HEALTH-AWARE CONTROL OF COMPLEX INDUSTRIAL SYSTEMS

Vicenç Puig

Advanced Control Systems

GDR Macs Workshop - ENSAM - Paris - 22/11/2023

Outline

ADVANCED CONTROL SYSTEMS

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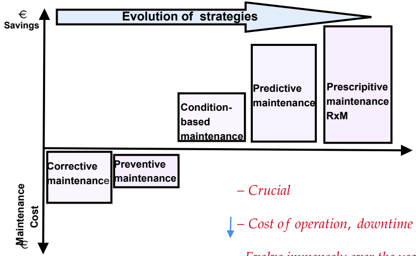
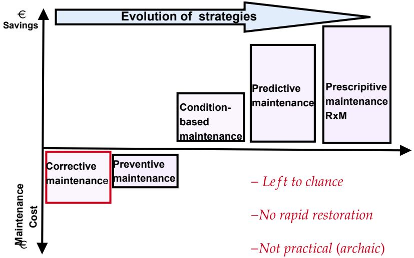


– *Crucial*

↓ – *Cost of operation, downtime*

–*Evolve immensely over the years*

Figure: Evolution of maintenance schemes.



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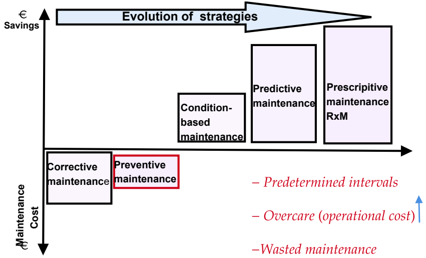

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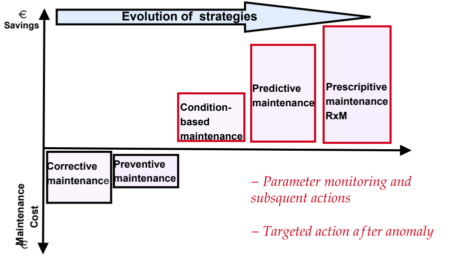

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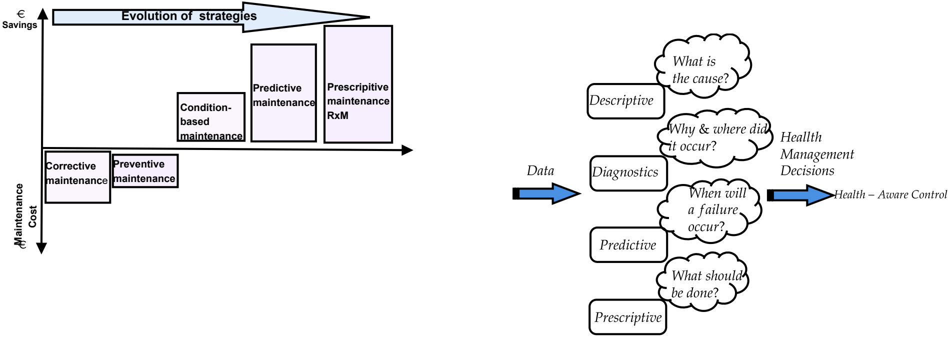

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Set-based Uncertainty Quantification For Prognostics.

Health-Aware Control Of a Wind Turbine

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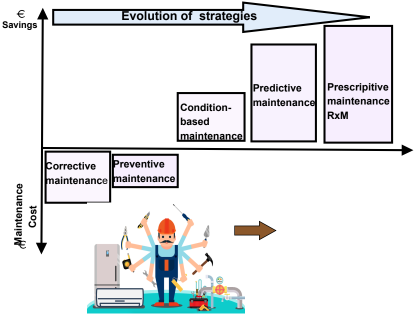

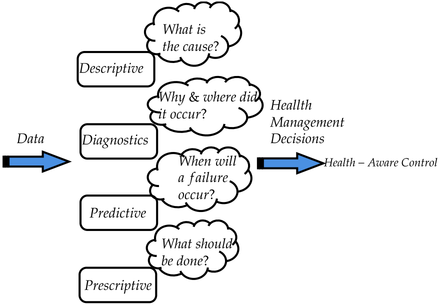

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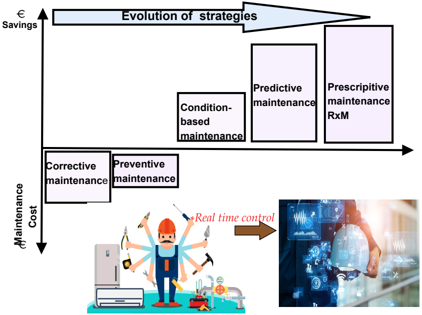

