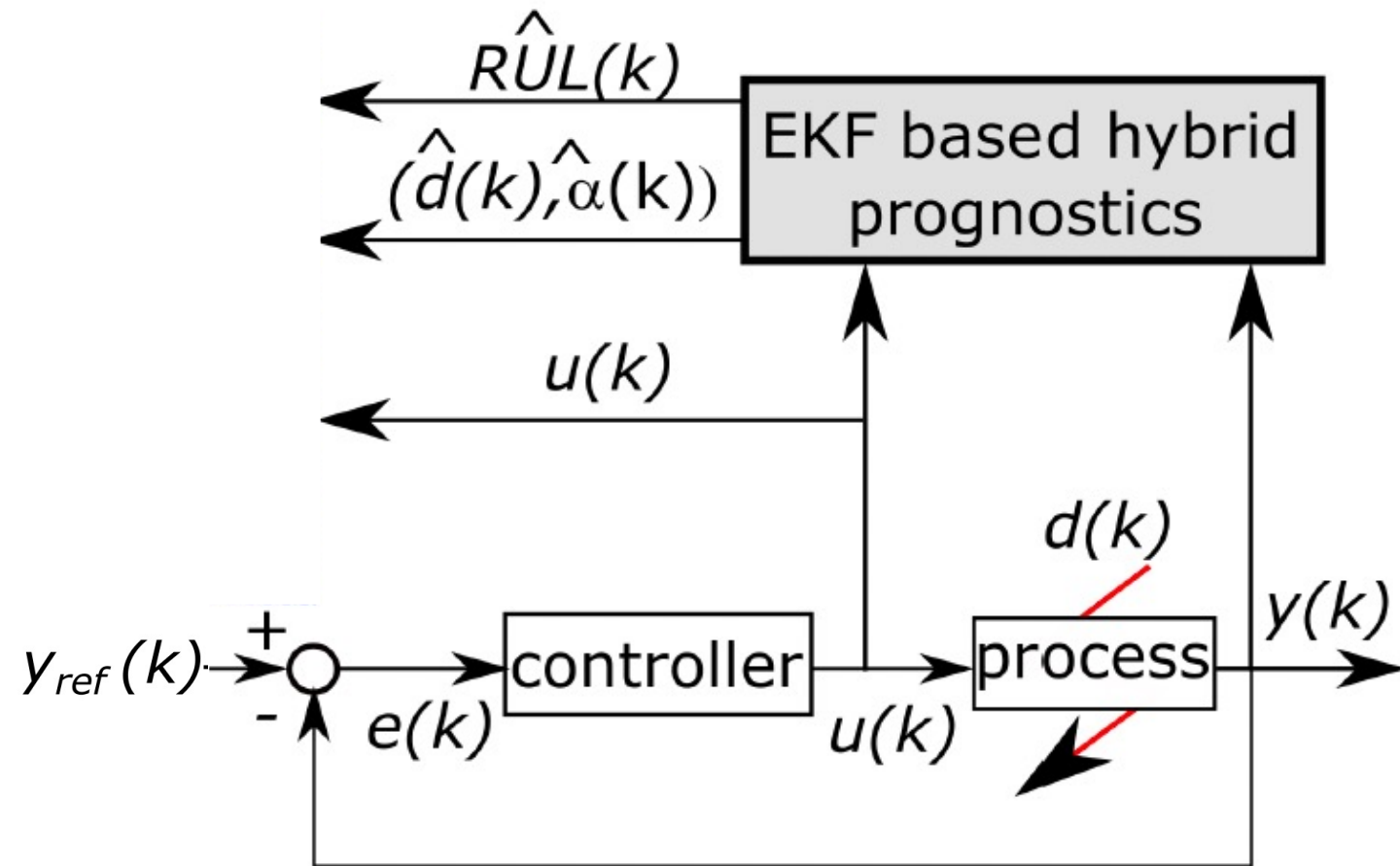
$$RUL(k) = (n_{fail} - k_p).T_s$$

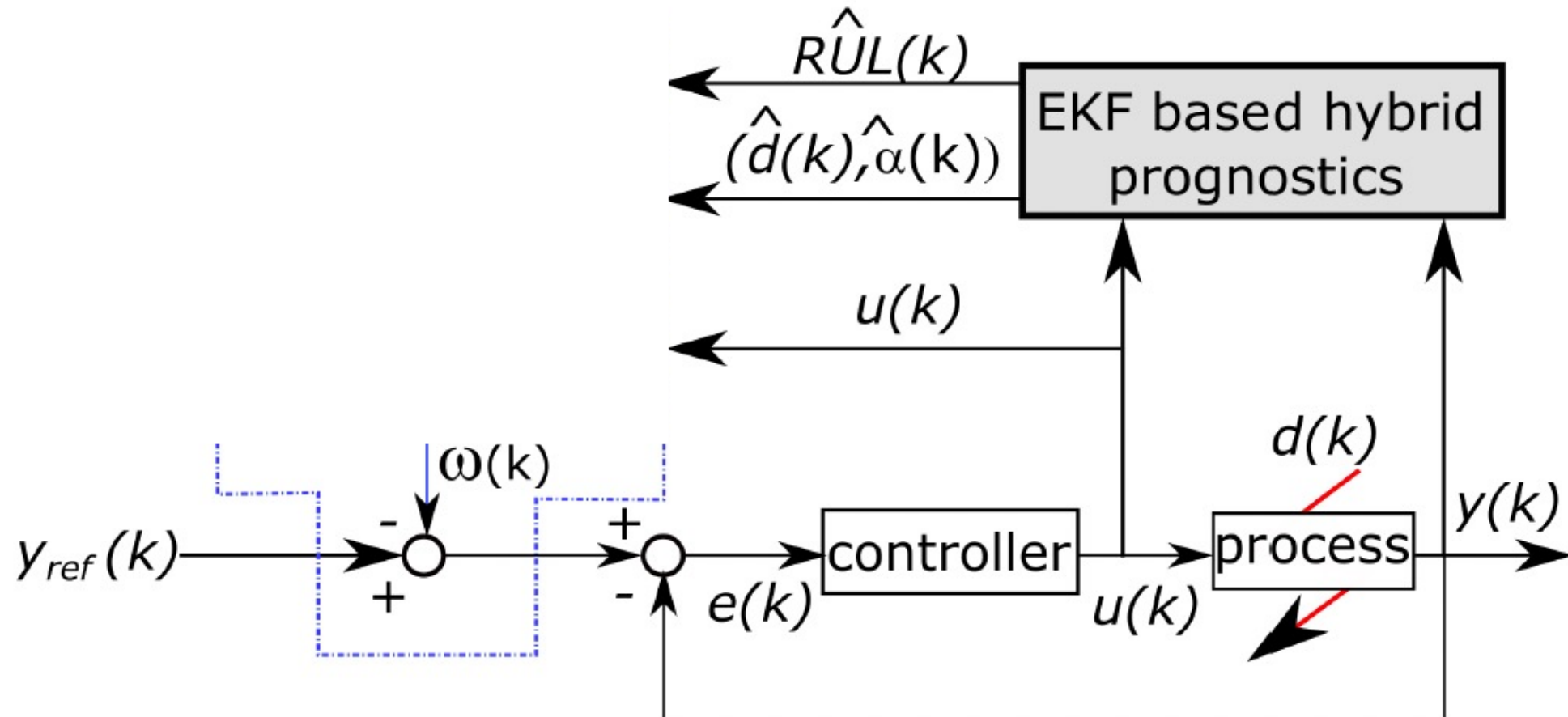
$$d(n_{fail}) = d(k).(1 + \alpha(k).u(k).T_s)^{n_{fail}-k}$$

$$d(n_{fail}) = d(k).(1 + \alpha(k).u(k).T_s)^L \quad d(n_f) < D_{fail}$$

$$L > \frac{\log \left(\frac{D_{fail}}{d(k)} \right)}{\log (1 + \alpha(k).u(k).T_s)}$$

$$\widehat{RUL}(k) = L$$





$$\omega^*(k) = \operatorname{argmin} J(k) \quad \longrightarrow \quad J(k) = \gamma.\omega(k)^2 + (1 - \gamma). \left(RUL_{ref}(k) - \widehat{RUL}(k) \right)^2$$

$$\widehat{RUL}(k) = \Phi(\hat{d}(k), ((y_{ref}(k) - \omega(k)) - y(k)))$$

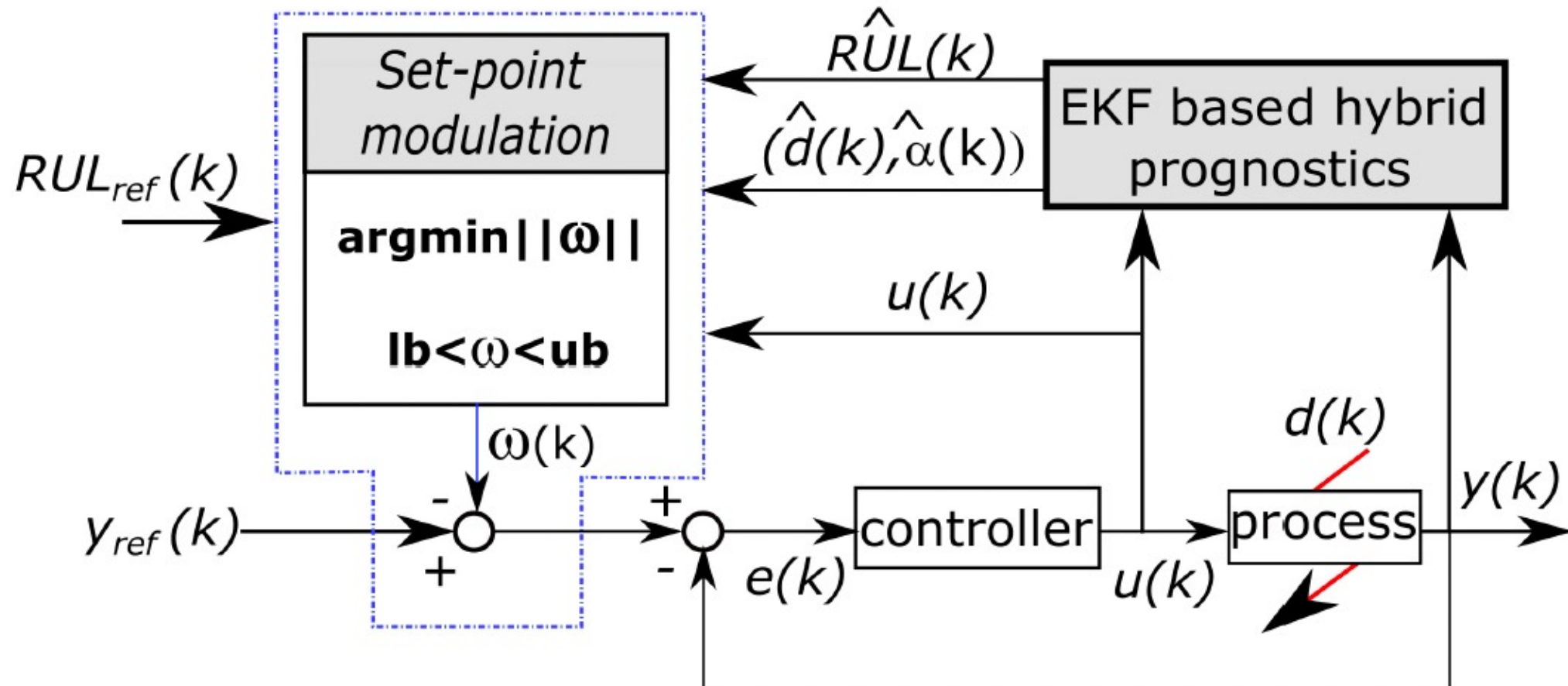
Algorithm 3 Set point modulation based reconfiguration strategy

