Remaining useful life prognosis of a deteriorating feedback control system and application to controller reconfiguration

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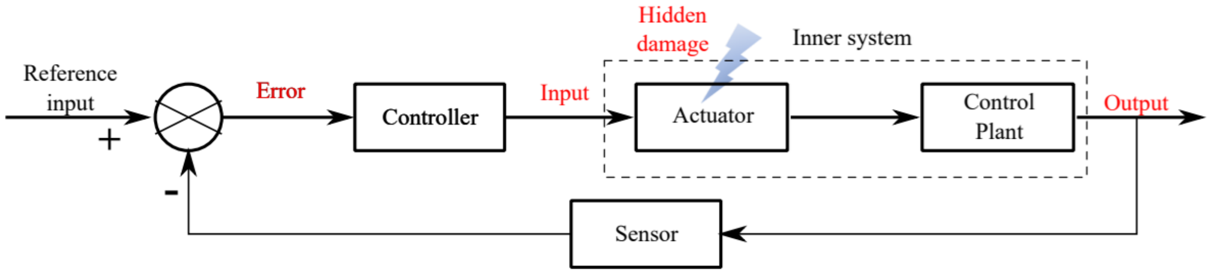
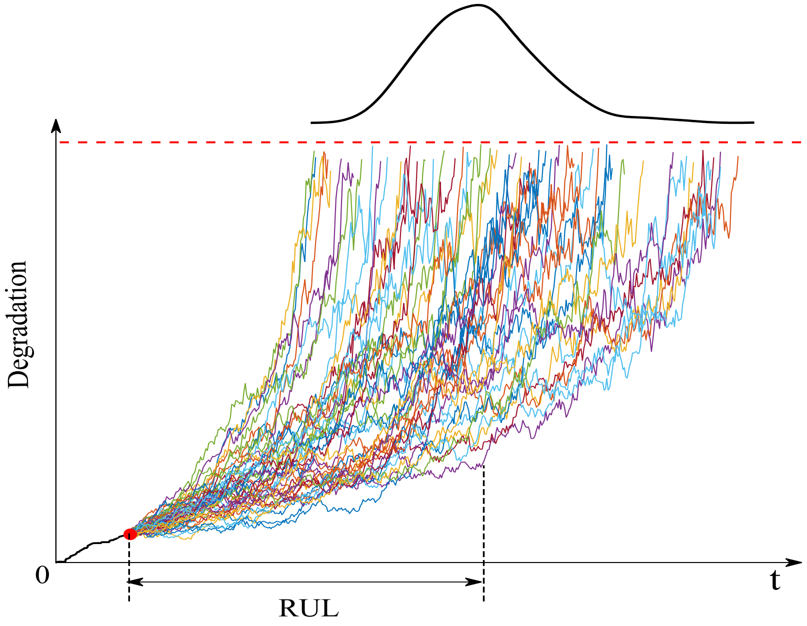
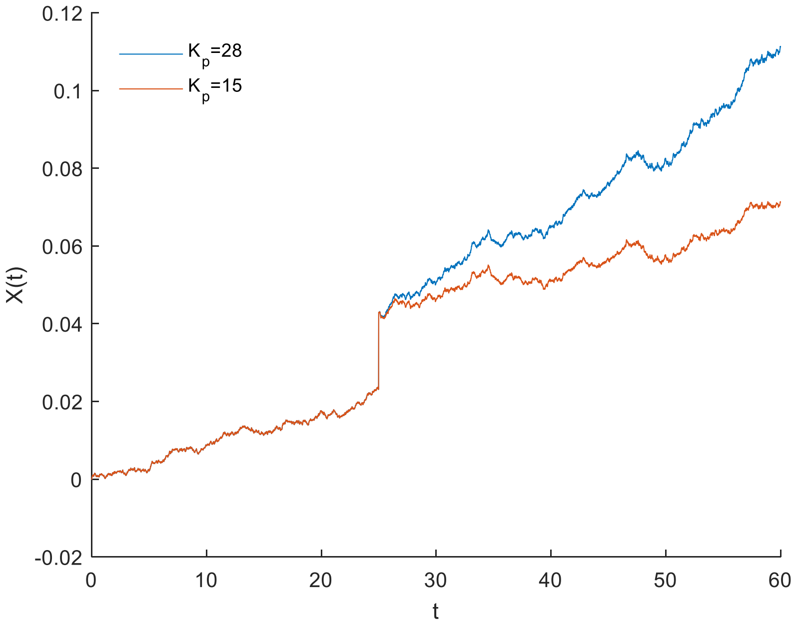
- Feedback control system (FCS)



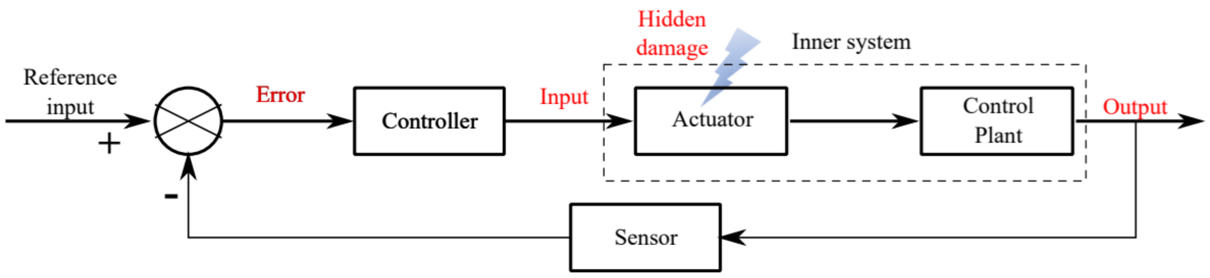
- Remaining useful life (RUL) prognosis methodology developing for a deteriorating FCS



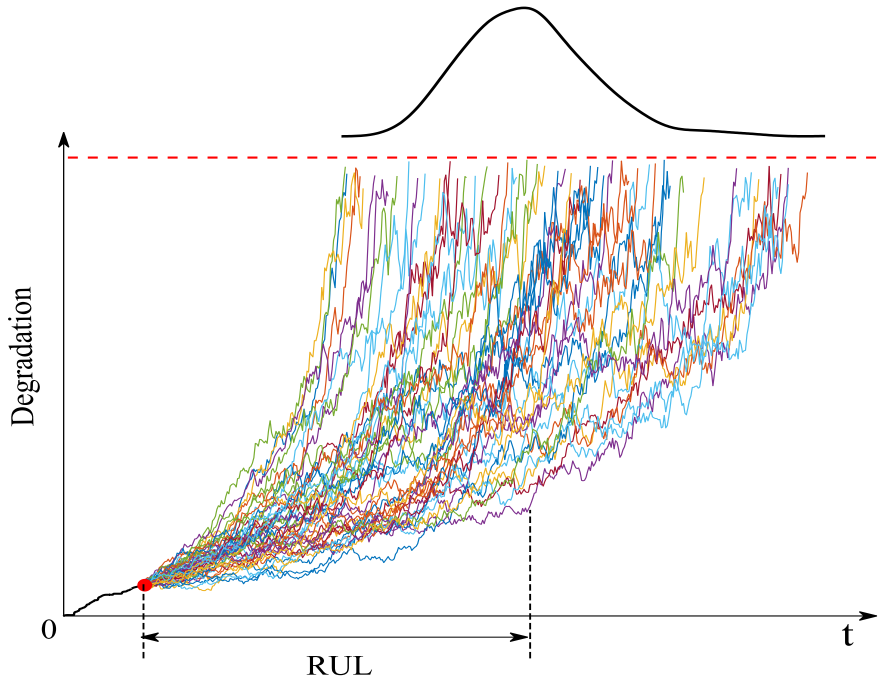
- RUL-based controller reconfiguration



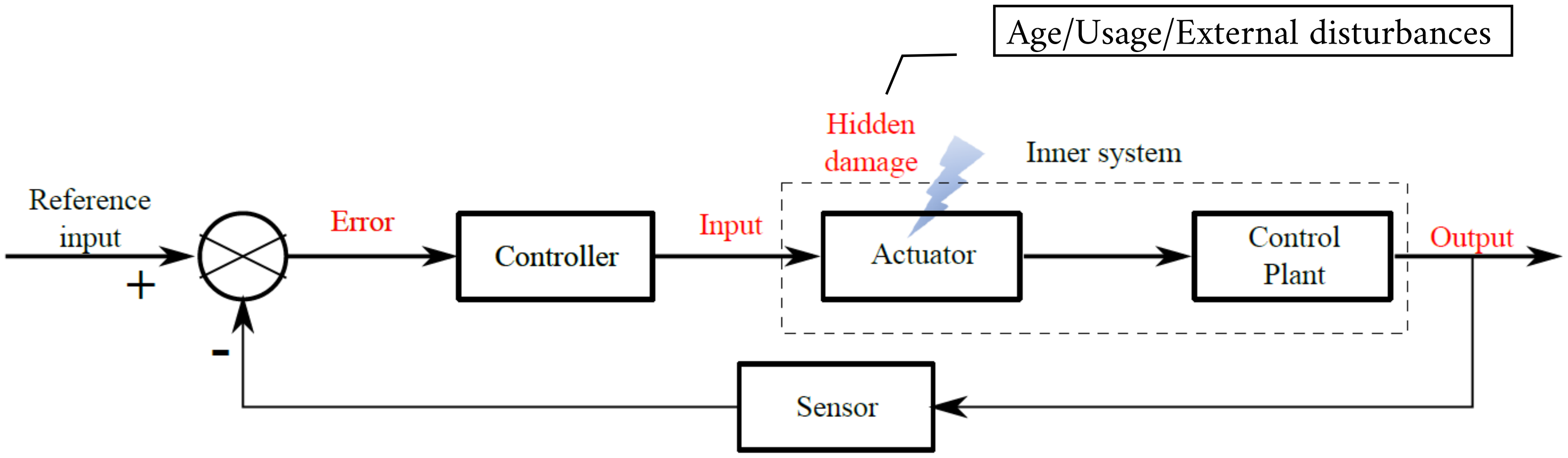
- RUL prognosis methodology developing for a deteriorating FCS

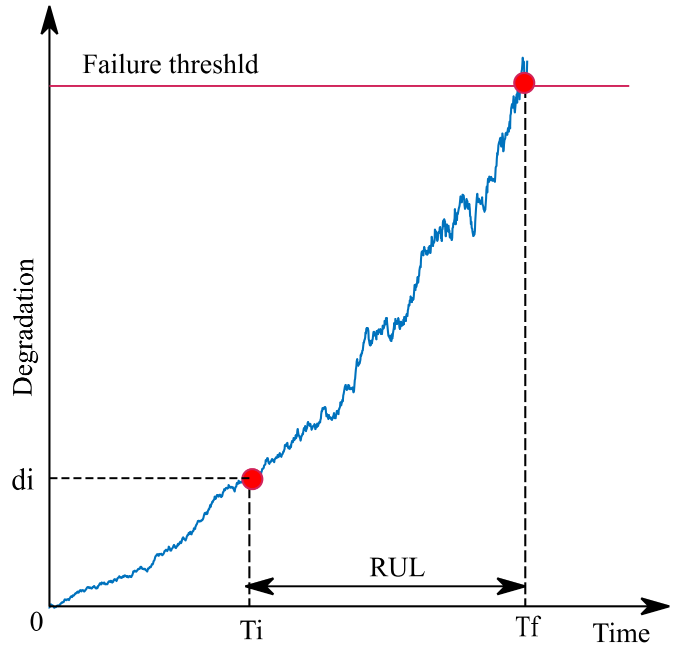


- Degradation evolution revealing for the overall FCS
- RUL prognosis based on degradation process