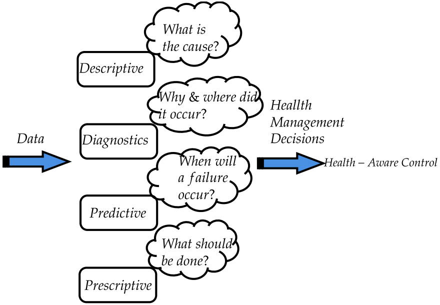

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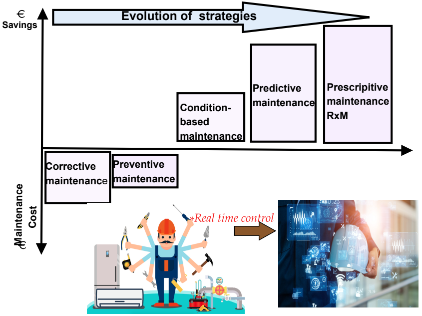

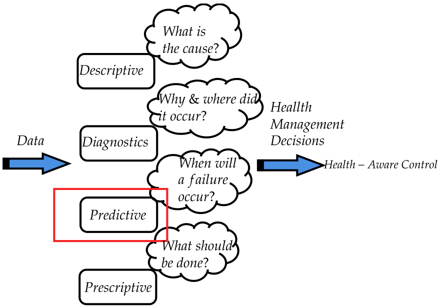

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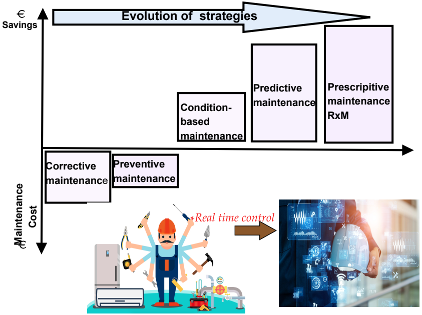

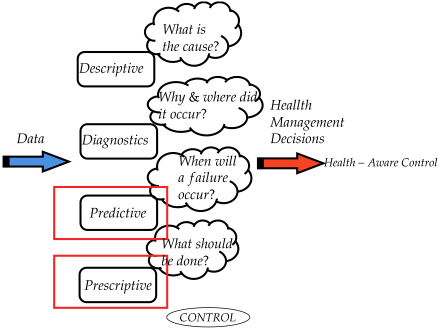

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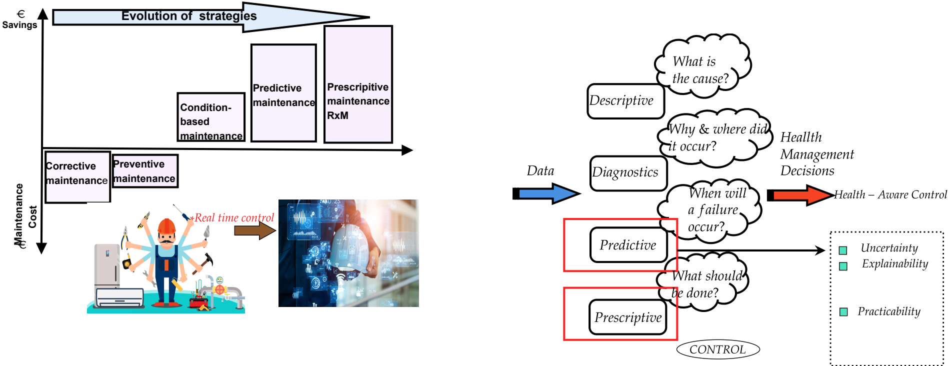