Input: $RUL_{ref}, y_{ref}, y, u_k,$

Output: ω^*

```

1:  $[\hat{d}(k), \alpha(\hat{k})] = \text{Algorithm 1}(z_{d_k}, P_{k|k}, u_k)$ 
2: if  $\widehat{RUL}(k) < RUL_{ref}(k)$  then
     $J(k) = \gamma.\omega(k)^2 + (1 - \gamma).(RUL_{ref}(k) -$ 
     $\text{Algorithm 2}(u(k))^2$ 
4:   such  $lb \leq \omega \leq ub$ 
     $\omega^*(k) = \operatorname{argmin}(J(k))$ 
6: end if
   return  $\omega^*(k)$ 
    
```



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Please consider the following academic example:

$$\begin{cases} \begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{pmatrix} -12 & -20.02 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(k) + w(k) \\ z(k) = \begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + v(k) \end{cases}$$

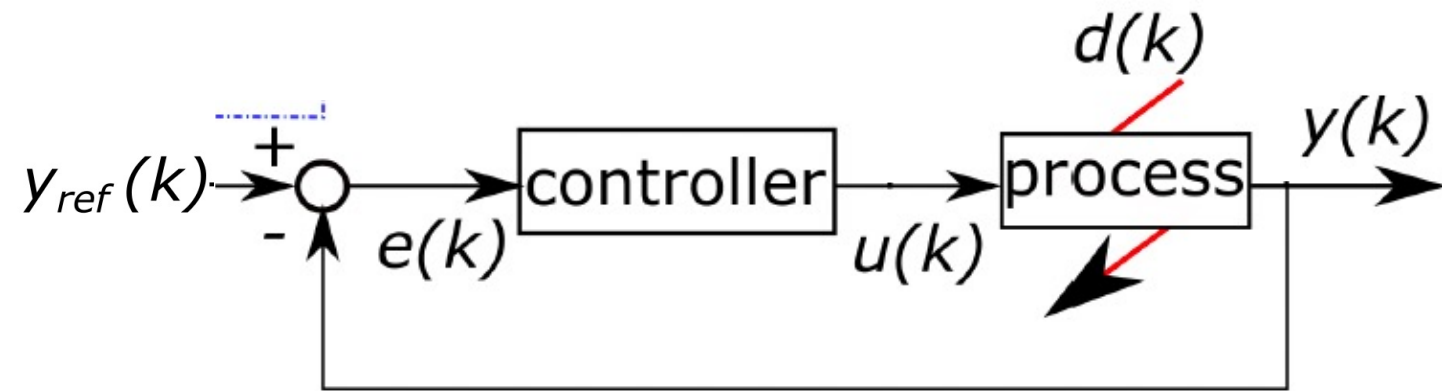
With degradation $x(k+1) = Ax(k) + B(u(k) - d(k)) + w(k)$

where
$$d(k) = \begin{cases} 0 & \text{if } \Phi(k) < K_{lim} \\ (\Phi(k) - K_{lim})^{K_{exp}} & \text{if } \Phi(k) > K_{lim} \end{cases}$$

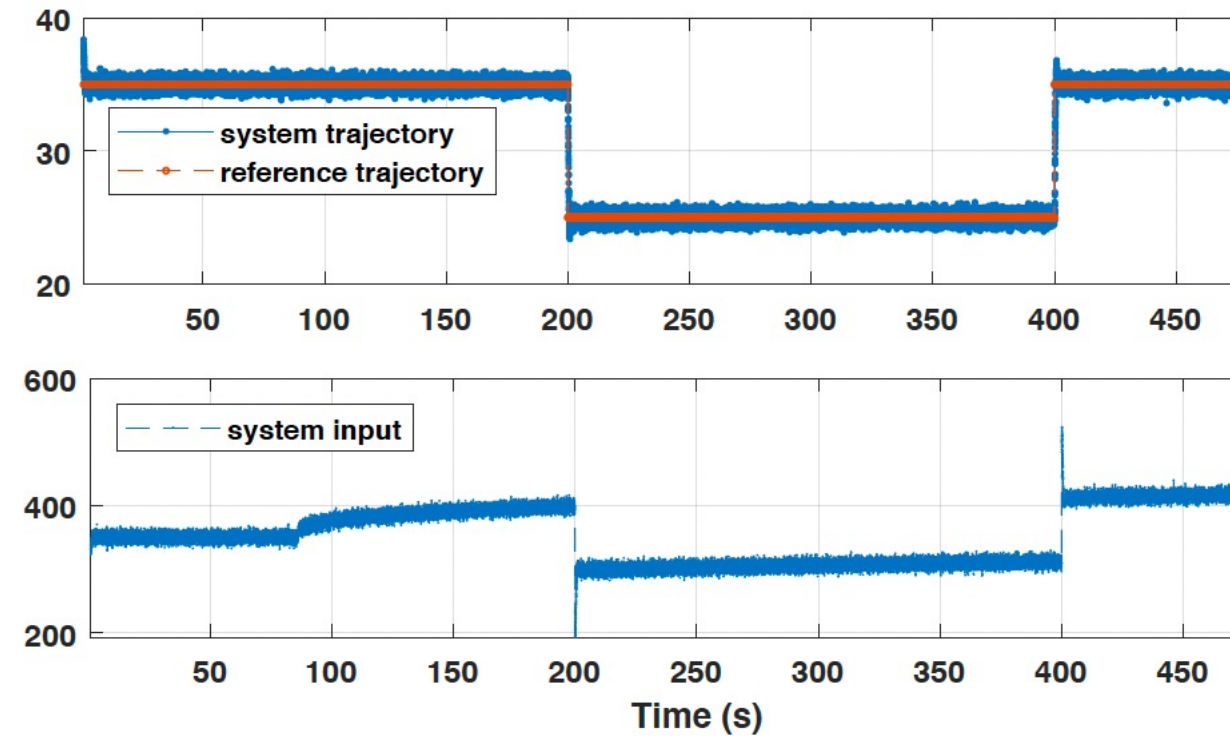
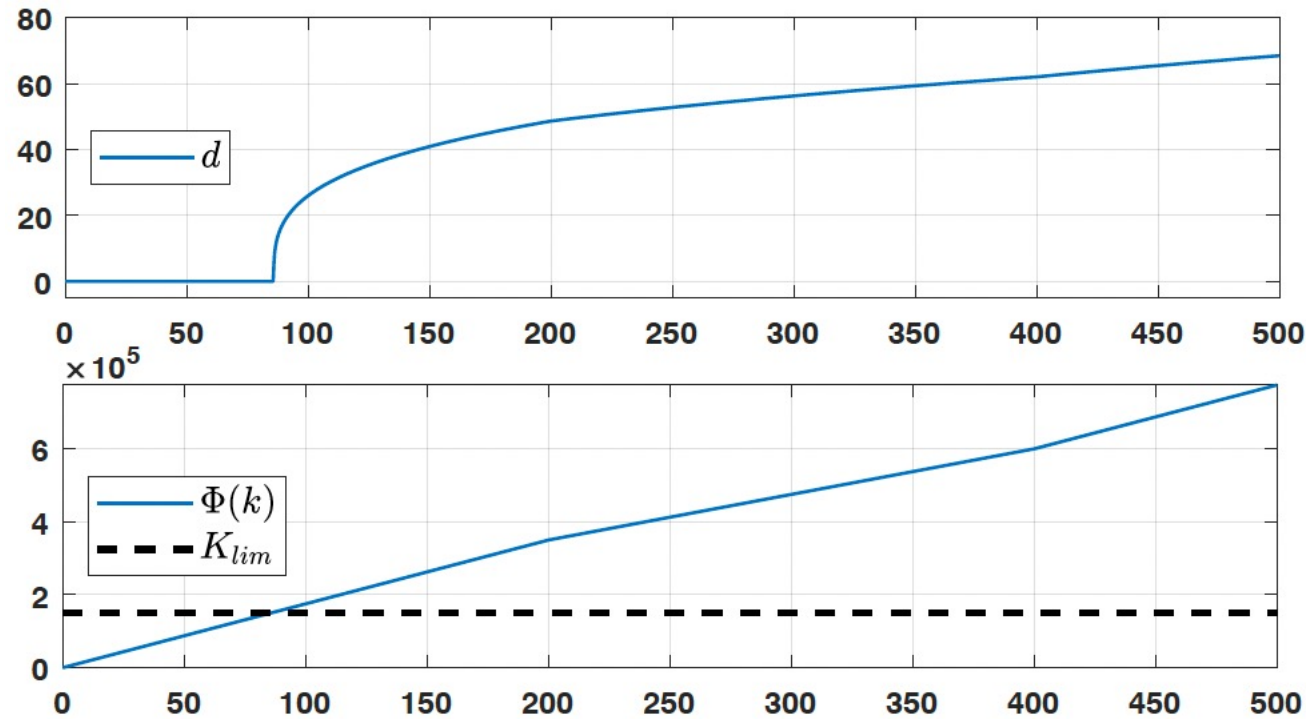
with $\Phi(k)$ defined as:

$$\Phi(k) = \sum_{k=1}^N x_1(k)$$

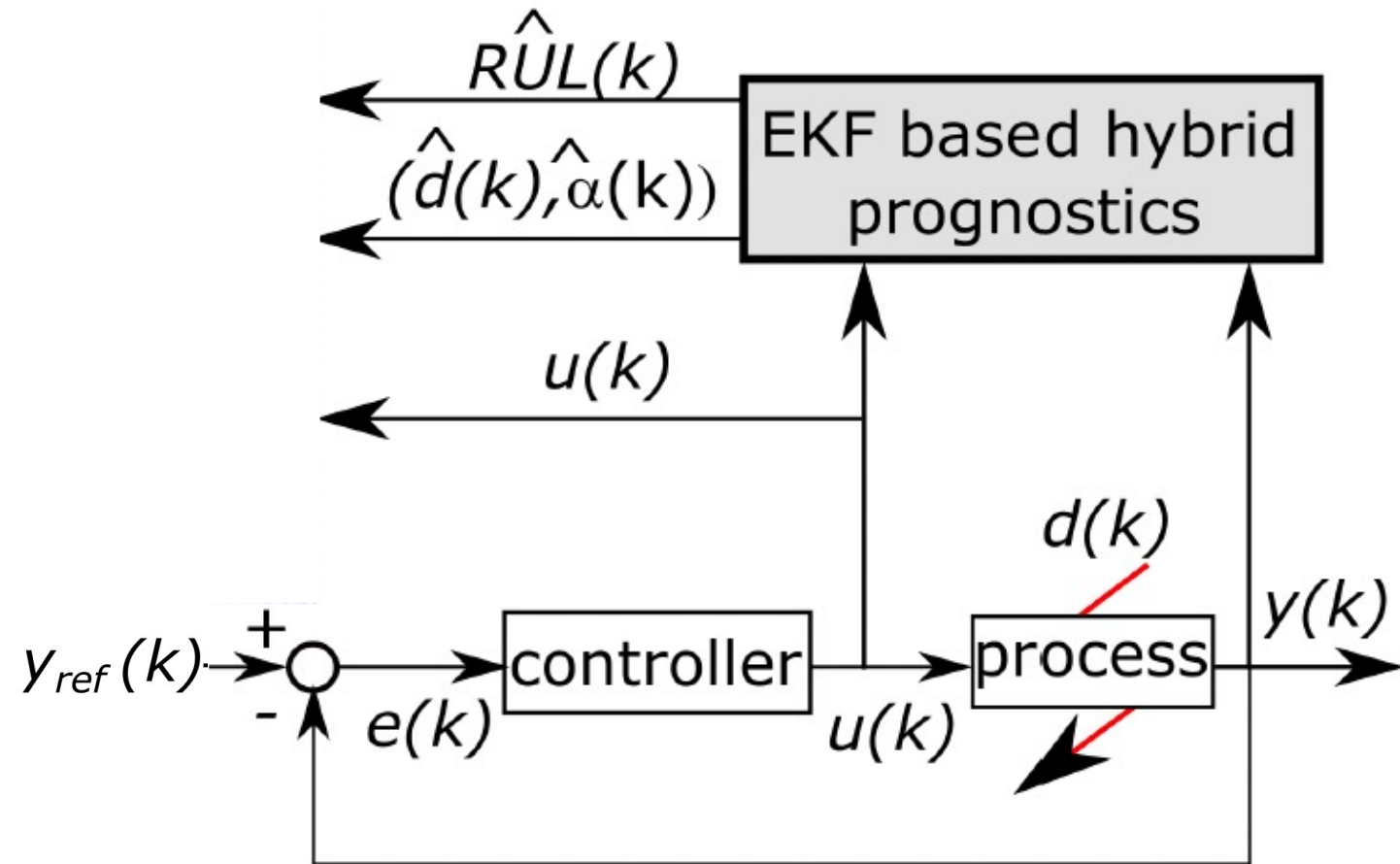
EXAMPLE 1 – CLOSED LOOP



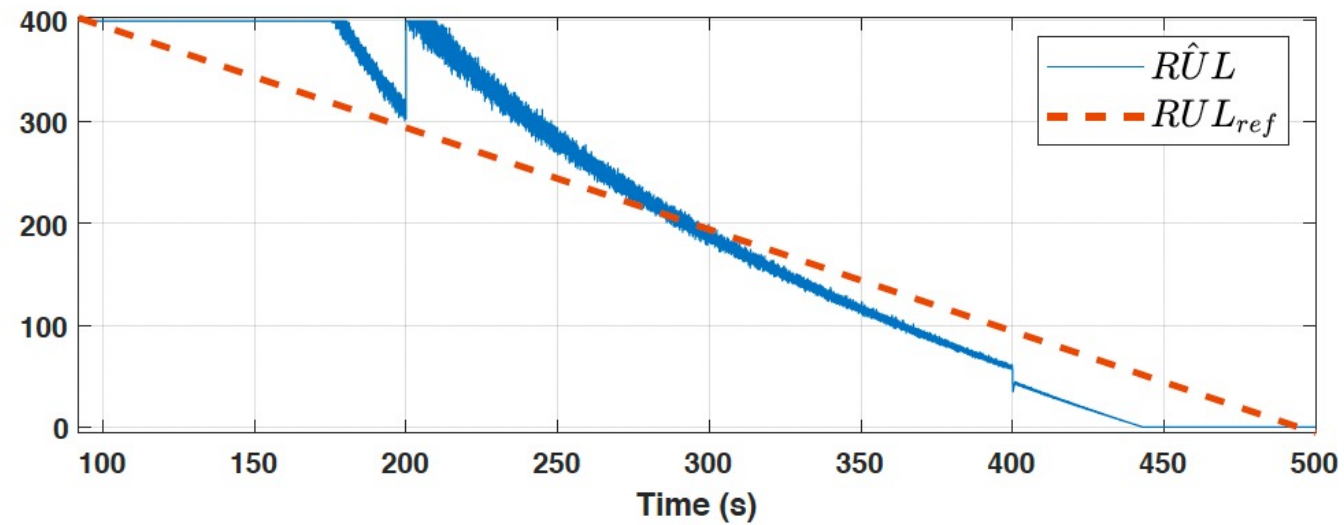
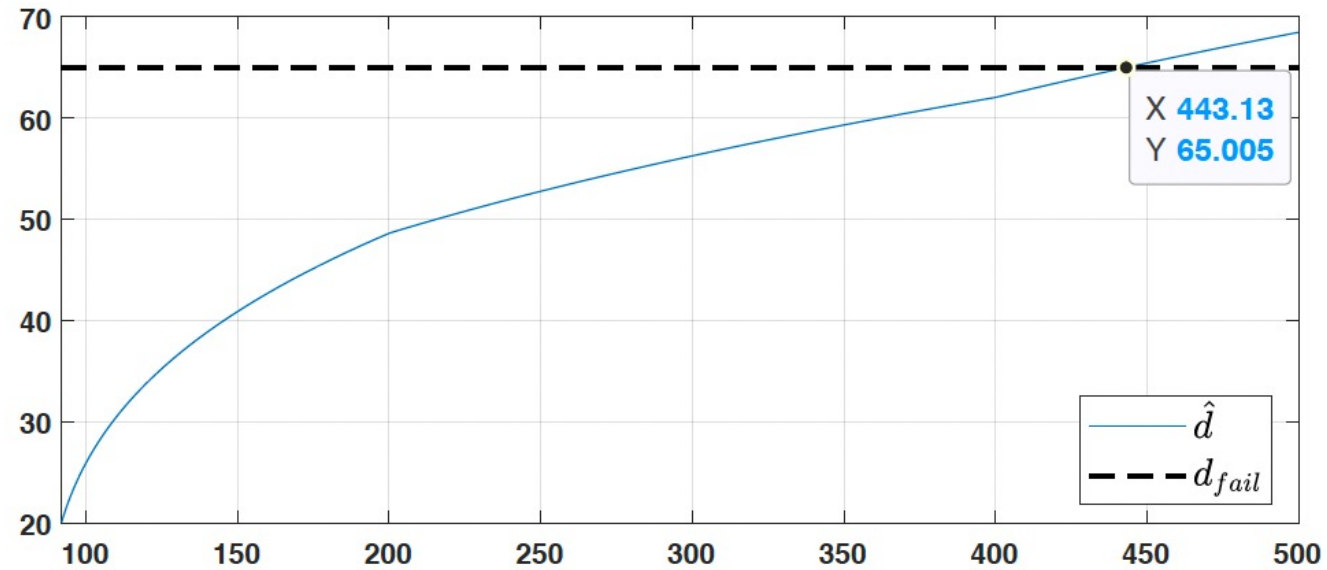
EXAMPLE 1 – DYNAMIC BEHAVIOR IN CLOSED LOOP