- A deteriorating FCS





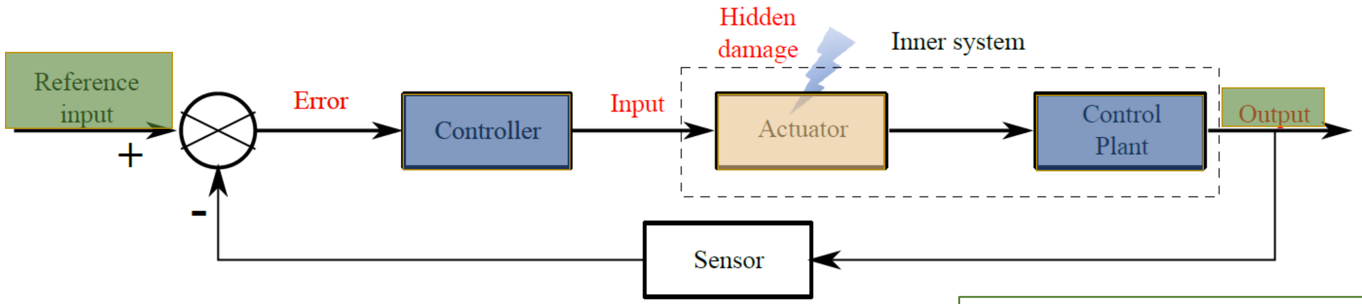
- FCSs are widely used in manufacturing system, railway systems, aircraft systems, power system and nuclear systems etc..
- Randomness of hidden damage
- Degradation
- Effect: performance loss of FCS
- Failure/failure threshold
- RUL prognosis can be used for post-prognosis activities: maintenance, controller reconfiguration

- Previous study

- **Component-level**

- Component observable
- Stochastic process modeling
- Probability calculation

- Langeron et al (2015,2017) ;Mabrouk et al (2019,2020) Xiao et al(2021)
- Mo et al (2015);Wu et al (2018);Alam et al (2014);Xu et al (2019)



- Physical knowledge dependent

- System structure
- Hidden damage estimation
- RUL estimation

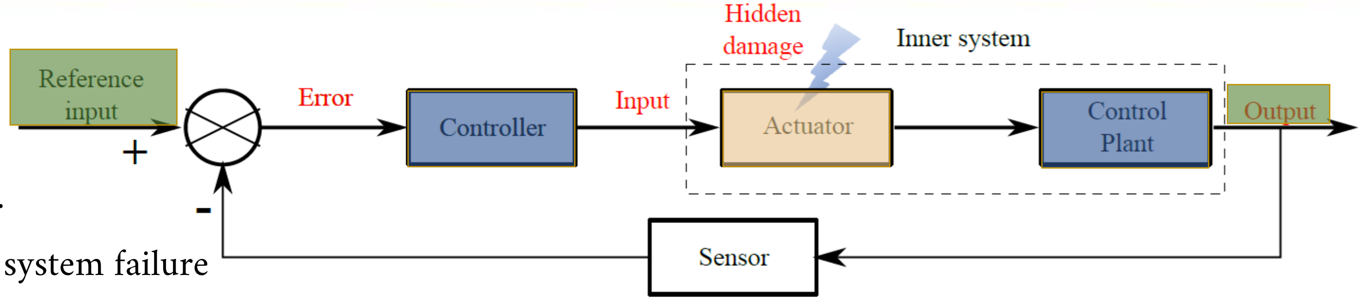
- Shi et al (2017);Si et al (2019);Nguyen et al(2015);Moulahi et al(2018,2022);Jayaprasanth et al (2023);Obando et al(2021)

- **System-level**
- Physical knowledge independent

- Input-Output
- **System performance-**based health index
- RUL prediction

- Aggab et al (2022);Gong et al (2022;2023)
- Chen et al(2023);Wang et al(2021)

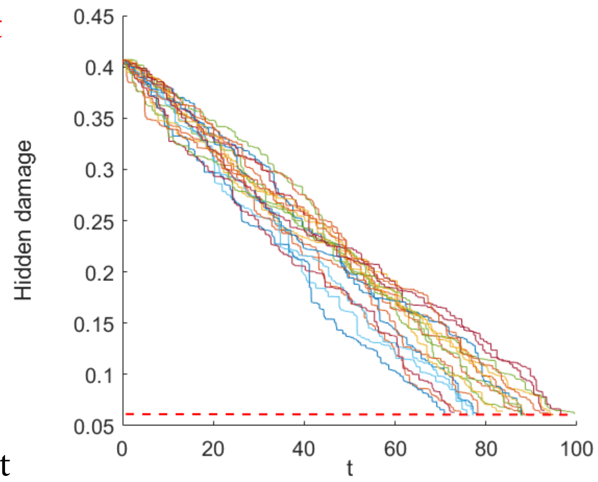
Remarks of previous method

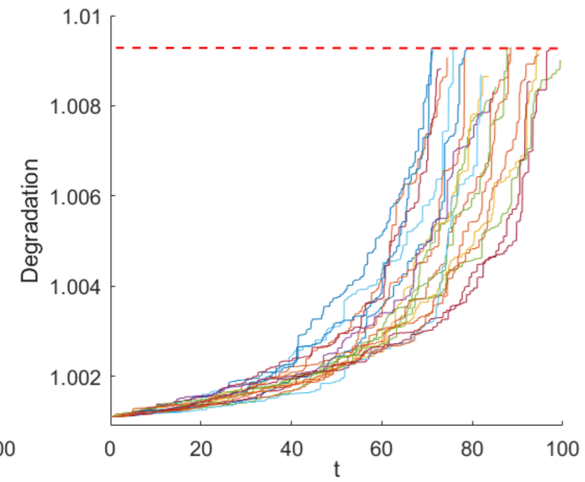


- Difficulty for the observation of component degradation evolution.
- Difficulty for the association between component degradation and system failure
- Insufficient system physical knowledge

Aim

- ➢ Construct a system-level degradation index only by reference input and output
- ➢ RUL prognosis

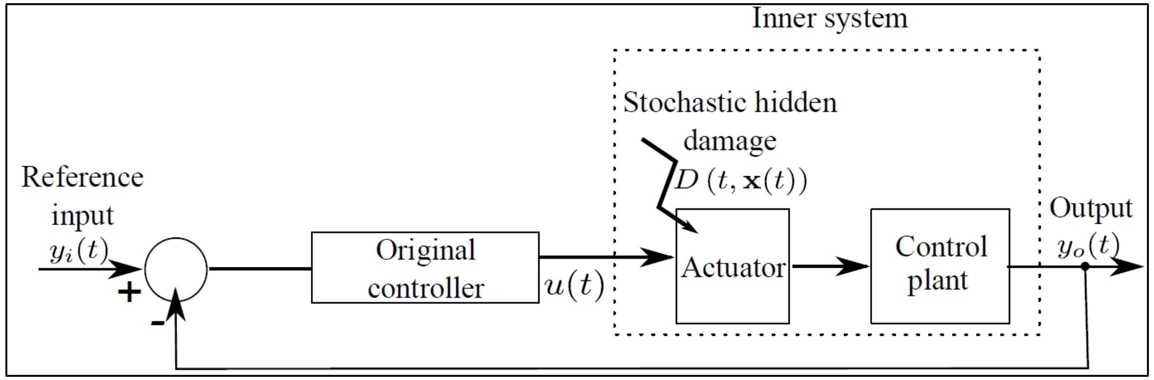


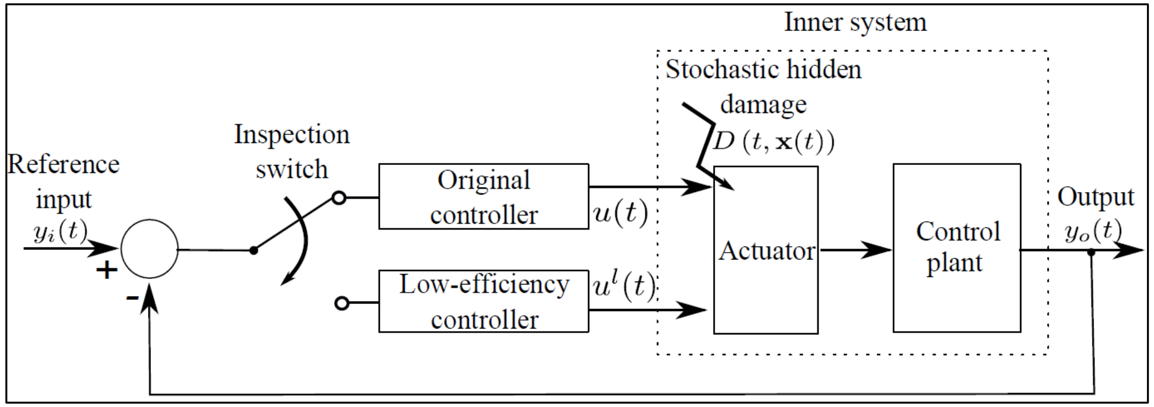


Difficulty

- Difficulty for degradation index construction with reference input and output
- Fault-tolerant property of controller cannot reveal hidden damage information

- Inspection method: for reducing controller effect





- System

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(D(t,\mathbf{x}(t)))\mathbf{x}(t) + \mathbf{B}(D(t,\mathbf{x}(t)))u(t), \\ y_o(t) = \mathbf{C}(D(t,\mathbf{x}(t)))\mathbf{x}(t), \end{cases}$$