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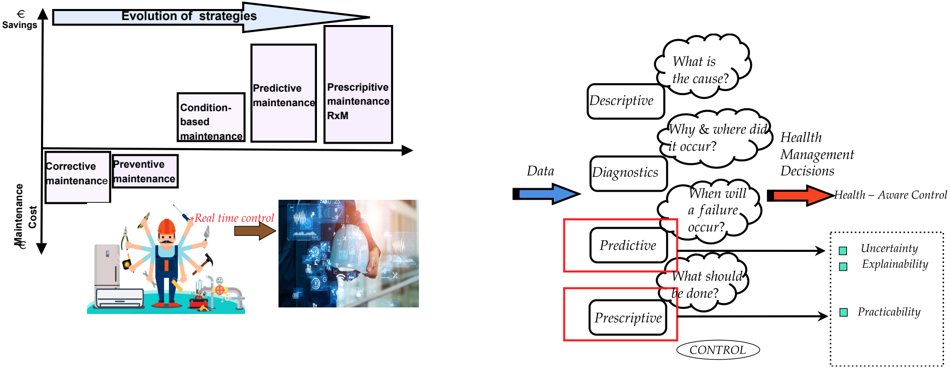

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This presentation aims to develop robust and comprehensive solutions to the development of Health-aware control.

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This presentation aims to develop robust and comprehensive solutions to the development of Health-aware control.

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4. Incorporate the developed prognostic methodology in 3 into a control framework.
5. Propose and develop a controller that preserves the health of an interconnected network.

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Introduction

Background-Degradation

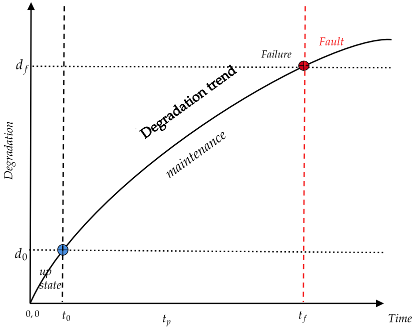

Figure: Degradation trend.

- Degradation is a precursor to failure(fault)

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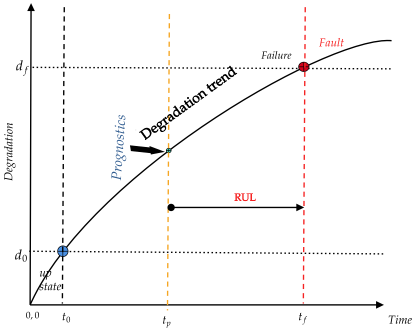

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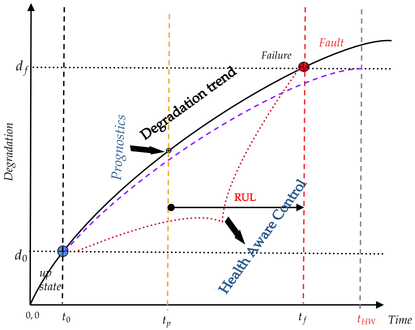

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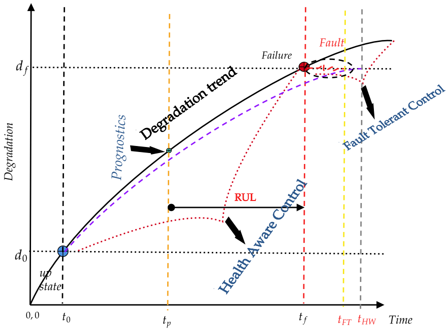
- HAC control to manage degradation.



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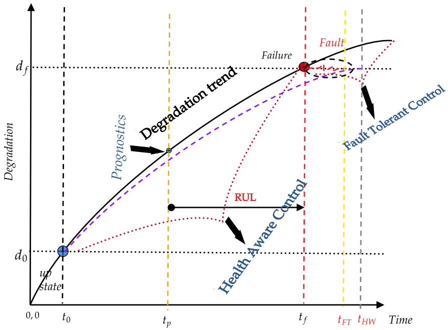

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Design controller based on the concept of degradation.

- Prognostics is the estimation of the RUL of the system.

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- ”Calculated” guess of a future event.

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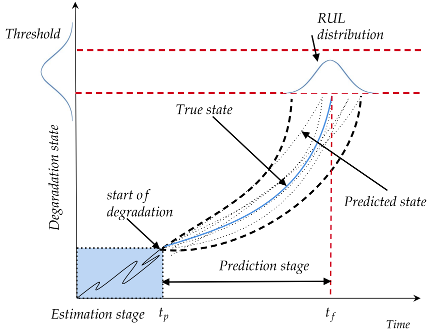

Figure: Illustration of prognostics methodology.