EXAMPLE 1 – EKF

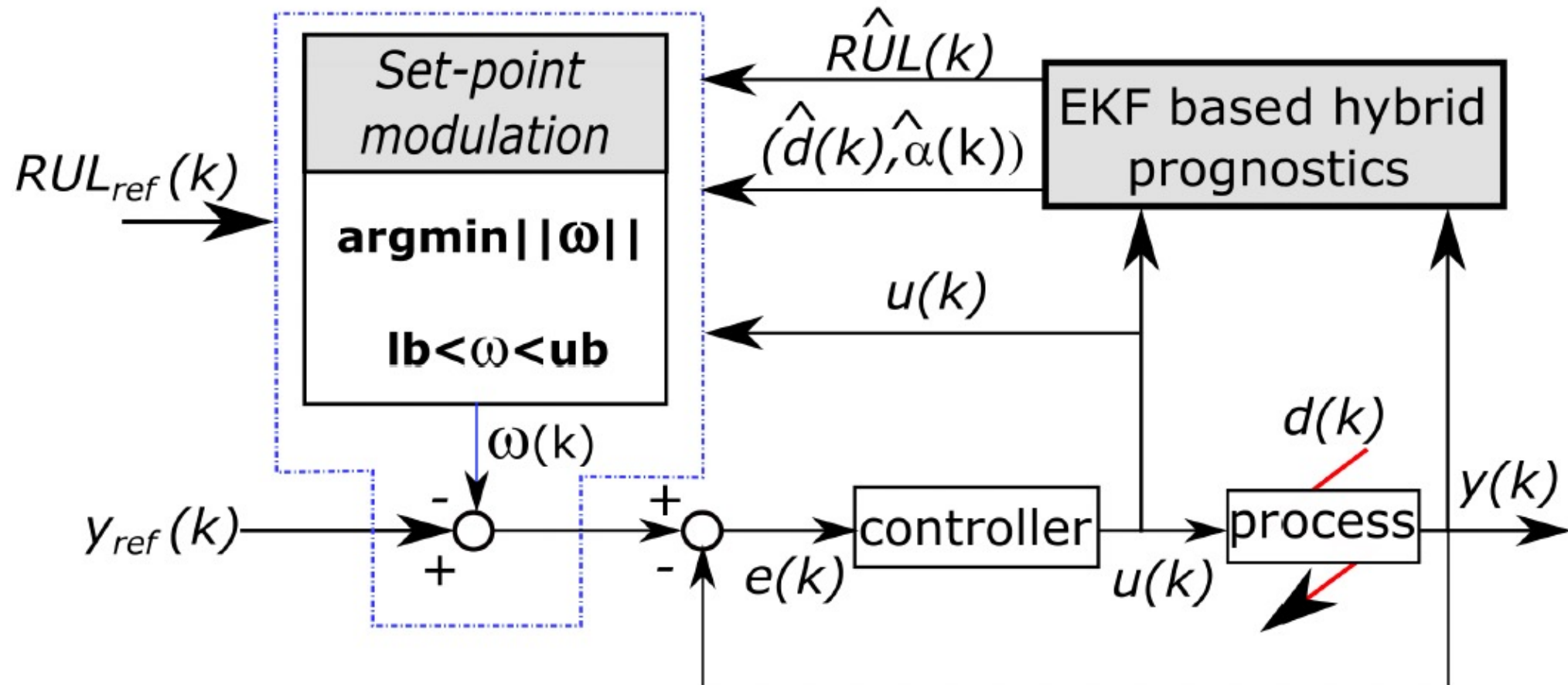


EXAMPLE 1 – DEGRADATION ESTIMATION & RUL

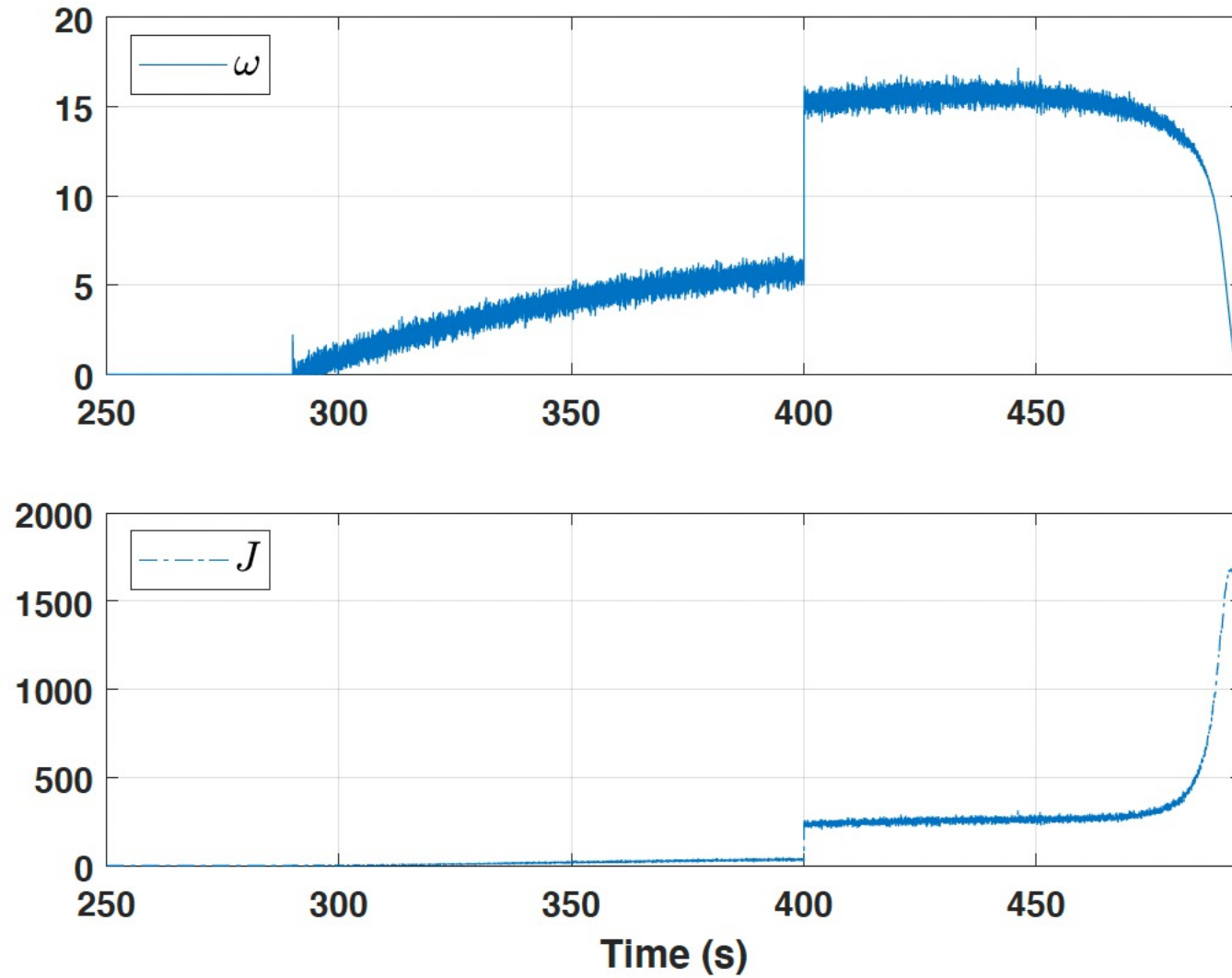


HYBRID PROGNOSTICS ENABLED SET POINT MODULATION STRATEGY

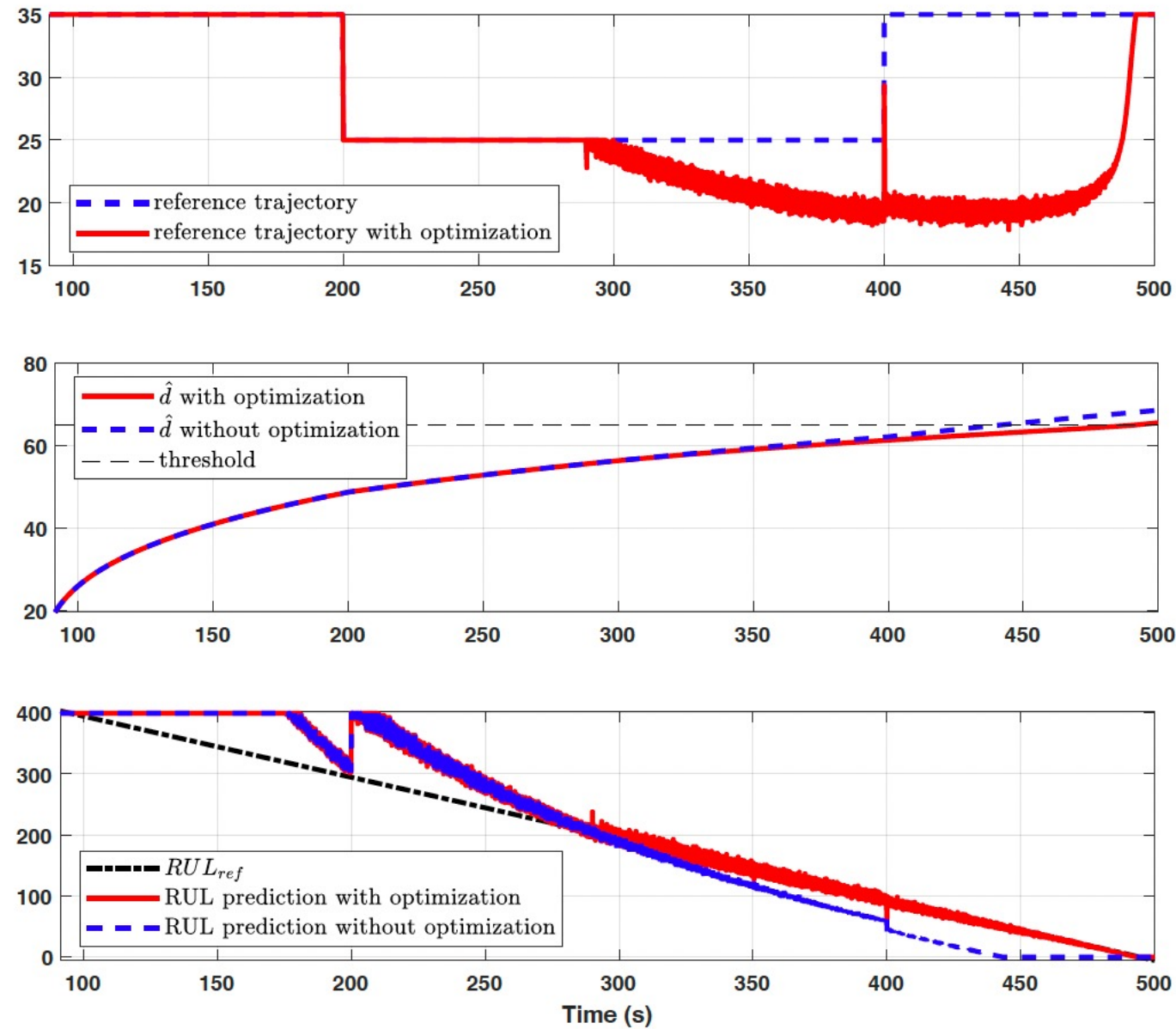
... cnes ...



EXAMPLE 1 – DESIGN PARAMETER & CRITERIA



EXAMPLE 1 – DYNAMIC BEHAVIOR & DEGRADATION & RUL



Two Criteria are defined:

$$I_H(\gamma, ub) = \frac{EOL_{WithStrategy}}{EOL_{WithoutStrategy}} \times 100$$

$$I_P(\gamma, ub) = \left(\sum_{i=1}^N \frac{y_{ref}(i) + \omega(i)}{y_{ref}(i)} \right) \times 100$$

$$\omega^*(k) = \operatorname{argmin} J(k)$$

$$J(k) = \gamma \cdot \omega(k)^2 + (1 - \gamma) \cdot \left(RUL_{ref}(k) - \widehat{RUL}(k) \right)^2$$