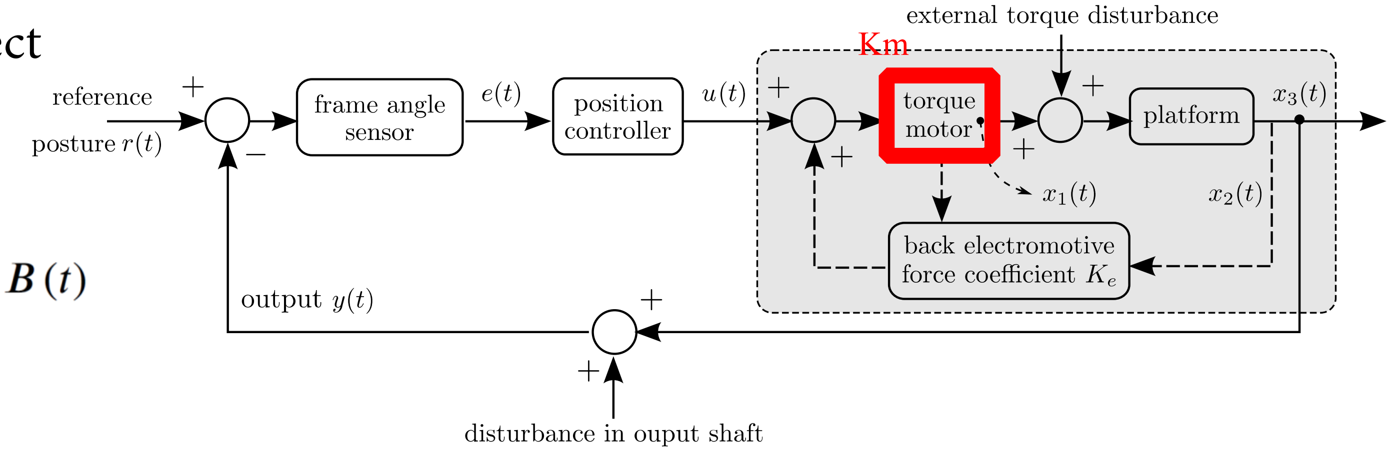
- Hidden damage

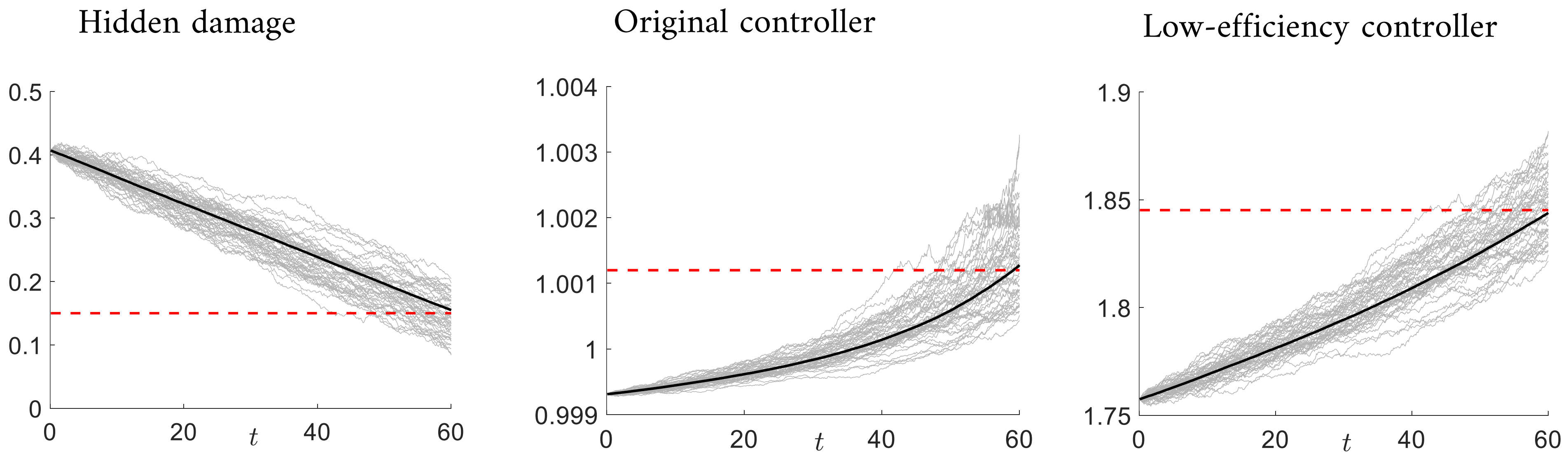
$$D(t, \mathbf{x}(t)) = D(0, \mathbf{x}(0)) + \int_0^t \mu_h(r, \mathbf{x}(r))\, dr + \sigma_h dB(r)$$

- Reference input and output acquisition

- Inspection method: for reducing controller effect
  - Hidden damage

$$K_m(t) = D\left(t, x_1(t)\right) = d_0 + \lambda \cdot \int_0^t x_1^2(r)\,dr + \sigma \cdot B(t)$$

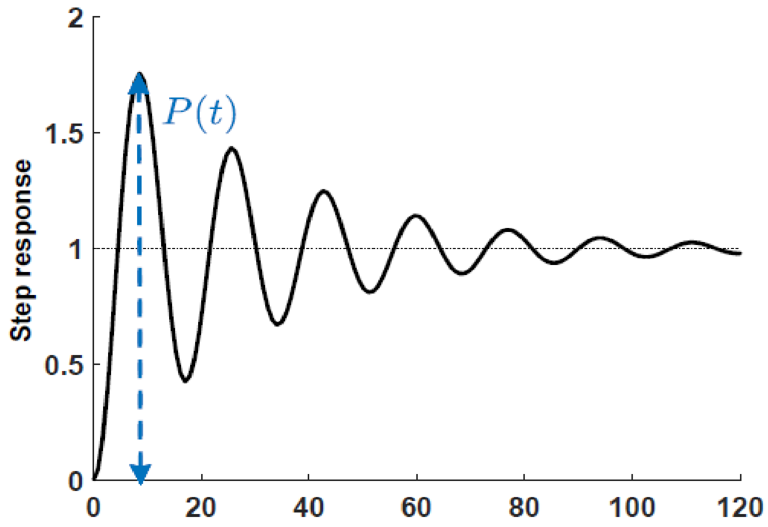


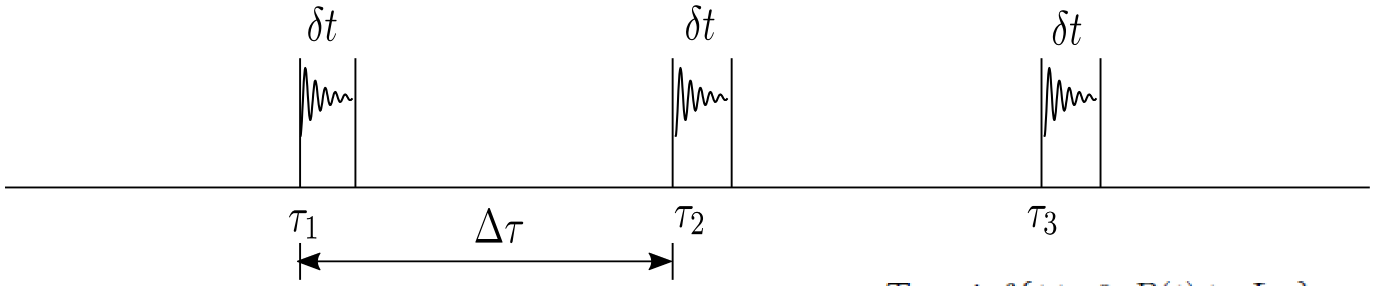


- Index construction: performance-based system-level index for revealing degradation evolution
  - System performance : stability, system response, ability to meet specific control objectives
  - Transfer function by Laplace transform

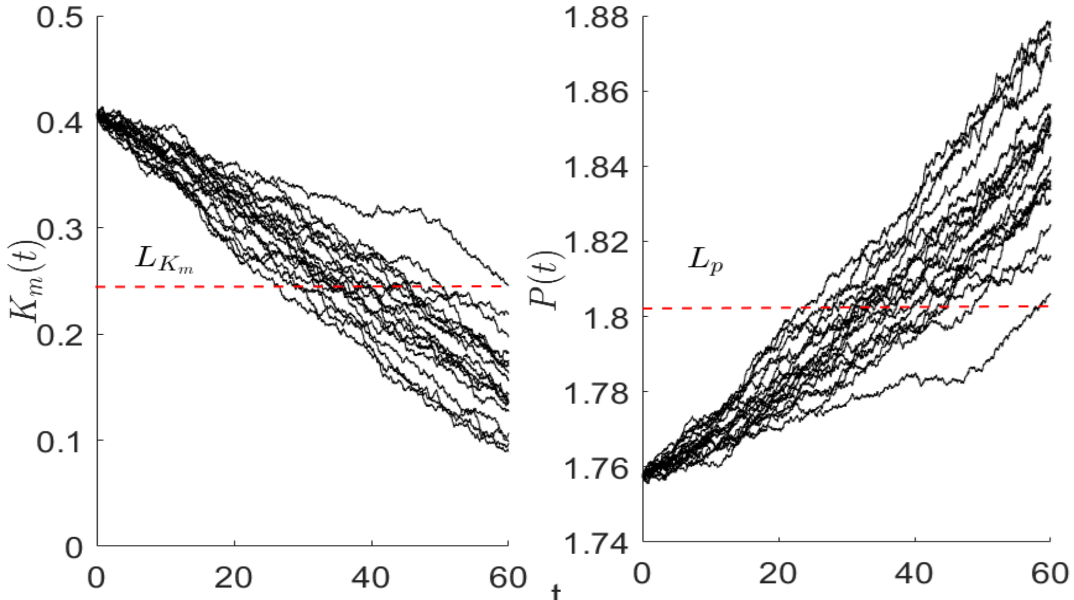
$$H(s) = \frac{Y_o(s)}{Y_i(s)}$$

  - Time-domain response peak





$$T_f = \inf\{t \geq 0, P(t) \geq L_P\}$$



➢ Degradation index modeling by stochastic process
➢ RUL distribution by probability calculation

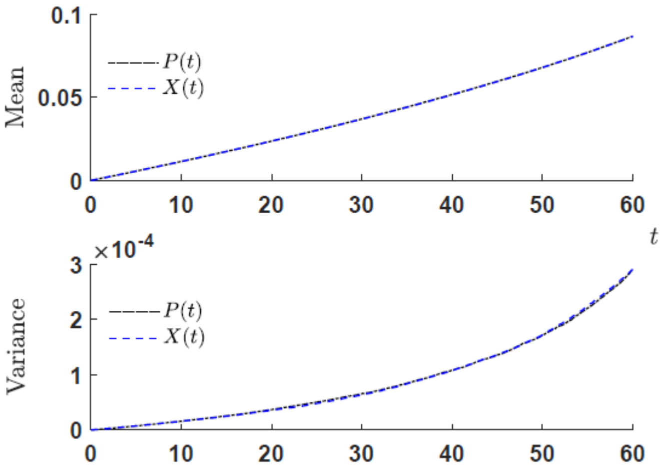
- Stochastic diffusion process (SDP) model

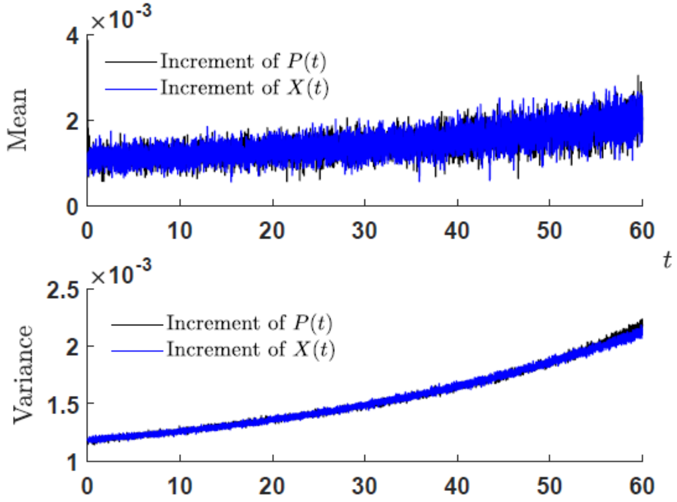
$$dX(t) = \mu\left(t, X(t)\right) dt + \sigma\left(t, X(t)\right) dB(t)$$

$$\mu(t, X(t)) = a_{t_\xi} t^{\xi} + \cdots + a_{t_1} t + a_{X_\xi} X^{\xi}(t) + \cdots + a_{X_1} X(t) + a_c$$
$$\sigma(t, X(t)) = b_{t_\eta} t^{\eta} + \cdots + b_{t_1} t + b_{X_\eta} X^{\eta}(t) + \cdots + b_{X_1} X(t) + b_c$$