3 stages of Prognostics:

- State estimation:

$$x_{k+1} = f(x_k, \rho_k, u_k, w_k),$$
$$y_k = h(x_k, \rho_k, u_k, v_k).$$

Predict x_{k_*} and/or ρ_{k_*} and accompanying uncertainty.

- Prognostics is the estimation of the RUL of system.
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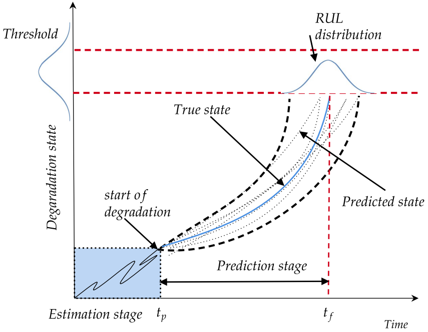

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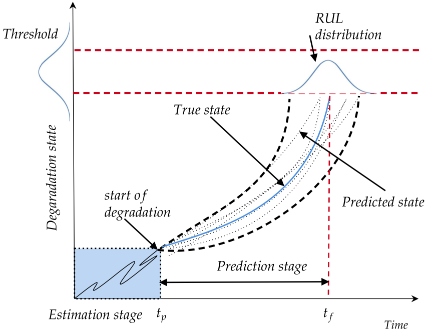

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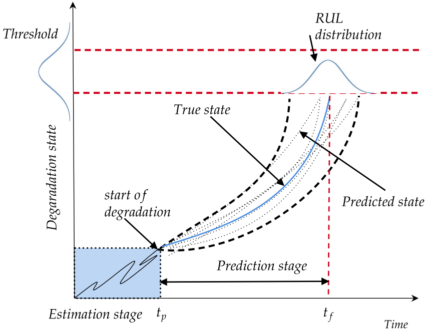

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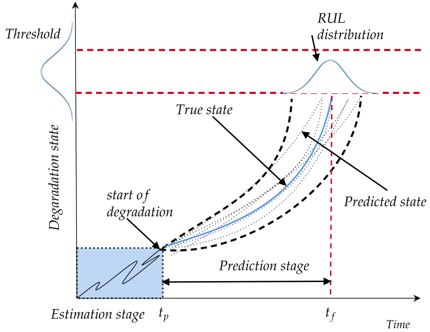

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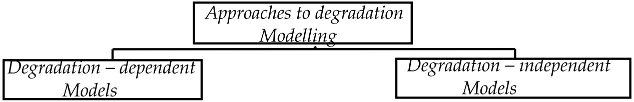
3 types of Prognostics

- Model, Data and Hybrid Prognostics

Introduction

Background-Health management control



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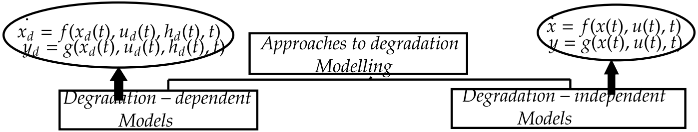

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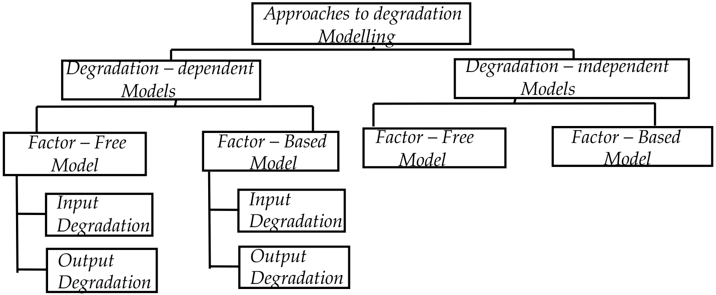

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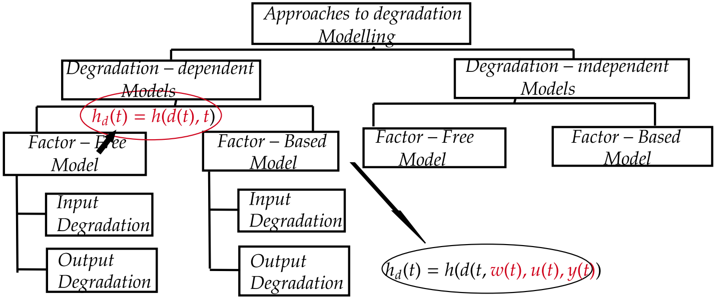
State of Art-Health management control