Parameter Sensitivies

test	1	2	3	4	5	6	7	8
γ	0.001	0.01	0.1	0.25	0.5	0.75	0.9	0.99

test	1	2	3	4	5	6	7	8
ub	1.5	3	4.5	6	7.5	9	10.5	12

EXAMPLE 1 – ANALYSIS

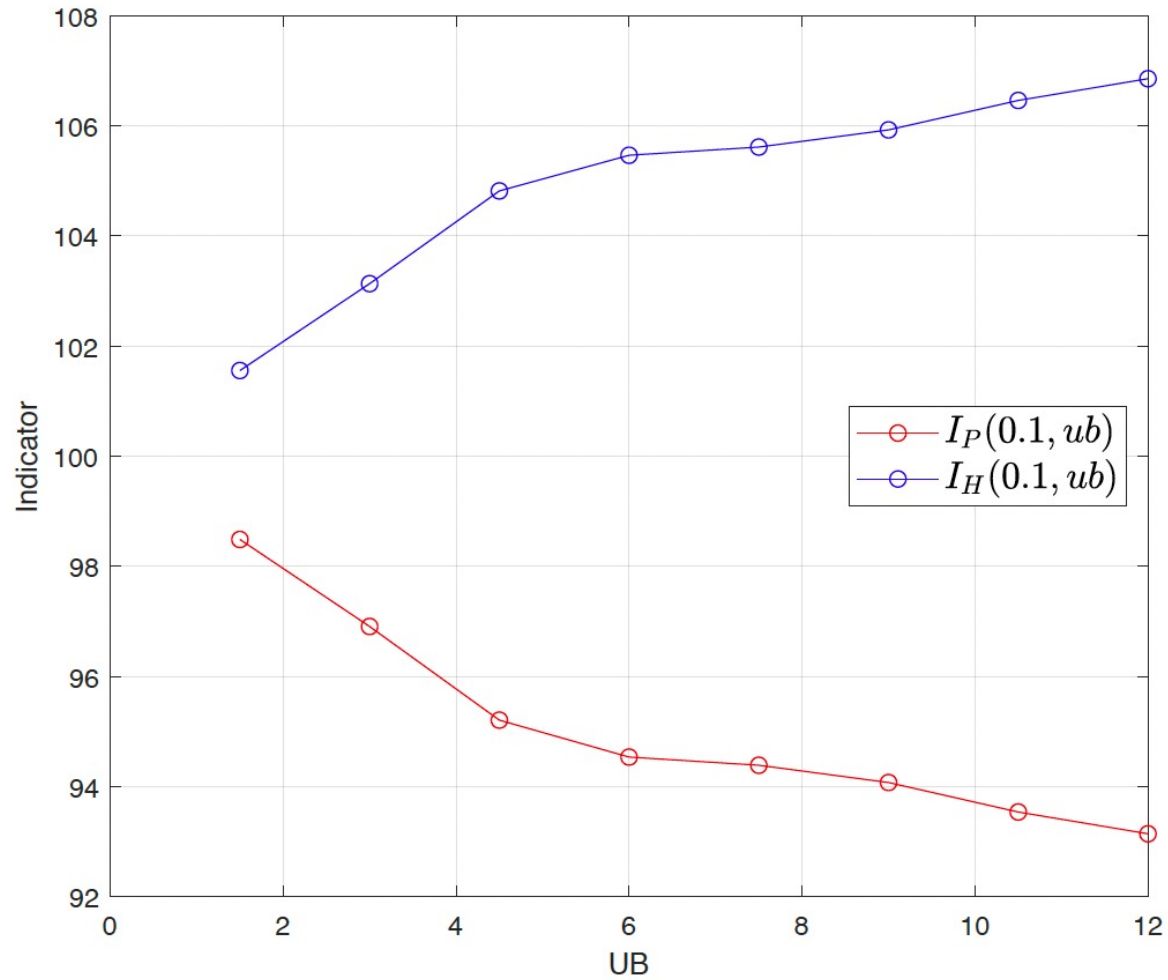


Figure 8. Indicators evolution for ub variations

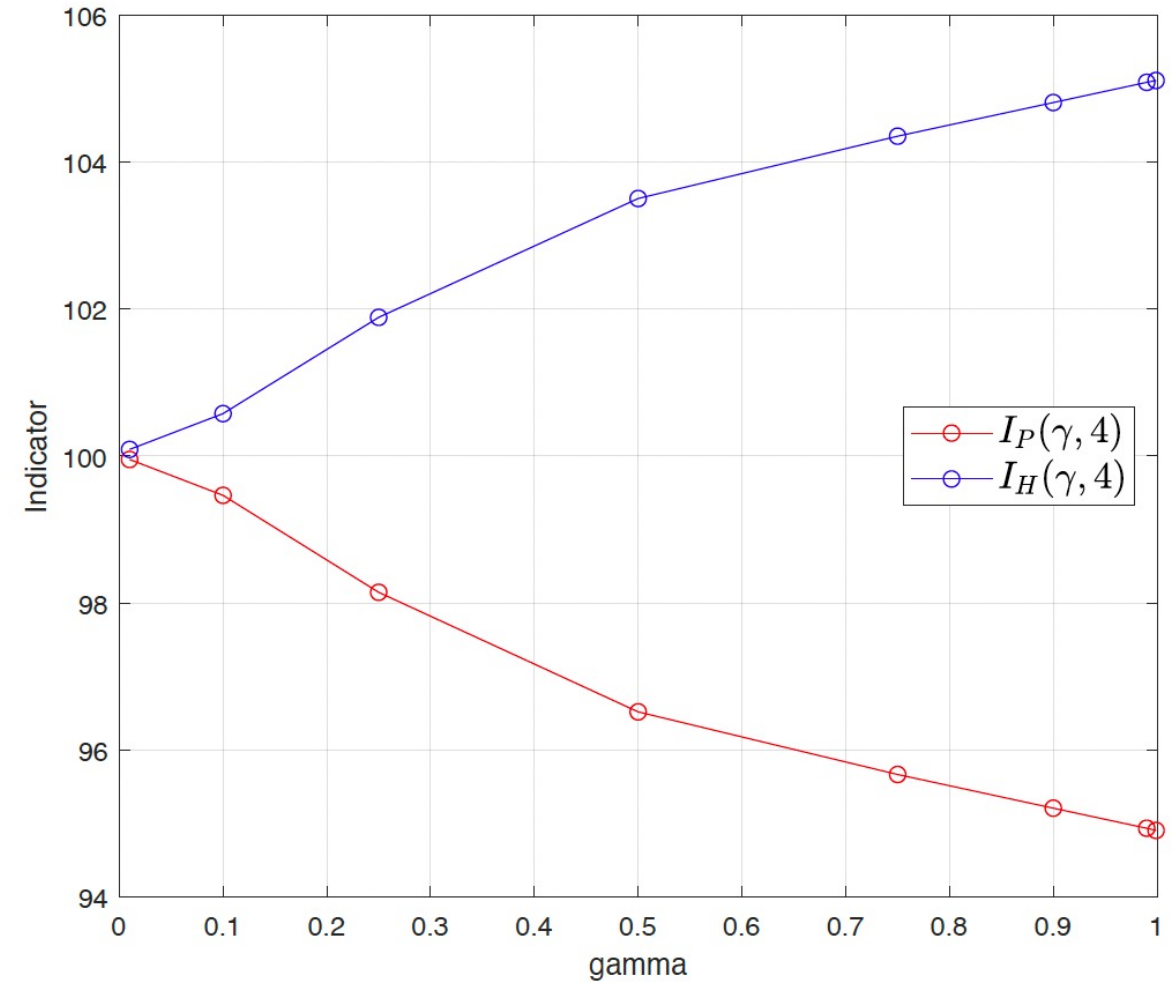
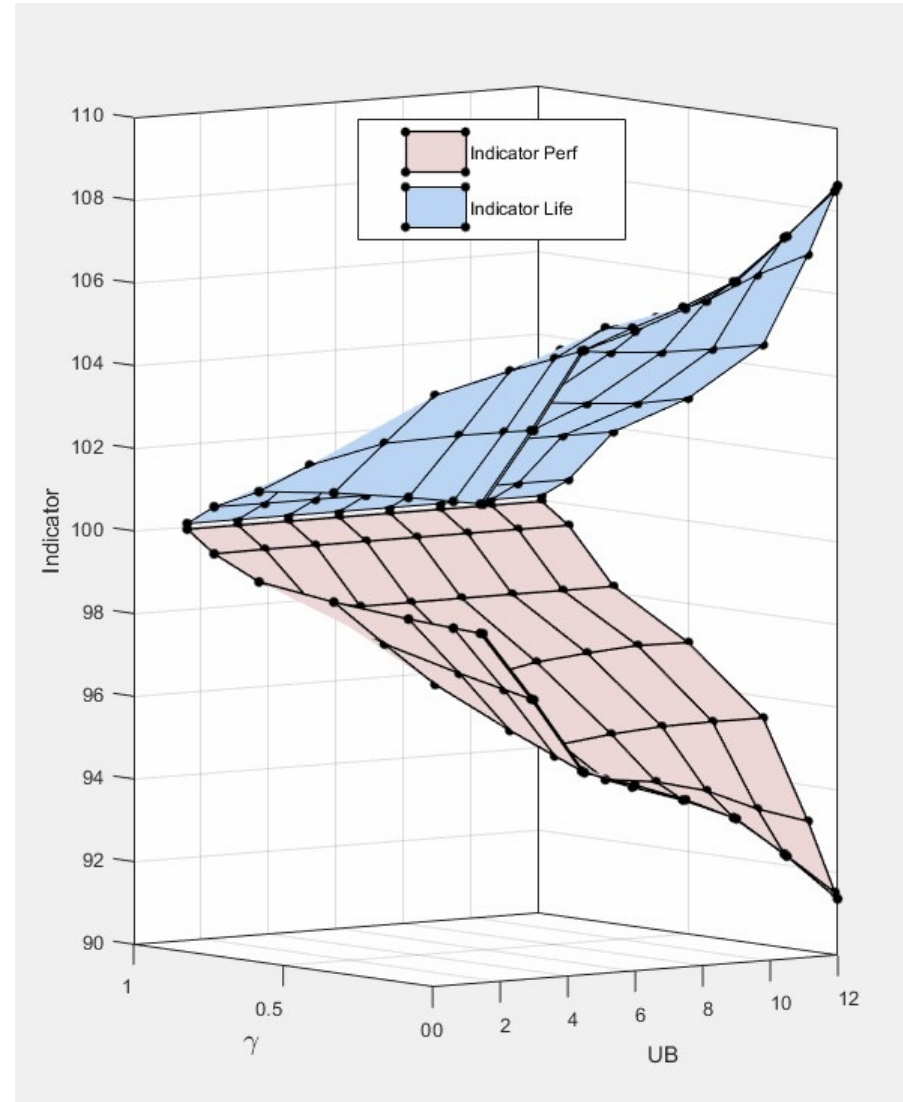