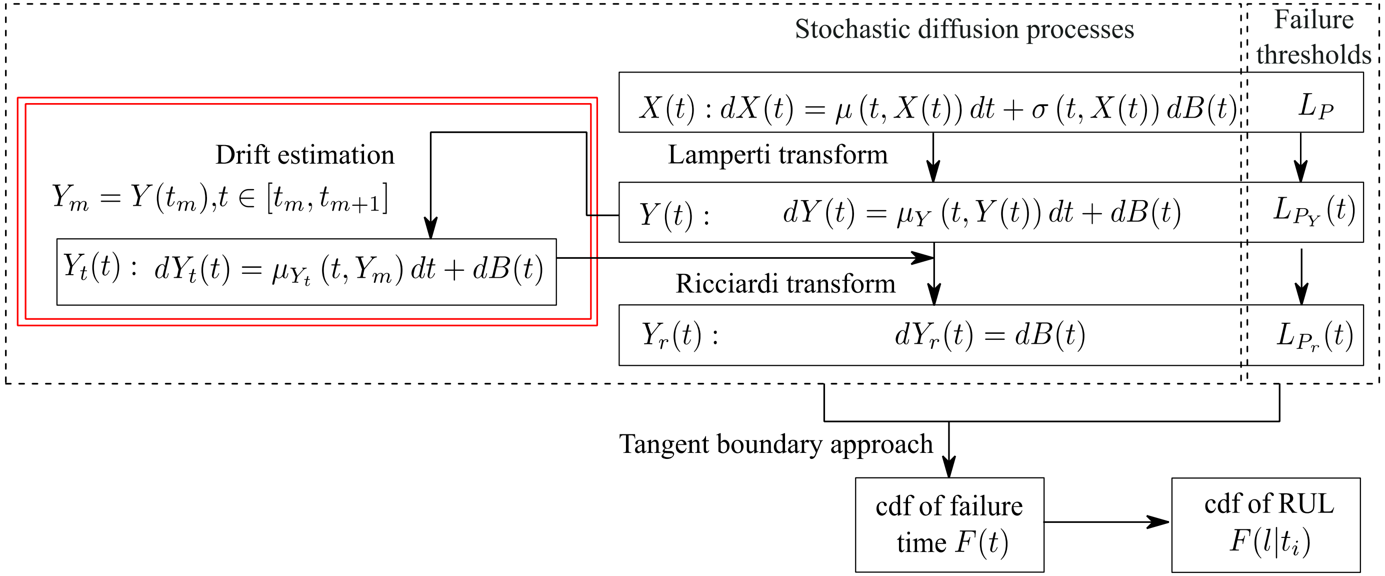
- Maximum likelihood estimation

$$\begin{aligned} l_\theta = \sum_{k=0}^{T-1} \Big[& \frac{1}{2} \log \left|2\pi\sigma(t_k, X(t_k); \theta_b)\sigma^T(t_k, X(t_k); \theta_b)\Delta t\right| \\ & + \frac{1}{2}\left(X(t_{k+1}) - X(t_k) - \mu\left(t_k, X(t_k); \theta_a\right)\Delta t\right)^T \\ & \times \left[\sigma(t_k, X(t_k); \theta_b)\sigma^T(t_k, X(t_k); \theta_b)\right]^{-1} \\ & \times \left(X(t_{k+1}) - X(t_k) - \mu\left(t_k, X(t_k); \theta_a\right)\Delta t\right) \Big] \end{aligned}$$





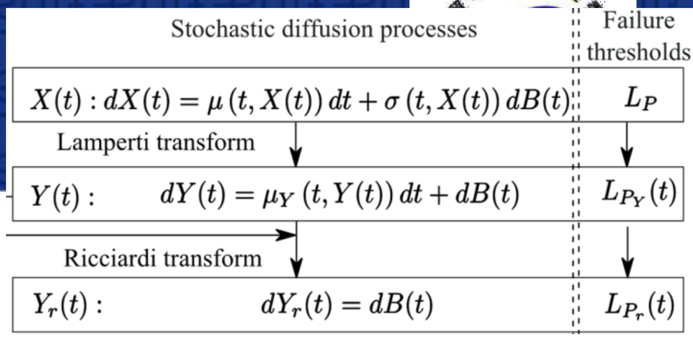
- RUL distribution obtaining procedure

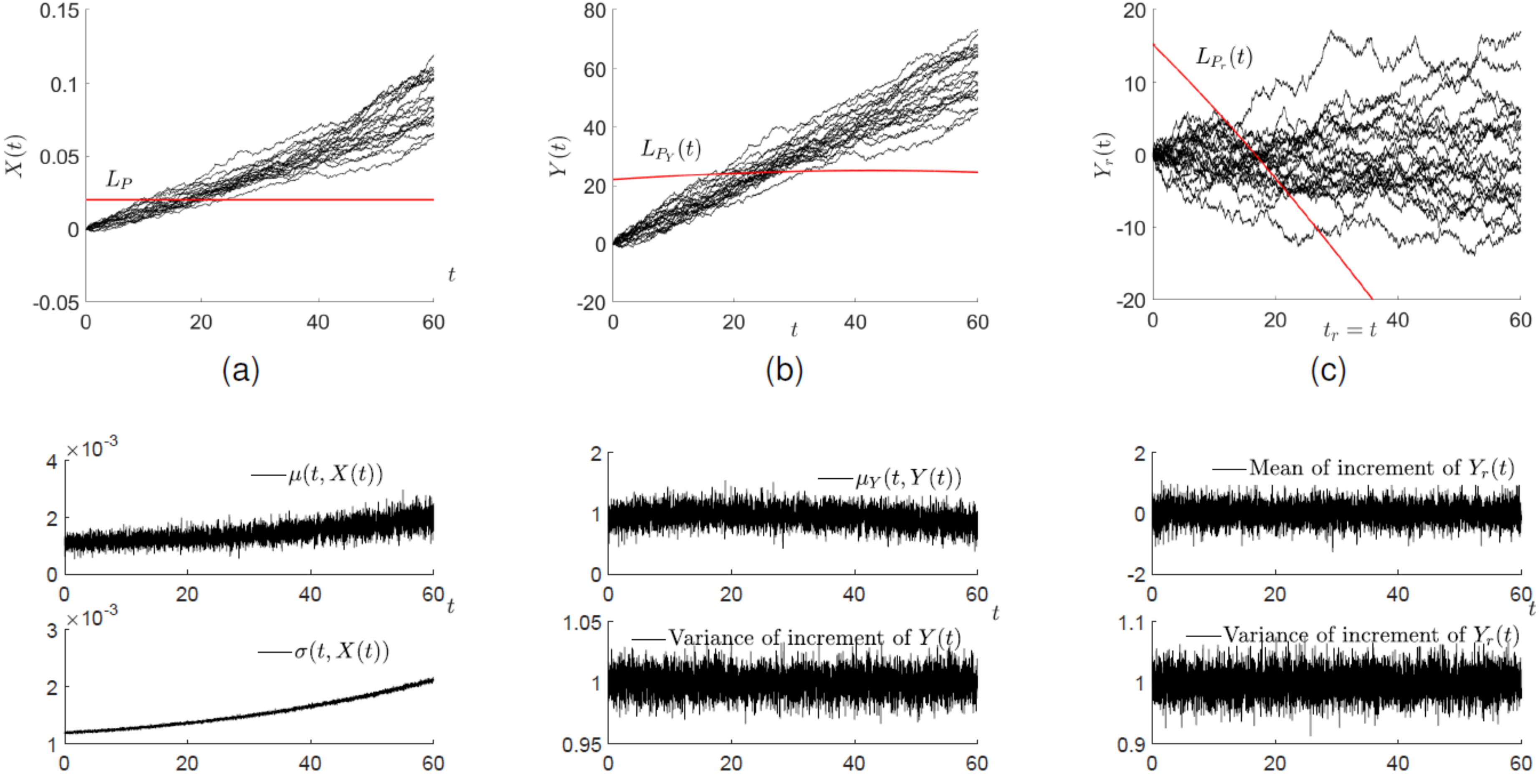


- Transform to a standard Brownian motion (SBM)
- Drift approximation

- RUL distribution estimation based on standard Brownian with time-varying failure threshold

- From a stochastic diffusion process to a standard Brownian motion





(a) (b) (c)

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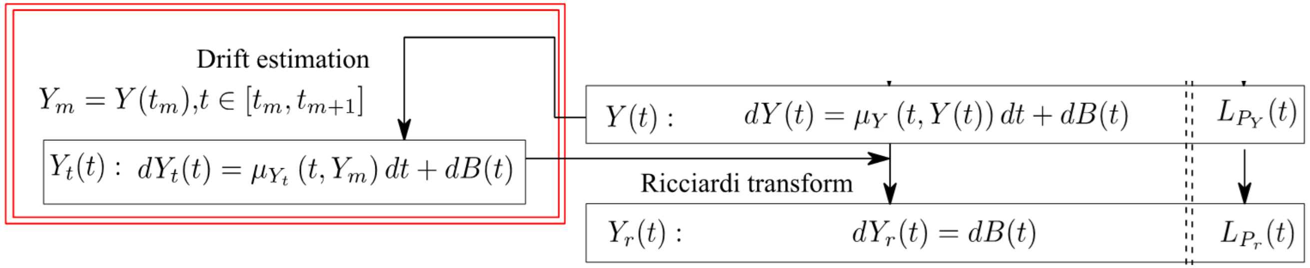


- Lamperti transform

$$dY(t) = \mu_Y(t, Y(t))\, dt + dB(t)$$

$$\mu_Y(t, Y(t)) = \frac{\varphi(t, X(t))}{\partial t} + \frac{\mu(t, X(t))}{\sigma(t, X(t))} - \frac{1}{2}\partial\sigma_X(t, X(t))$$

$$Y(t) \triangleq \varphi(t, X(t)) = \int \frac{1}{\sigma(t, x)} dx \Big|_{x = X(t)}$$

$$L_{P_Y}(t) \triangleq \varphi(t, L_P) = \int \frac{1}{\sigma(t, x)} dx \Big|_{x = L_P}$$

- Ricciardi transform

$$\mu_Y(t, Y(t)) = \frac{1}{2}(c_1(t) + c_2(t) y)$$

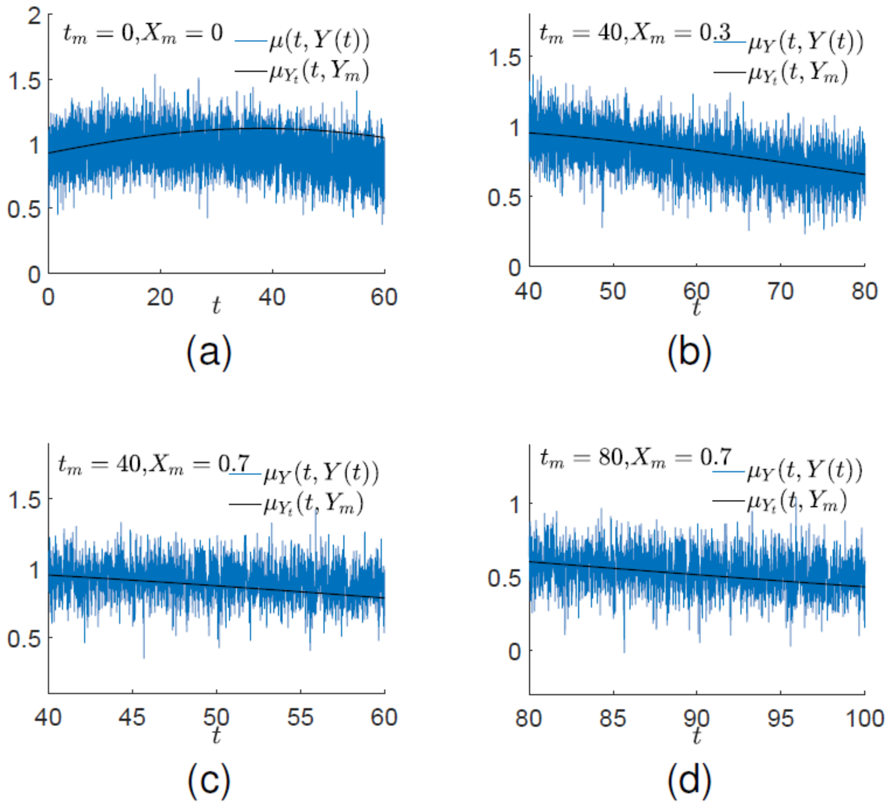
$$\begin{cases} Y_r(t) \triangleq \psi(t, y) = exp\left[-\frac{1}{2}\int_0^t c_2(\tau) d\tau\right] y \\ \quad - \frac{1}{2}\int_0^t c_1(\tau) exp\left[-\frac{1}{2}\int_0^\tau c_2(r) dr\right] d\tau \\ t_r \triangleq \phi(t) = \int_0^t exp\left[-\int_0^\tau c_2(r) dr\right] d\tau \end{cases}$$