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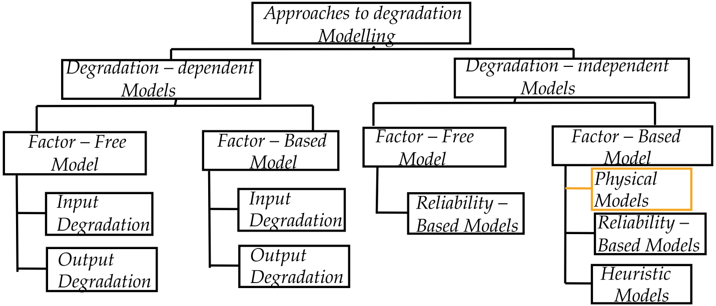

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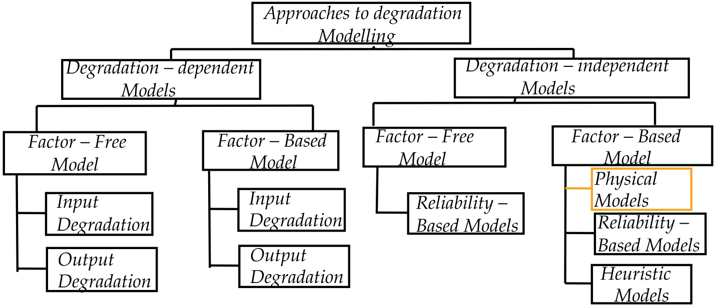

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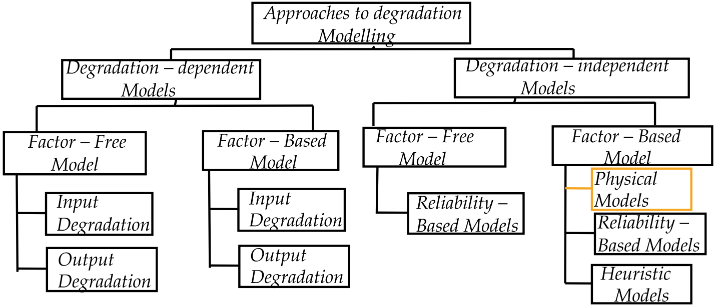

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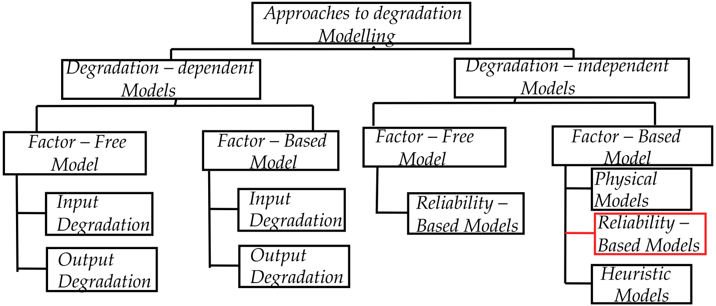

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  - Manipulates plants' variables to prolong lifetime or achieve mission targets.
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Set-based Uncertainty Quantification for Prognostics

Set based prognostics

- Uncertainty is assumed unknown but bounded.
- Set-based estimation for bounded state or parameter.
- Initial bounded set is propagated under model dynamics.
- $T_{EOL} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_\theta} \rightarrow \mathbb{B}$, the $EOL(k_p)$

$$EOL(k_p) = \inf\{k \in \mathbb{N} : k \geq k_p \wedge T_{EOL}(x(k), \theta(k), u(k)) = 1\},$$

- $$□RUL(k_p) = □EOL(k_p) - k_p.$$

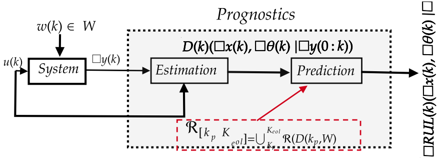

Figure: : Set-based Prognostics methodology.

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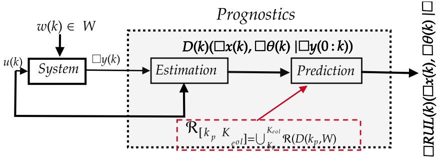
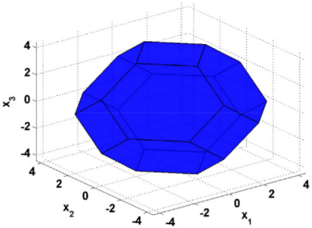


Figure: : Set-based Prognostics methodology.