Figure 10. Indicators evolution for γ variations

EXAMPLE 1 – ANALYSIS

Combinaison of both parameters



EXAMPLE 1 – ANALYSIS

Extension in seconds of system life for various combinaisons

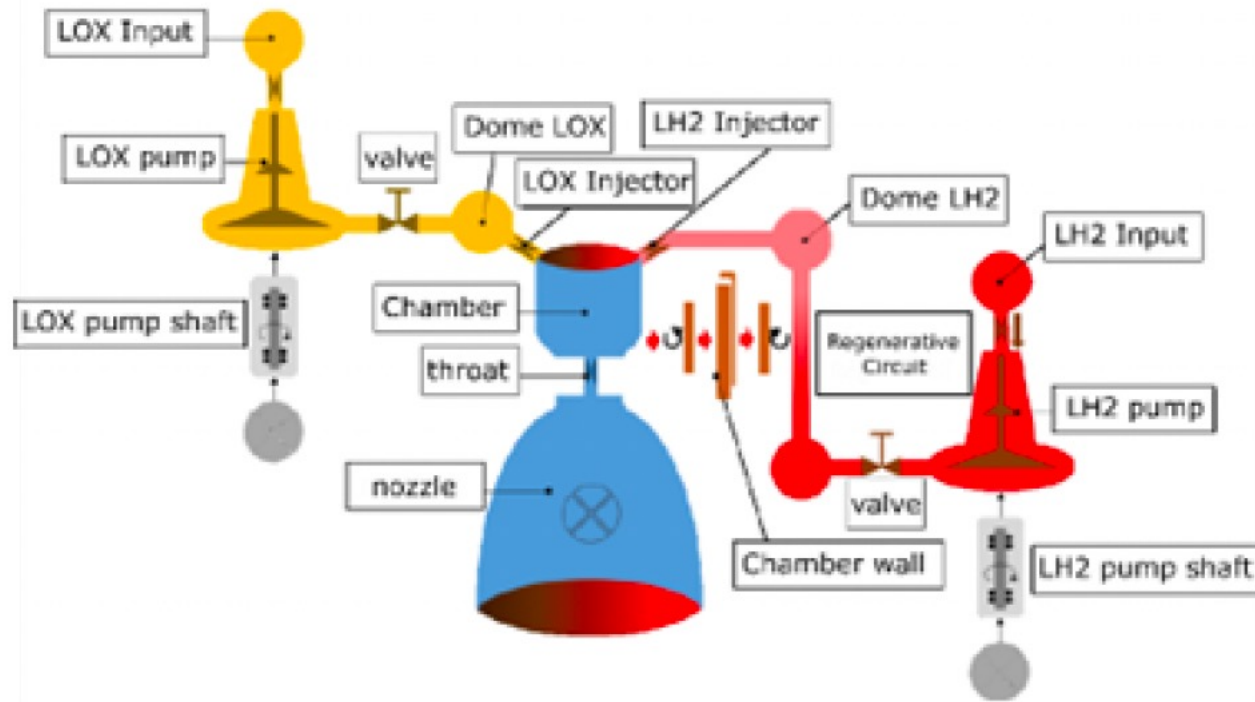
γ / UB	1.5	3,0	4.5	6,0	7.5	9,0	10.5	12,0
0.010	6.49	12.7	18.30	22.53	26.10	30.07	34.53	39.54
0.100	6.49	12.69	18.25	22.40	25.87	29.70	33.89	38.41
0.250	6.43	12.40	17.44	20.64	23.24	25.77	28.13	30.06
0.500	6.29	11.76	15.64	17.18	18.28	18.88	18.91	18.92
0.750	5.88	9.77	10,00	10,00	10,00	10,00	10,00	10,00
0.900	4.64	4.75	4.75	4.75	4.75	4.75	4.75	4.75
0.990	1.85	1.81	1.81	1.81	1.81	1.81	1.81	1.81
0.999	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17

Legen

<10s	<20s	<30s	<40s

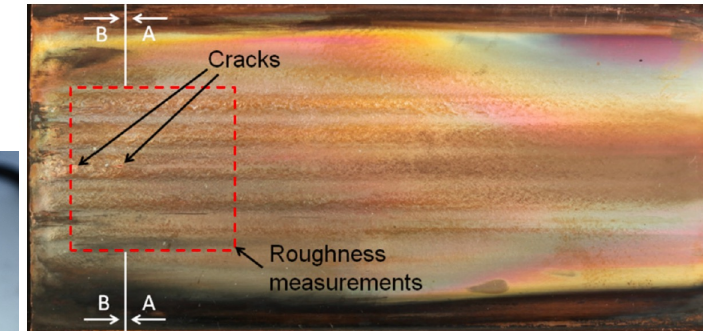
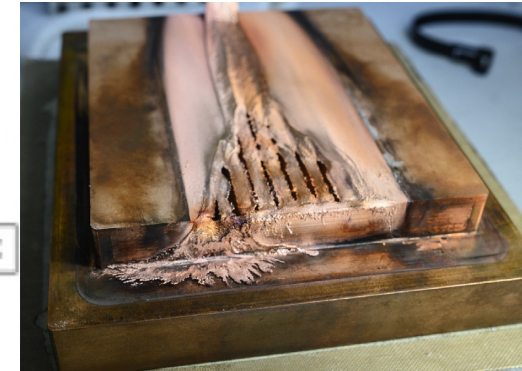
Note: The system without HAC fails after 443s.

REUSABLE LIQUID ROCKET ENGINE - RLRE

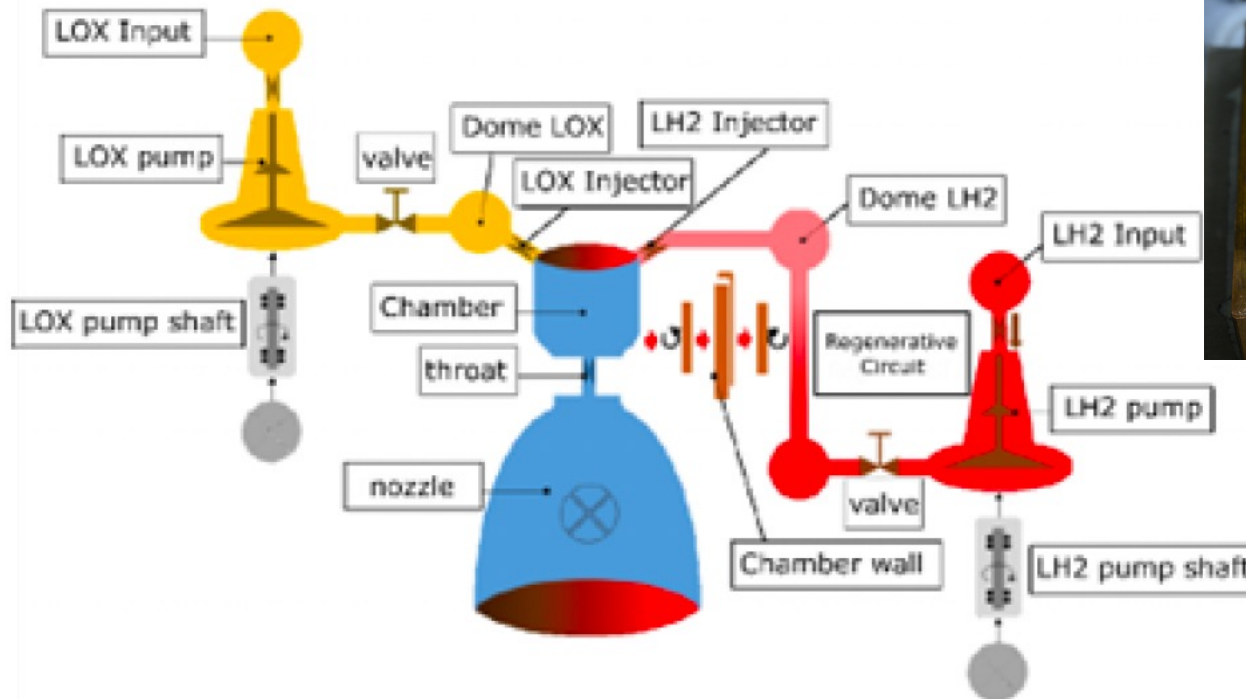


Simulation Engine: CNES RT-NT-2510000-CNES

Degradation



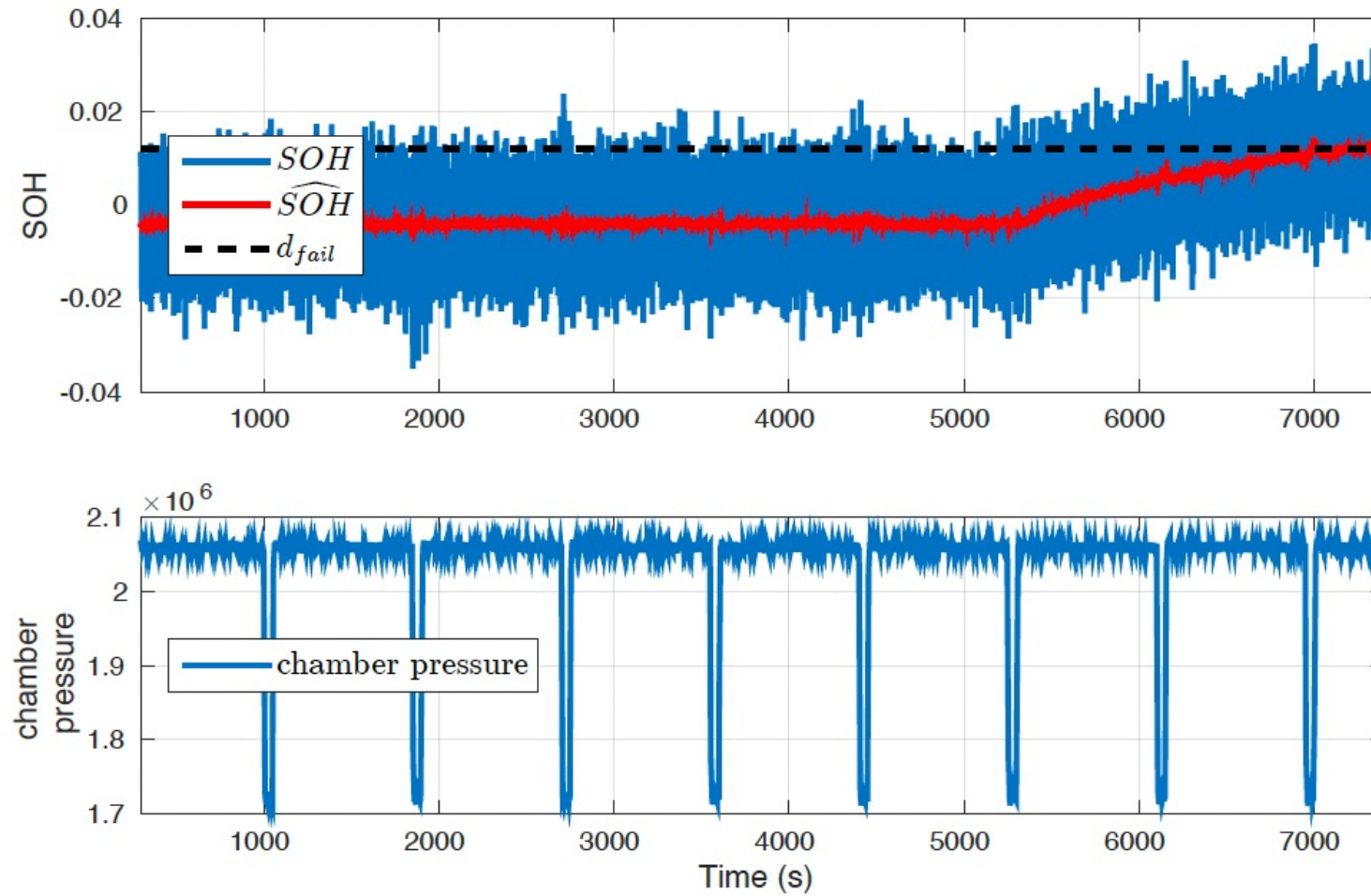
Hötte, F., Sethe, C. V., Fiedler, T., Haupt, M. C., Haidn, O. J., & Rohdenburg, M. (2020). Experimental lifetime study of regeneratively cooled rocket chamber walls. *International Journal of Fatigue*, 138, 105649.