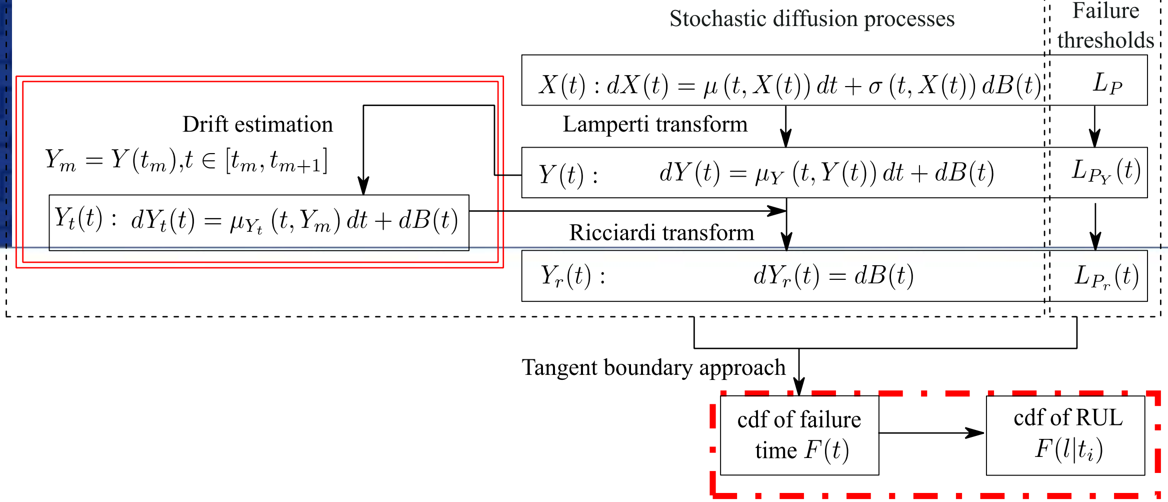
$$L_{P_r}(t) \triangleq \psi(t, \varphi(t, L_P)) = \varphi(t, L_P) - \varphi(t, X_m) - \int_0^t \left(\frac{\mu(\tau, X_m)}{\sigma(\tau, X_m)} - \frac{1}{2}\partial\sigma_X(\tau, X_m)\right) d\tau.$$

- Drift approximation

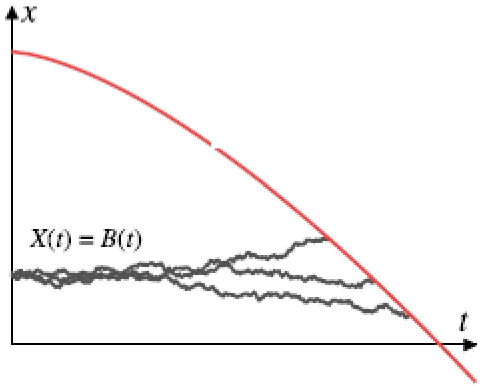
$$\mu_Y(t, Y(t)) \approx \mu_Y(t, Y(t_m)), \; t \in [t_m, t_{m+1}]$$

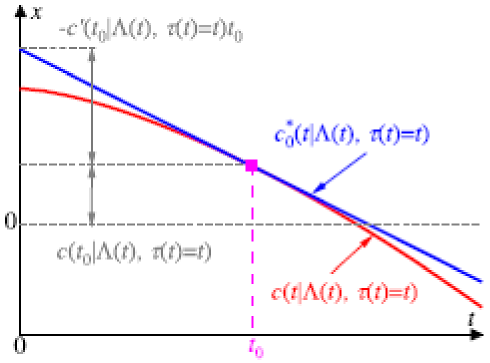
$$dY_t(t) = \mu_{Y_t}(t, Y_m)\, dt + dB(t)$$





- RUL distribution estimation: tangent boundary approach
  - Current literature have limitation when boundary is curvy





- For any time t, failure time cumulative distribution function (CDF) is

$$F(t) = \Phi\left(\frac{-\Delta_1 + \Delta_2}{\sqrt{t}}\right) + \exp(2\Delta_2\Delta_1) \cdot \Phi\left(\frac{-\Delta_1 - \Delta_2}{\sqrt{t}}\right)$$

with

$$\Delta_1 = L_{P_Y}(t) - \int_0^t \mu_{Y_t}(\tau, Y_m)d\tau - L'_{P_Y}(t)t + \mu_{Y_t}(t, Y_m)t$$

and

$$\Delta_2 = \left(\mu_{Y_t}(t, Y_m) - L'_{P_Y}(t)\right)t$$

- l as RUL $\quad F(l|t_m, X_m) = \Phi\left(\frac{-\Delta'_1 + \Delta'_2}{\sqrt{l}}\right) + \exp(2\Delta'_2\Delta'_1) \cdot \Phi\left(\frac{-\Delta'_1 - \Delta'_2}{\sqrt{l}}\right)$

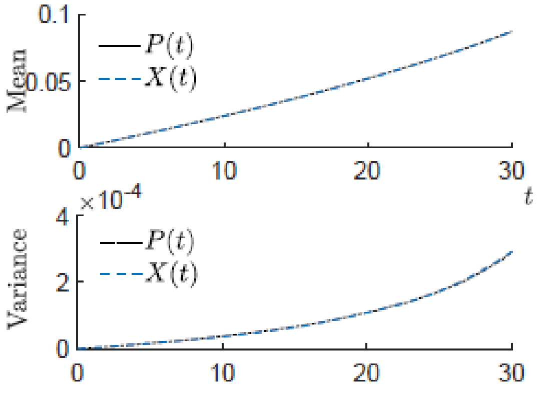
with

$$\Delta'_1 = L_{P_Y}(t_m + l) - \int_0^{t_m+l} \mu_{Y_t}(\tau, Y_m)d\tau - L'_{P_Y}(t_m) + \int_0^{t_i} \mu_{Y_t}(\tau, Y_m)d\tau - L'_{P_Y}(t_m + l)(t_m + l) + \mu_{Y_t}(t_m + l, Y_m)(t_m + l)$$

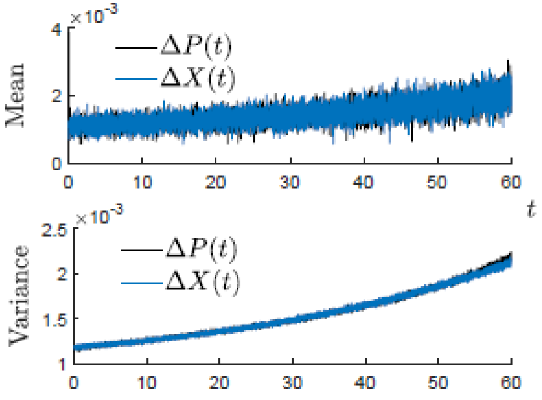
and

$$\Delta'_2 = \left(\mu_{Y_t}(t_m + l, Y_m) - L'_{P_Y}(t_m + l)\right)(t_m + l).$$

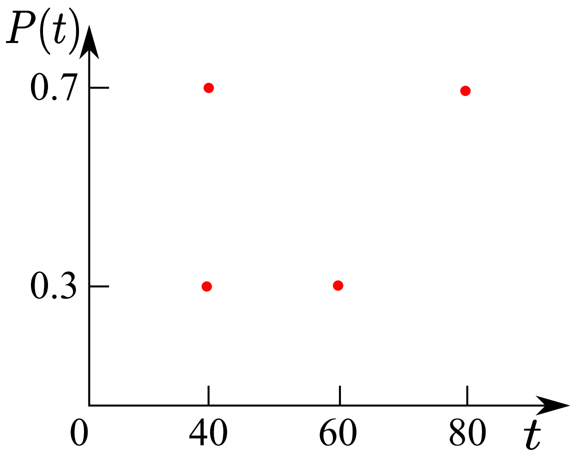
- Validation of degradation modeling

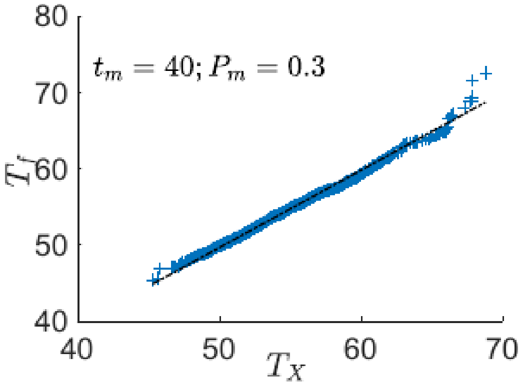


(a)

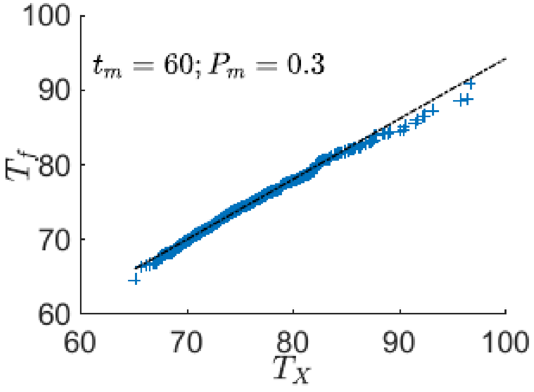


(b)





(a)



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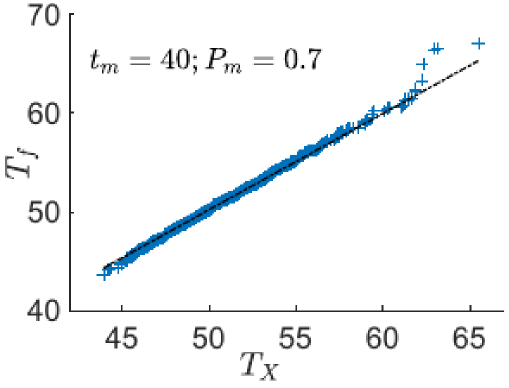
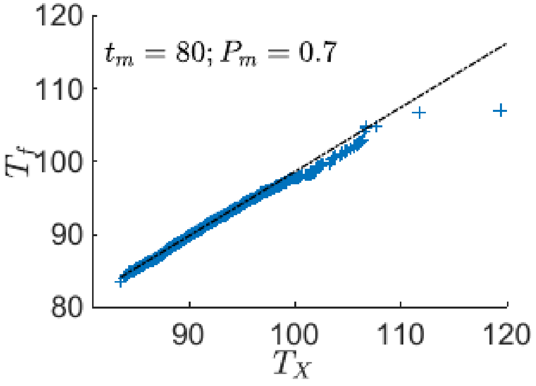
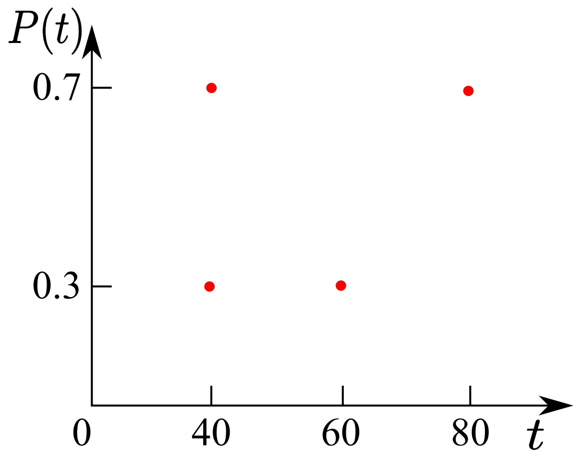