Zonotopic Kalman filter is used for estimation



$$\mathcal{Z}\langle p, H\rangle = p \oplus HB^r$$

$$\langle p_1, H_1\rangle \oplus \langle p_2, H_2\rangle = \langle p_1 + p_2, [H_1 H_2]\rangle$$

$$\mathcal{K} \odot \langle p, H\rangle = \langle \mathcal{K}p, \mathcal{K}H\rangle$$

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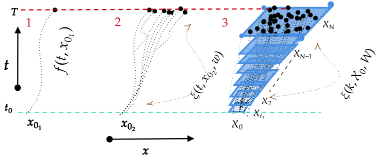
Reachability Analysis

Technique to evaluate resultant reachable sets under set bounded uncertainty description.

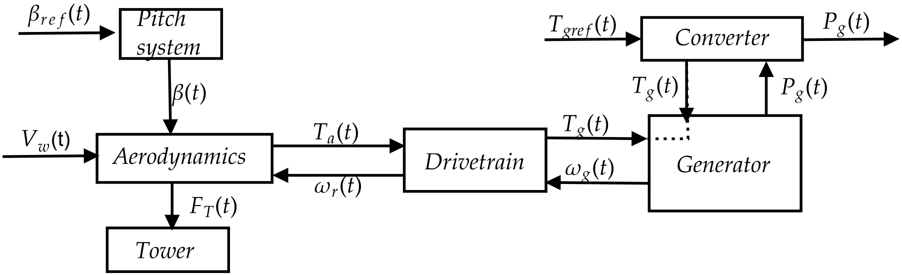
$$\mathcal{R}_{[t_0,\,T]}(\mathcal{X}_0,\,\mathcal{W}) = \bigcup_{t\in[t_0,\,T]} \mathcal{R}_t(\mathcal{X}_0,\,\mathcal{W})$$

$$\text{s.t.} w(t) \in \mathcal{W}$$

- Union of all feasible trajectories
- computationally feasible and efficient.
- Resultant set must be interpretable.



Application-Wind turbine dynamics



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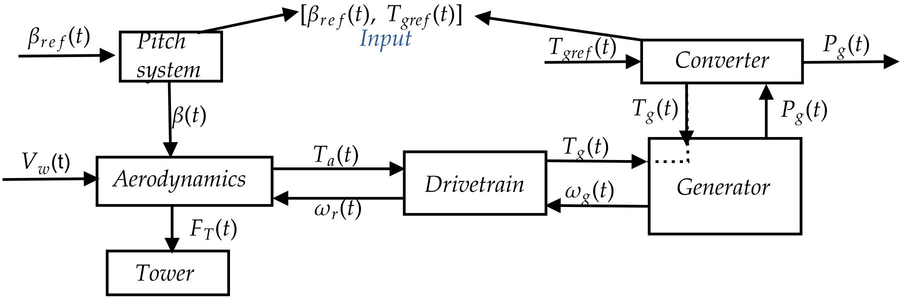
Set-based Uncertainty Quantification for Prognostics

Application-Wind turbine dynamics



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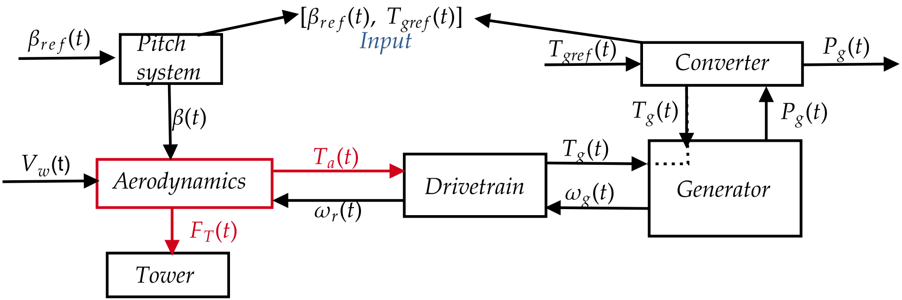