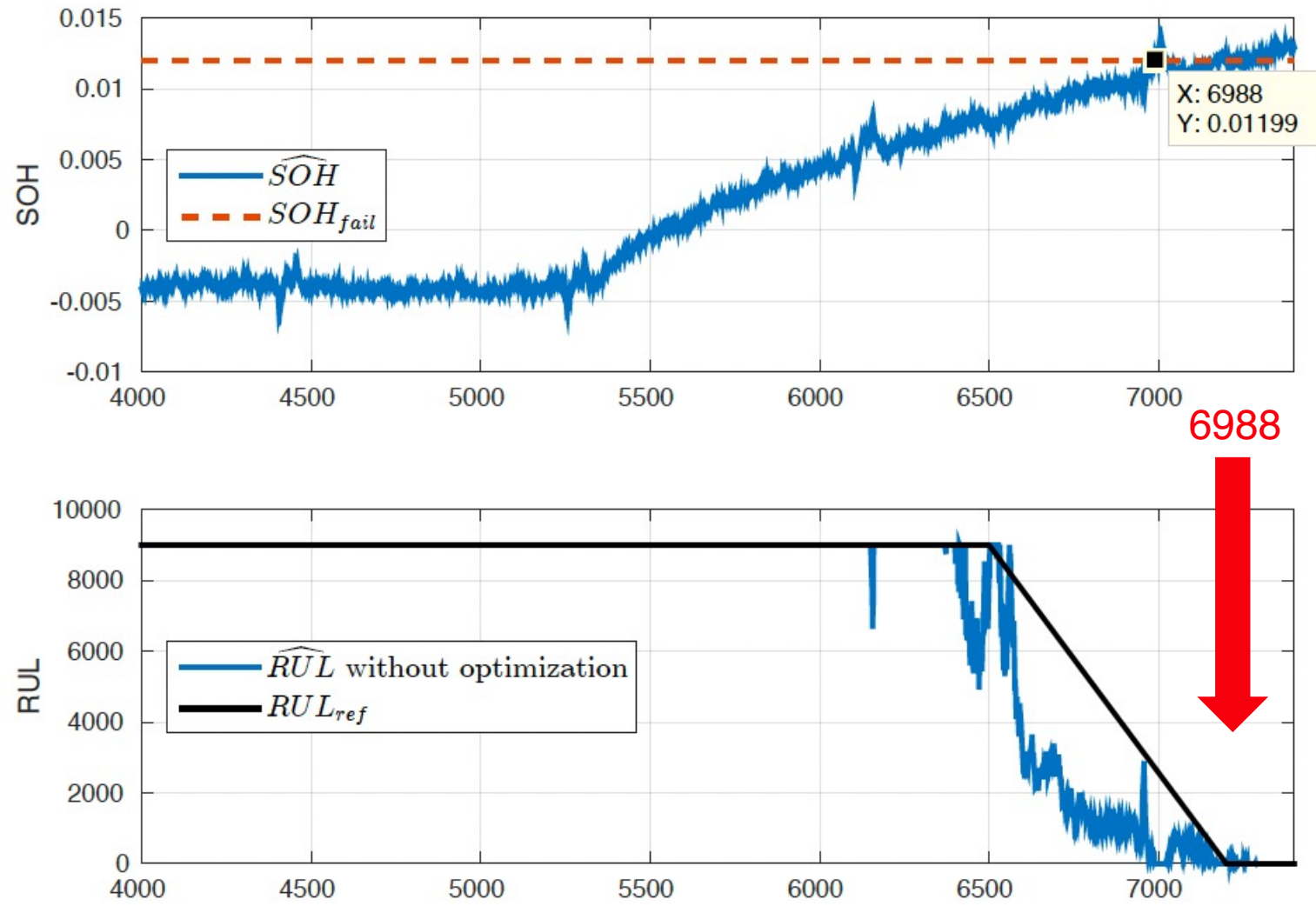
Degradation is the appearance of cracks in the combustion chamber.

This leads to fuel leaking from the regenerative circuit into the combustion chamber.

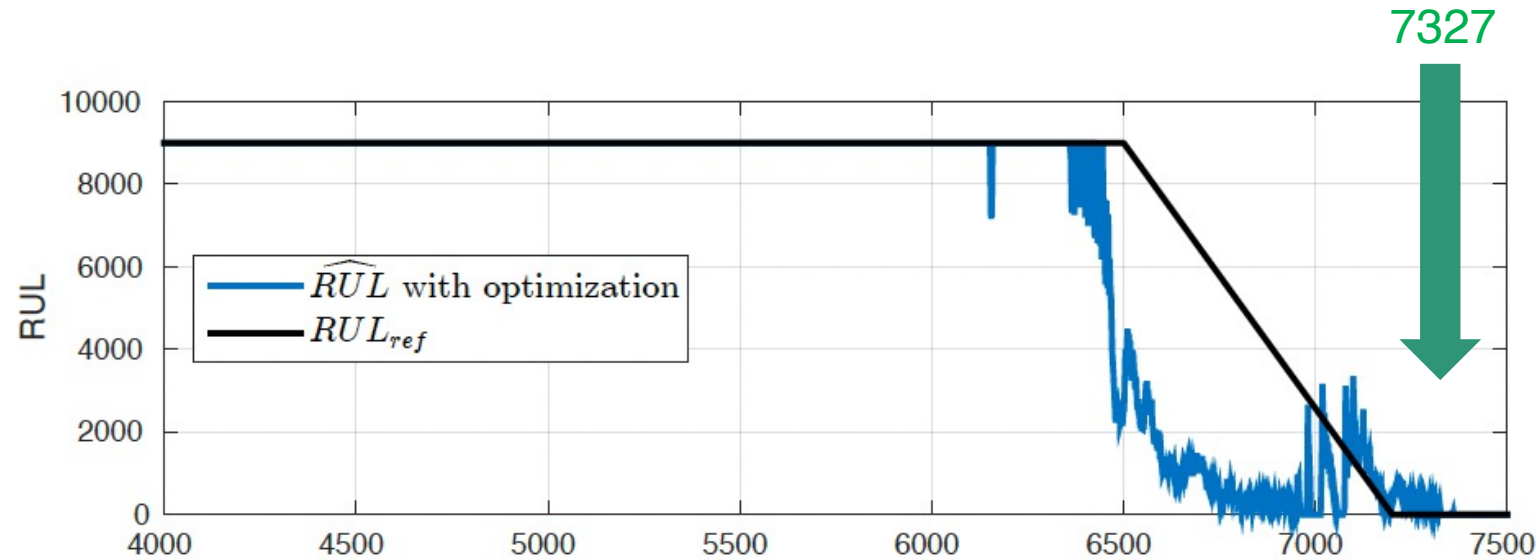
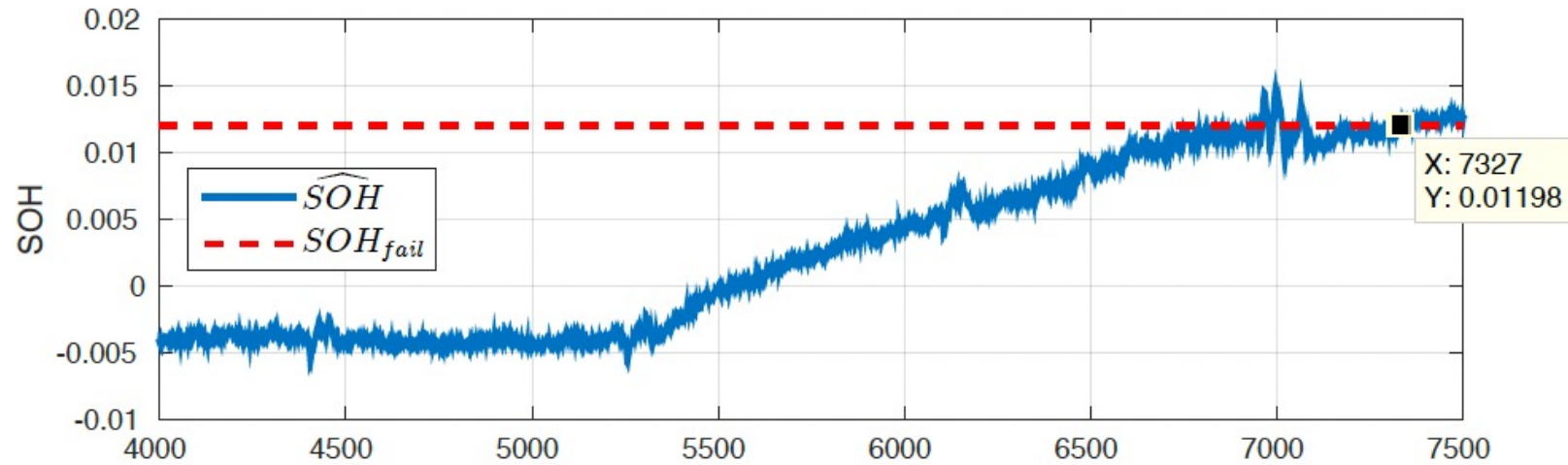
Leak will affect the efficiency on combustion and so the dynamic performance of the closed loop