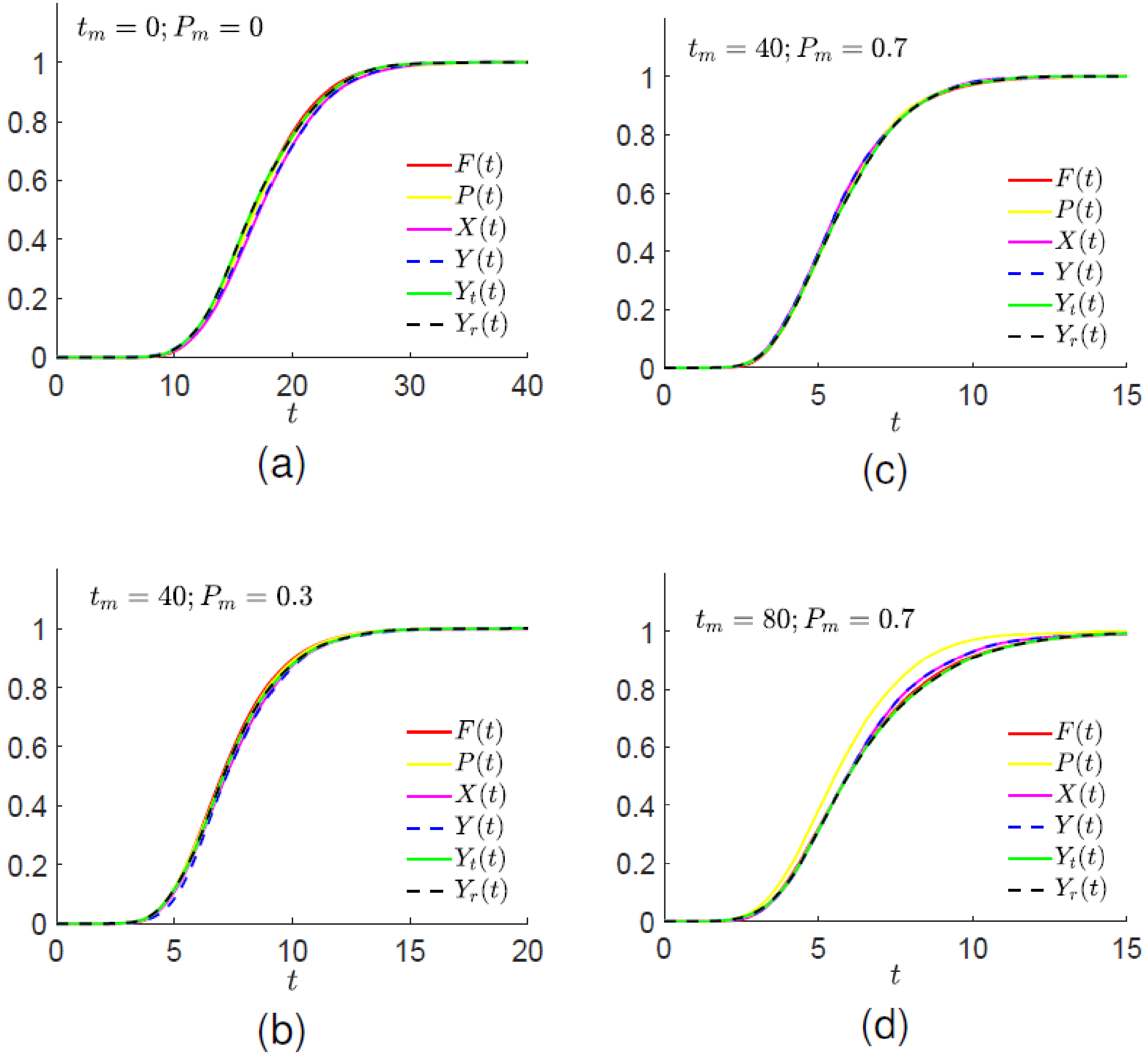
(c)



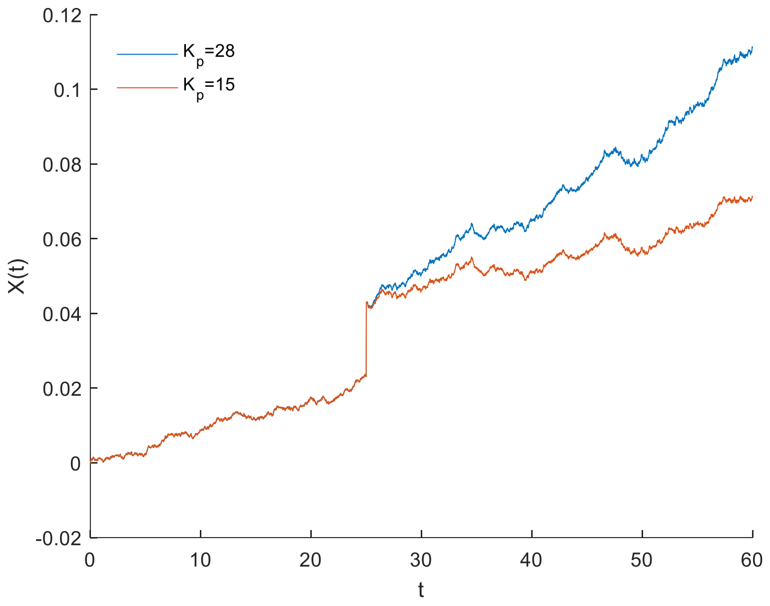
(d)

- Validation of RUL distribution estimation



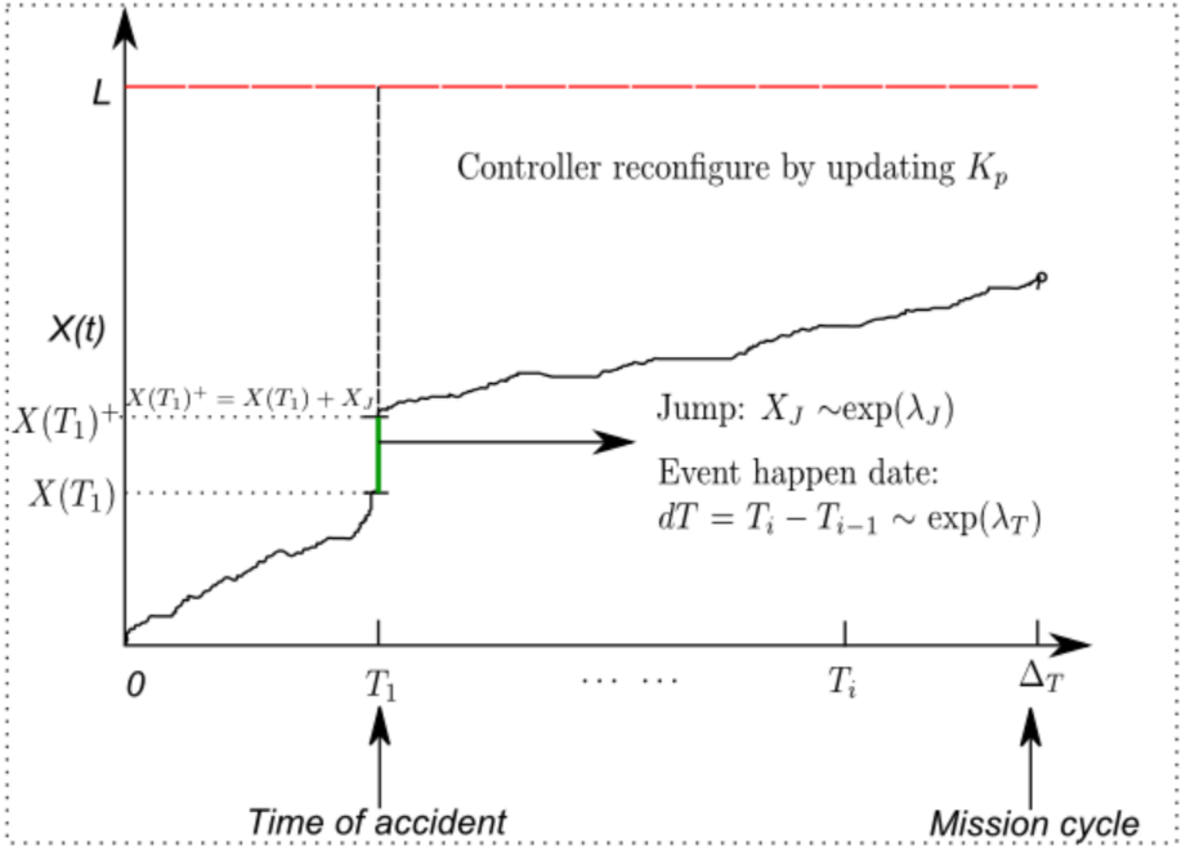


- ➢ RUL-based controller reconfiguration



  - ➢ Accident: random jump happening in a fixed duration mission
  - ➢ Controller reconfiguration method by balancing RUL and system performance

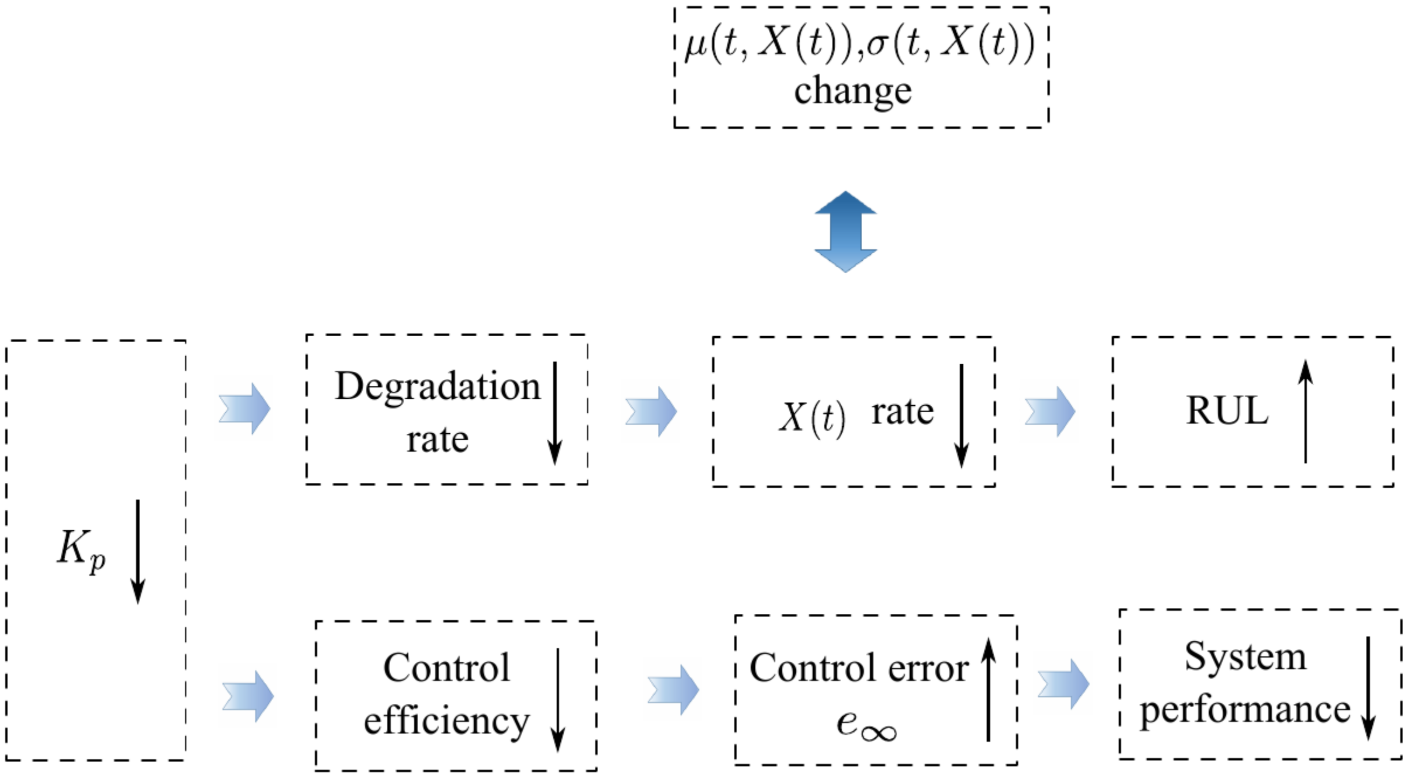
- # Mission description and objective
  - A fixed duration mission