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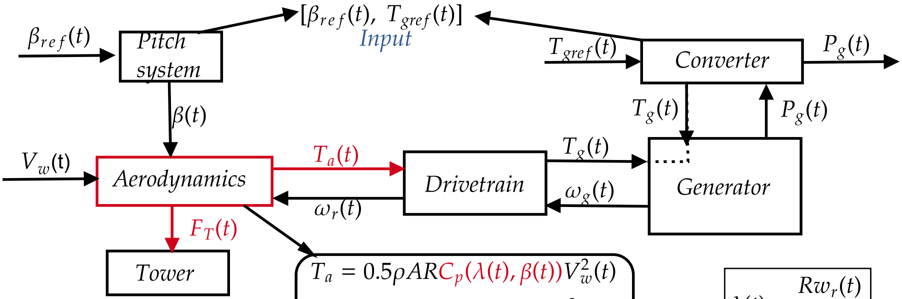


$$T_a = 0.5\rho A R C_p(\lambda(t),\beta(t))V_w^2(t)$$

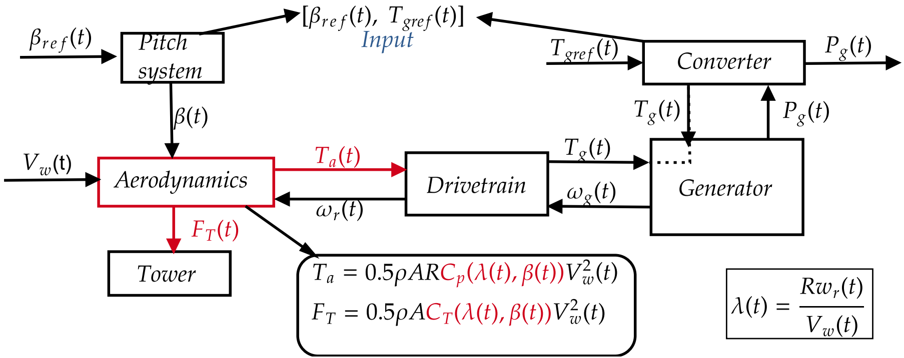
$$F_T = 0.5\rho A C_T(\lambda(t),\beta(t))V_w^2(t)$$

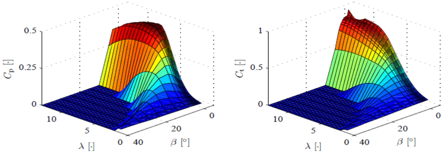
$$\lambda(t) = \frac{R\omega_r(t)}{V_w(t)}$$

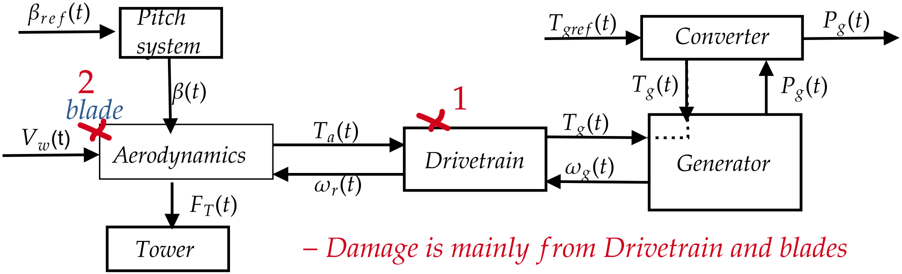
ADVANCED CONTROL SYSTEMS

Set-based Uncertainty Quantification for Prognostics

Application-Wind turbine dynamics





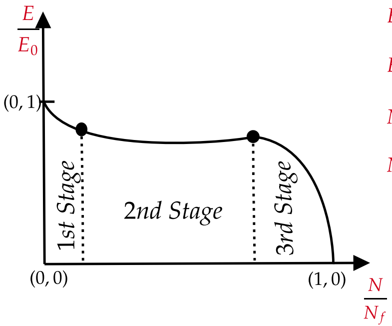


– *Damage is mainly from Drivetrain and blades*

–*Blade exposure to the elements*

– *Study to make WT tech. competitive*

–*Due to subsidy cuts*



$E -$ *stiffness at a specific point*

$E_0 -$ *Undamaged stiffness*

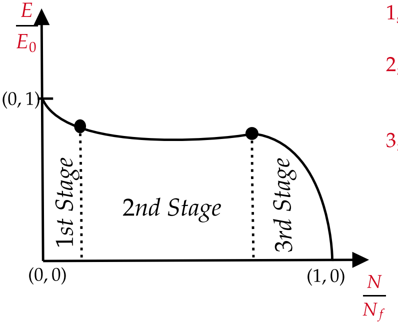
$N -$ *Total test cycles*

$N_f -$ *Fatigue life in cycles*

Figure: : Stages of stiffness degradation.