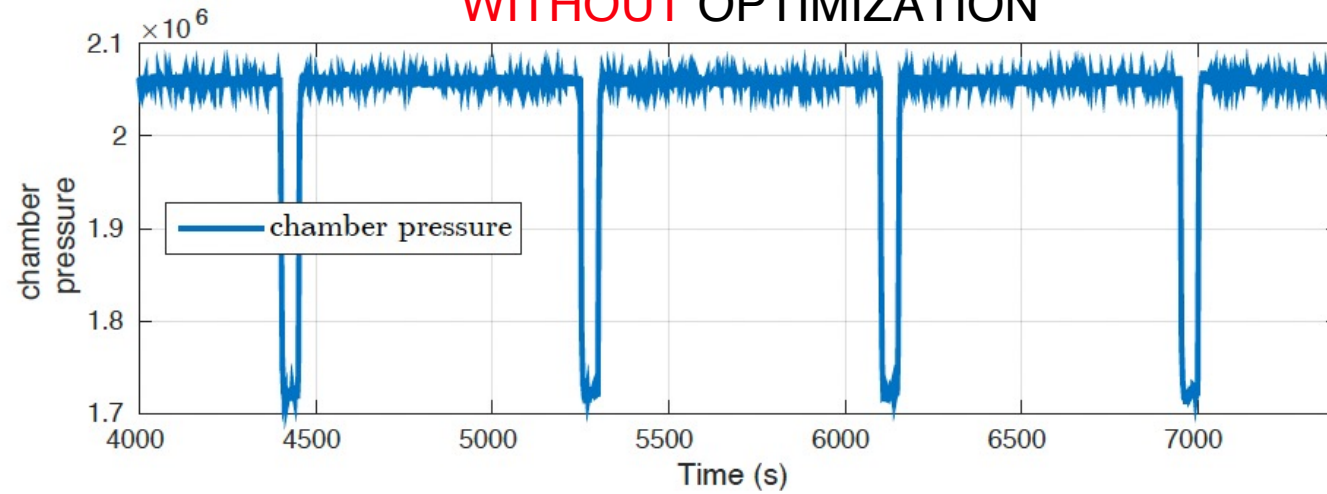
RLRE - **WITHOUT** OPTIMIZATION



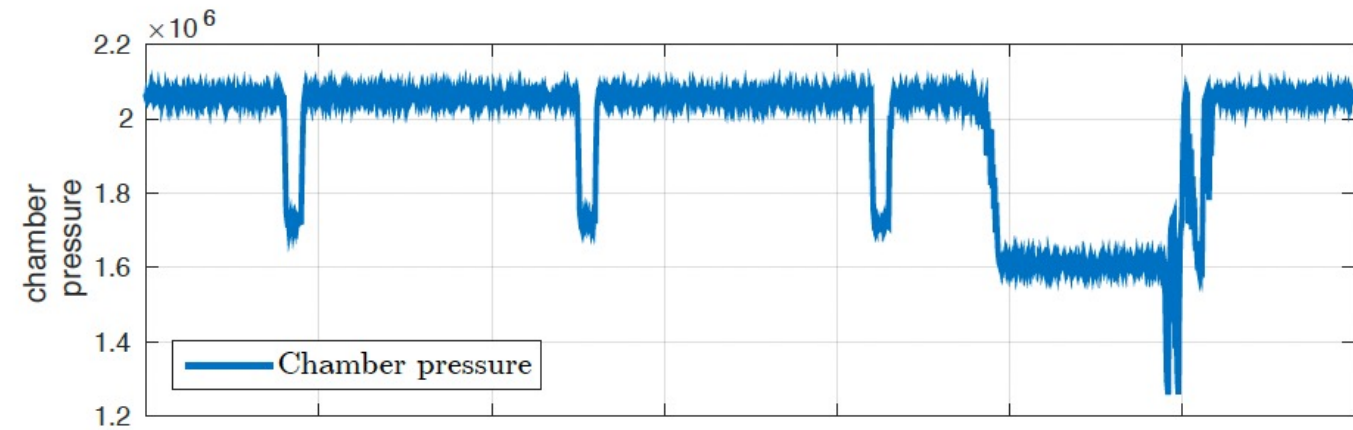
RLRE - WITH OPTIMIZATION



WITHOUT OPTIMIZATION



WITH OPTIMIZATION

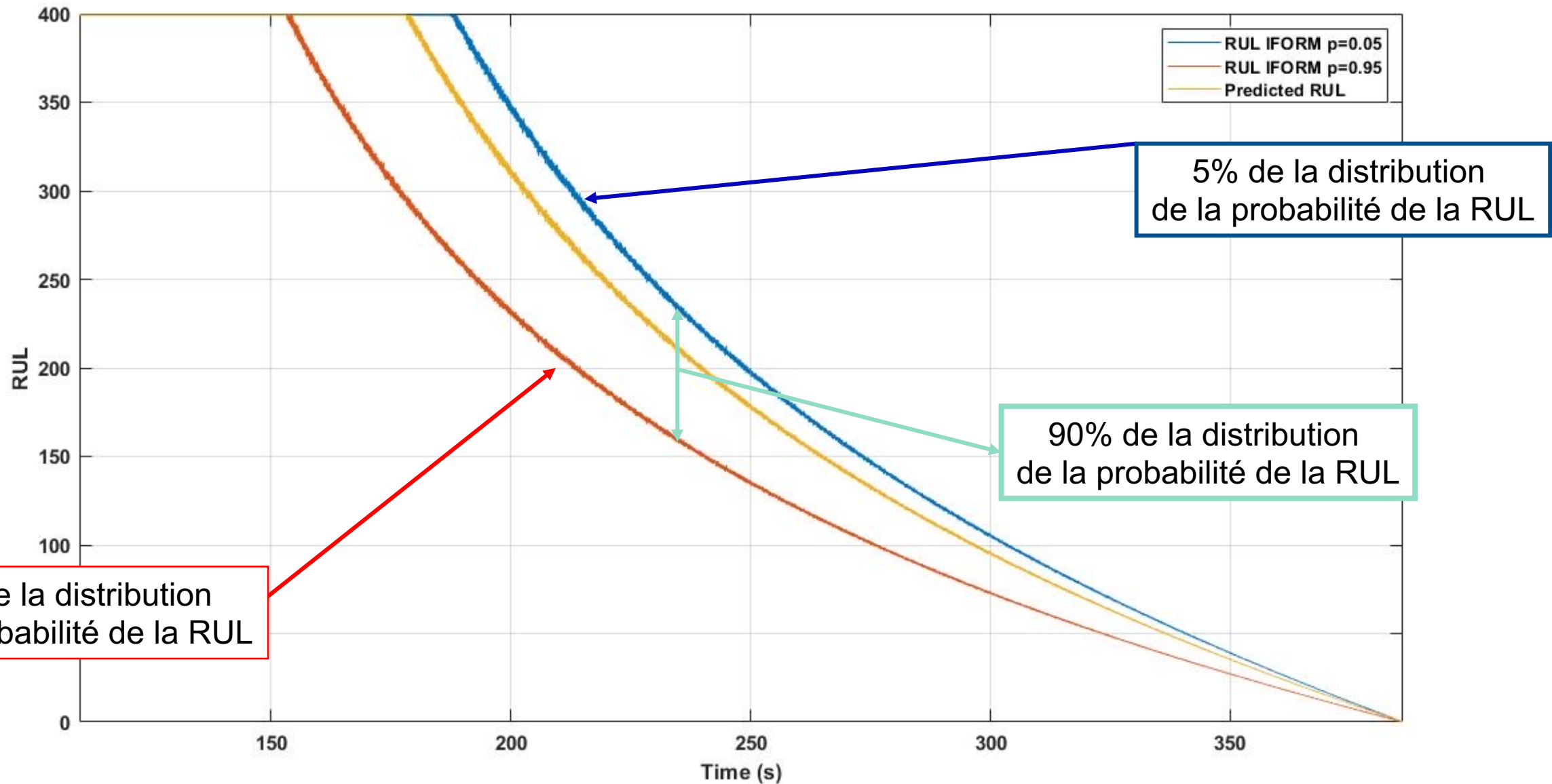


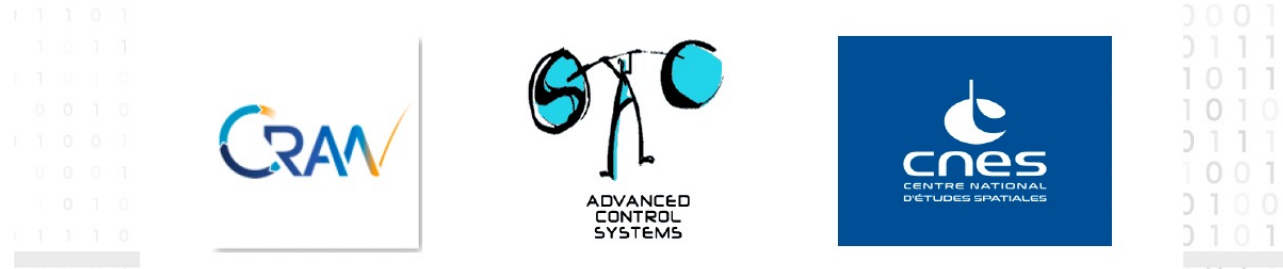
- Introduction / Contexte
- Problem Statement
- Our Solution
- Illustrative Examples
- **Conclusions / Perspectives**

- **Trade-off between dynamic performance and RUL has been proposed**
- **Degradation model estimation**
- **FDI Robustness has been assumed**
- **Uncertainties on RUL estimation have been omitted**
- ...

PERSPECTIVES – RUL with uncertainties

JL





Assessing a Statistical and a Set-based Approach for Remaining Useful Life Prediction.

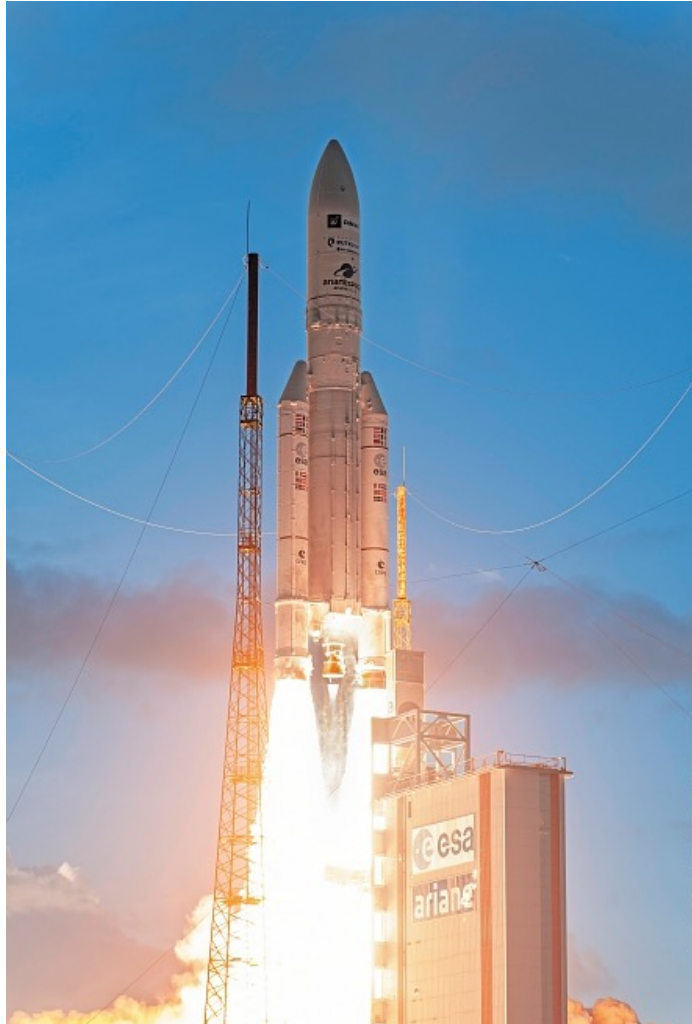
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MED 2023

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Prognostics aware control design for extended remaining useful life: Application to Liquid Propellant Reusable Rocket Engine

HAC Meeting 22th November 2023

Julien Thuillier, Mayank Shekhar Jha,
Sebastien Le Martelot, and Didier Theilliol