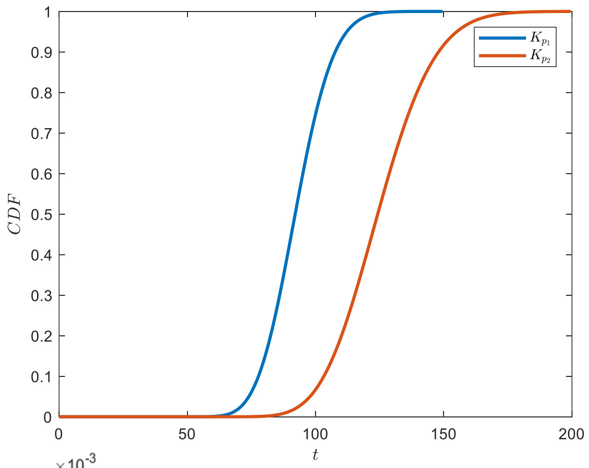


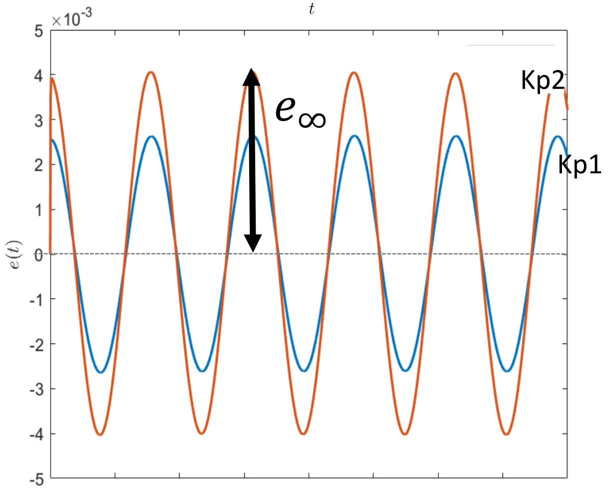
$$u(t) = K_p \cdot \left(e(t) + \frac{1}{T_i} \cdot \int_0^t e(z)dz + T_d \cdot \frac{de(t)}{dt} \right)$$

- When accident happens, we update the controller gain Kp appropriately to increase the probability that the FCS can accomplish this mission.

- Controller reconfiguration: update the controller gain Kp

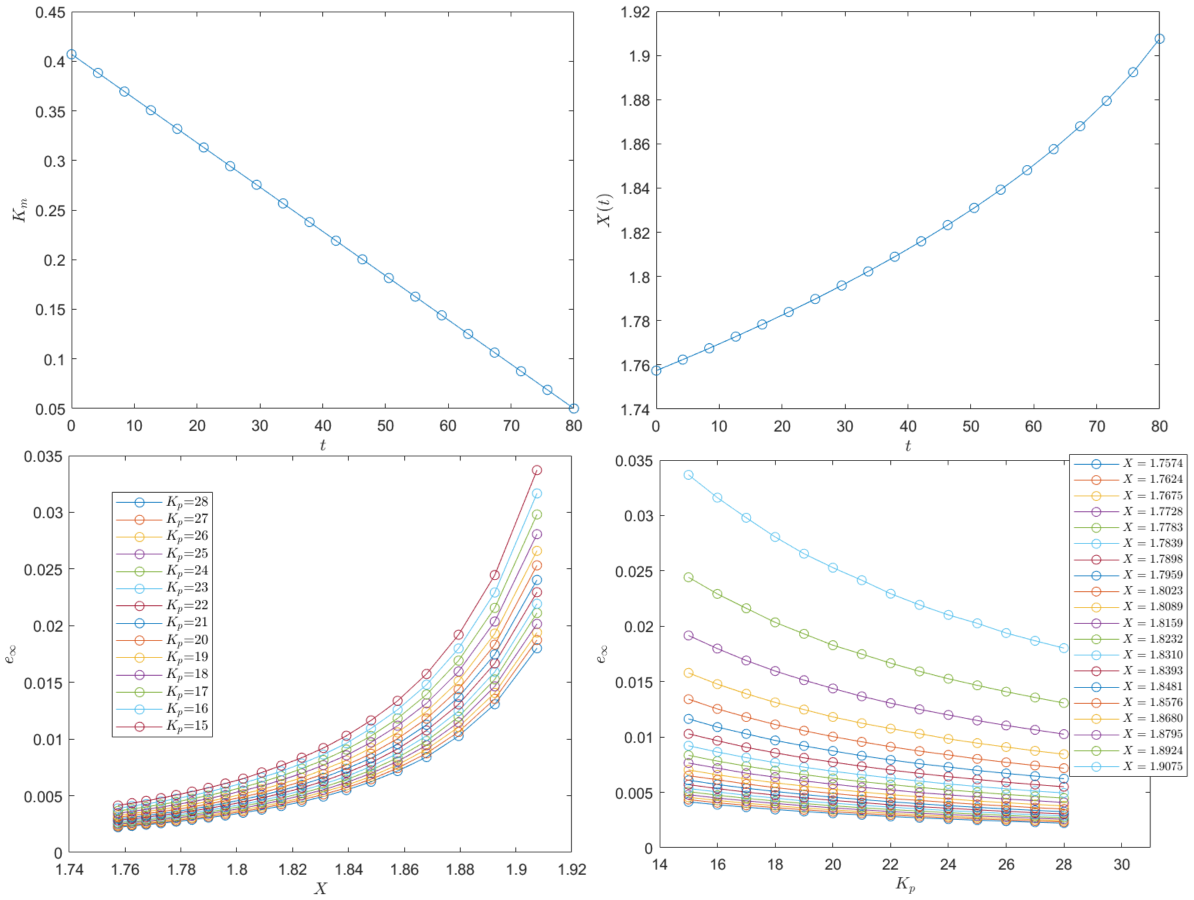






➢ The selection of the appropriate controller gain is carried out by balancing the system degradation level (RUL prognosis via developed method) with the system performance (How to calculate system performance?).

- How to introduce e_∞ into objective function
  - e_∞ function



- As analyzed, e_∞ relates to degradation level and controller gain

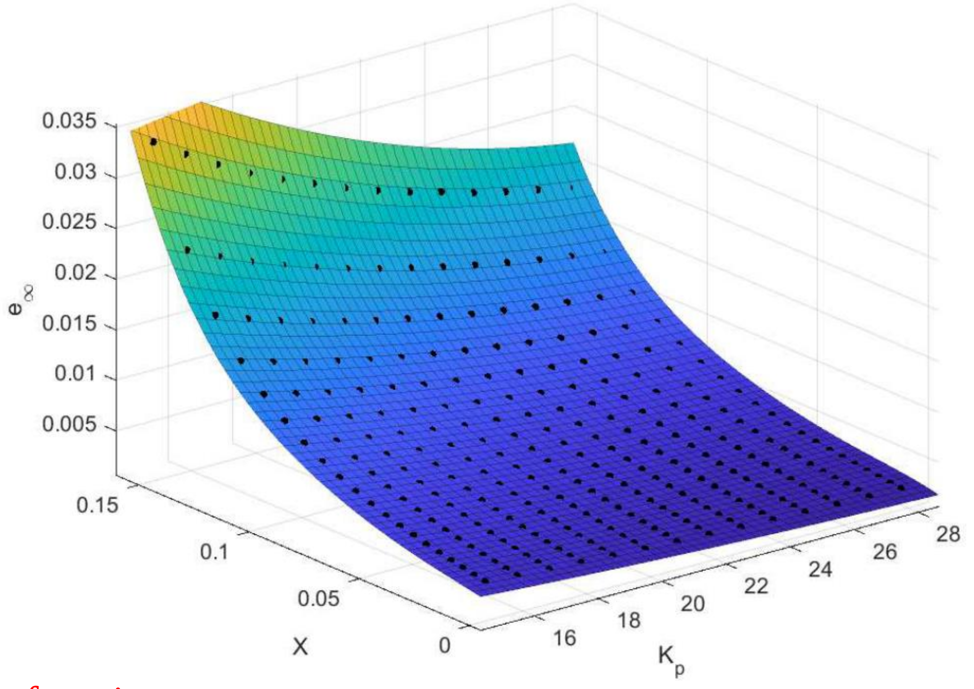
➢ A mapping from X,Kp to e_∞ to be given

- How to introduce e_∞ into objective function

$$\boxed{K_p, X \rightarrow e_\infty}$$

- We build this mapping via least square

$$e_\infty = f(K_p, X) = \sum_{i}^{m} \sum_{j=0}^{i} p_{ij} {K_p}^i X^{i-j}$$

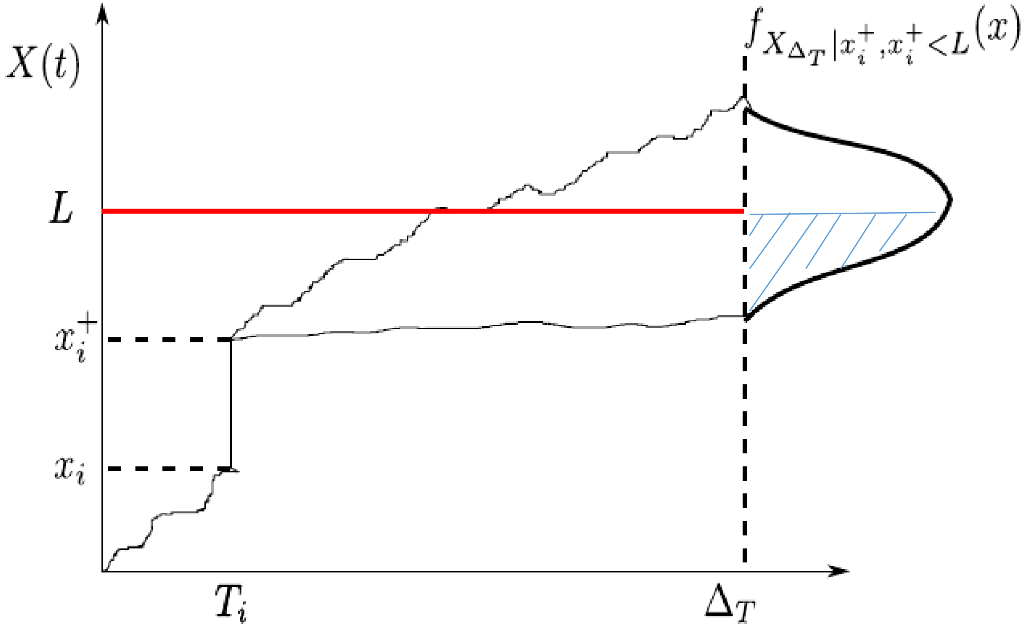


➢ e_∞ function and RUL distribution integrate into objective function

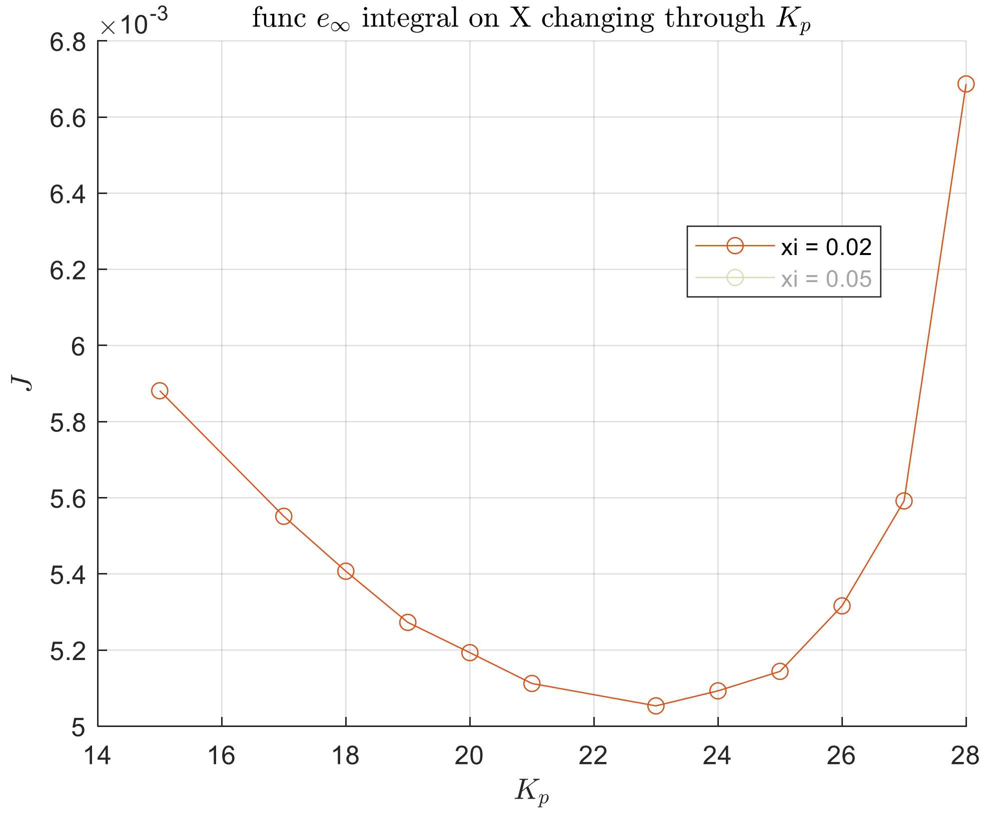
- Objective function: give failure threshold L

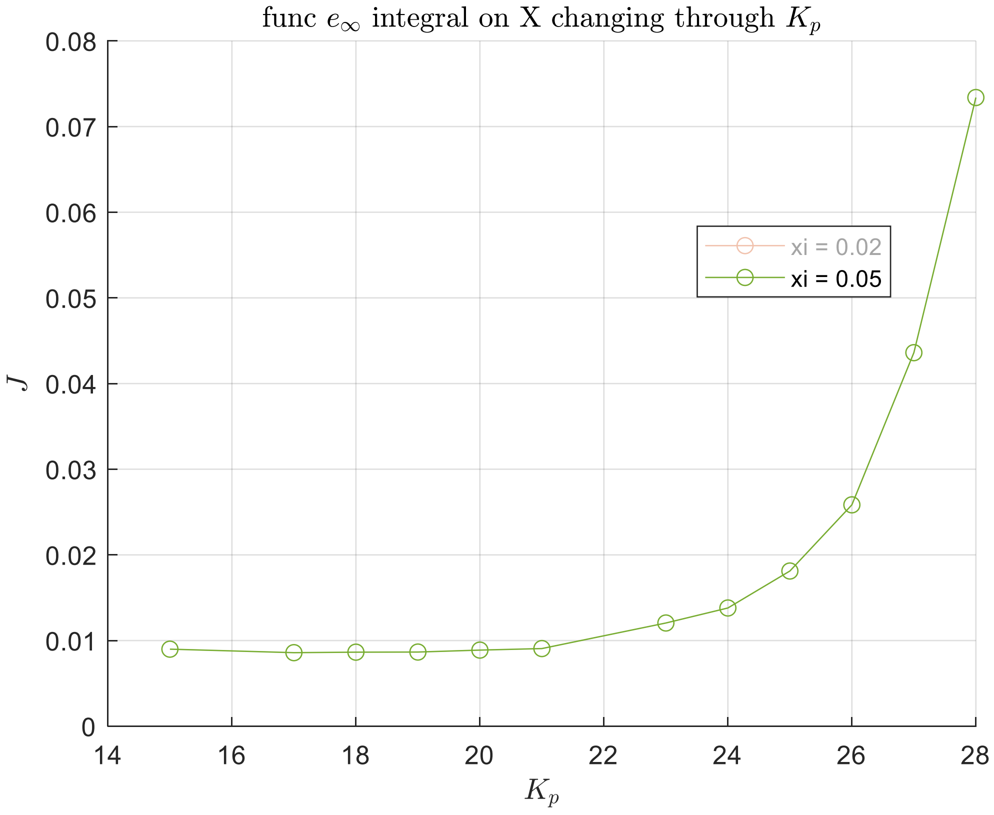
$$\mathrm{J}(K_p) = \frac{\int_0^L e_\infty(K_p, x) \cdot f_{X_{\Delta_T}|x_i^+, x_i^+ < L}(x)dx}{1 - F(X_{\Delta_T}|X_{T_i} = x_i^+)}$$

$$K_p^{opt} = \arg\min_{K_p} \mathrm{J}(K_p)$$

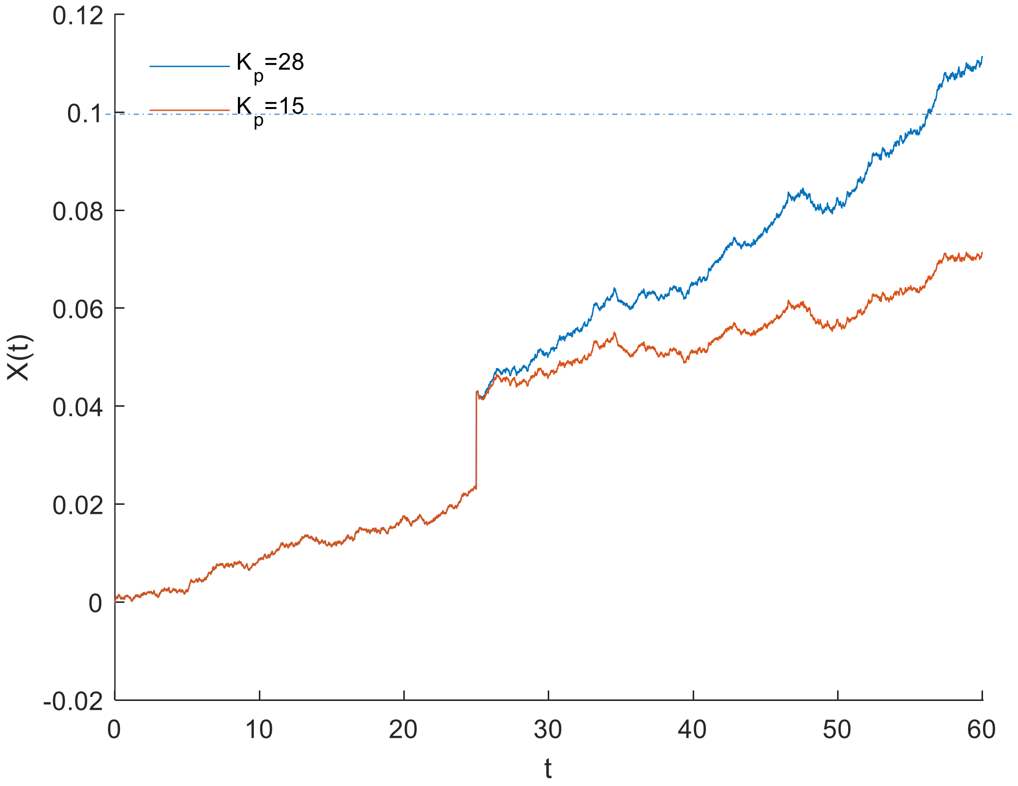


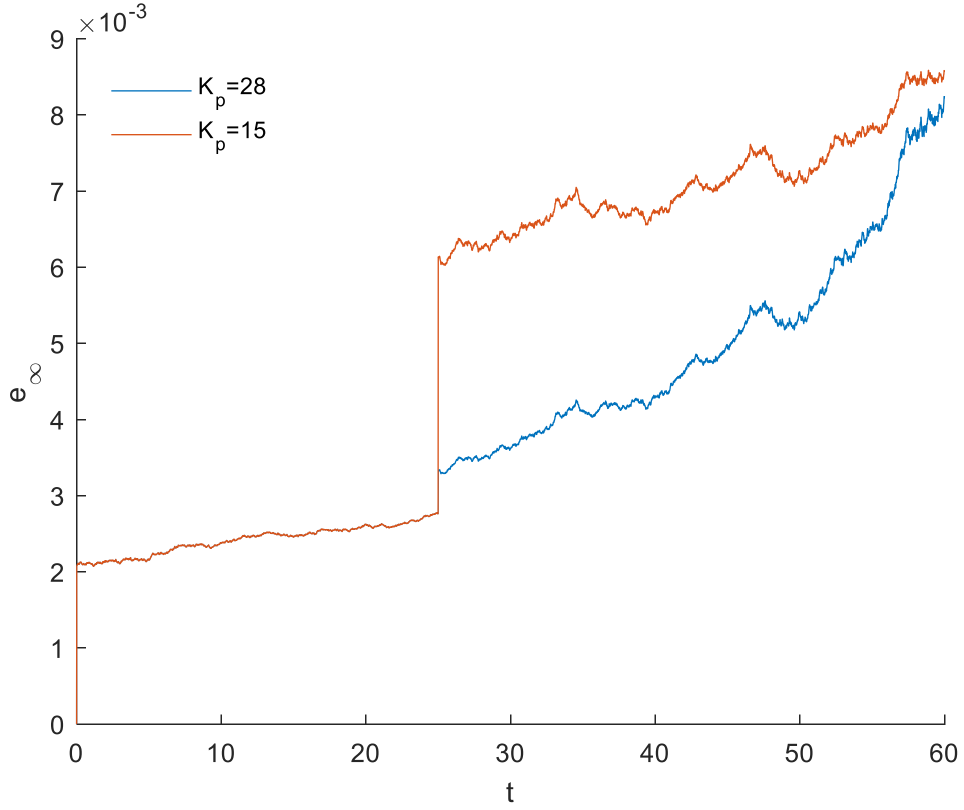
- Some results: objective function application at Ti=20,$\Delta_T = 60, L = 0.1$;





- Some results: controller reconfiguration





- Conclusion and perspective

  - A degradation index, the peak of the output response under a unit step input signal, is constructed from the output information observed from a low-efficiency controller-driven FCS.
  - A RUL prognosis method is developed based on a stochastically modeled peak
  - A controller reconfiguration method is proposed, wherein the controller gain is updated based on the optimized objective function built by balancing the steady control error and RUL.

  - Maintenance policies incorporating controller reconfiguration, preventive maintenance and corrective maintenance.

Thanks for your attention