Set-based Uncertainty Quantification for Prognostics

Modelling wind turbine blade’s degradation



1_{st} – *Microscopic cracks* (2 – 5%)

2_{nd} – *Edge laminations and longitudinal cracks (gradual)*

3_{rd} – *Fast, abrupt steps*

ADVANCED CONTROL SYSTEMS

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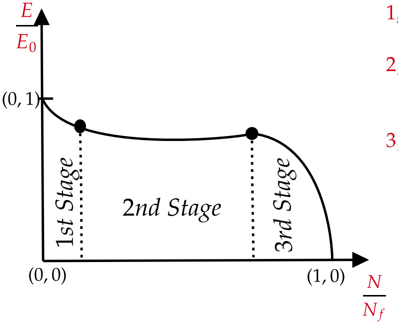

1_{st} – *Microscopic cracks* $(2-5\%)$

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$$\boxed{\frac{dD}{dN} = f_i(\phi, D) + f_p(\phi, D)}$$

ϕ = *stress magnitude*

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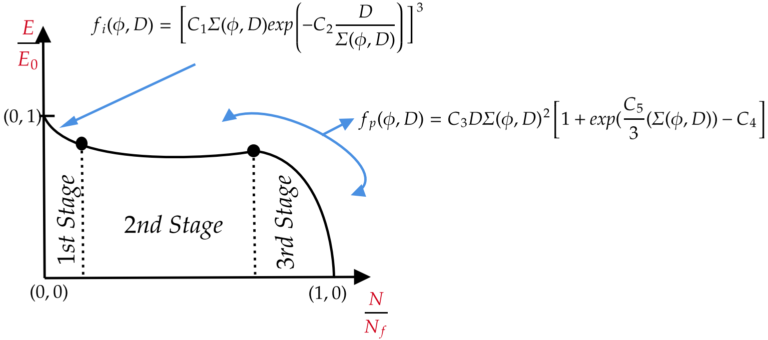

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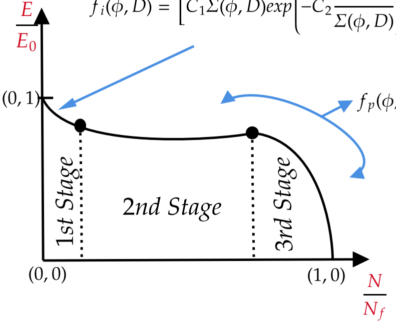
Set-based Uncertainty Quantification for Prognostics

Modelling wind turbine blade's degradation

$$f_i(\phi, D) = \left[C_1\Sigma(\phi, D)exp\left(-C_2\frac{D}{\Sigma(\phi, D)}\right)\right]^3$$



$$f_p(\phi, D) = C_3D\Sigma(\phi, D)^2\left[1 + exp(\frac{C_5}{3}(\Sigma(\phi, D)) - C_4\right]$$

$$\boxed{\Sigma(\phi, D) = \frac{\phi}{(1 - D)X_c}}$$

ϕ is the stress taken as the flap-wise blade root moment

Set-based Uncertainty Quantification for Prognostics

Modelling wind turbine blade's degradation

- Degradation independent and a factor-based models
- Blade root moment is approximated with plant variables

*Plant model**

$$\dot{w}_r = \frac{1}{J}(T_a - N_g T_g)$$

$$\dot{\beta} = \frac{1}{\tau_p}(-\beta + \beta_{ref})$$

$$\dot{T}_g = \frac{1}{\tau_g}(-T_g + T_{ref})$$

Degradation model

$$\phi(t) = a_1\beta(t) + a_2 w(t)$$

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ADVANCED CONTROL SYSTEMS

Odgaard, P. et al. "Observer based detection of sensor faults in wind turbines." In EWEC,4421–4430. Marseille, France

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ADVANCED CONTROL SYSTEMS

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n_c

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$$\frac{dD}{dN} = f_i(\phi, D) + f_p(\phi, D)$$

$$\theta(t) = \frac{T_s}{t_c} \times n_c$$

$$\dot{D} = \theta(t)(f_i(\phi, D) + f_p(\phi, D))$$

– Discontinous and nonlinear
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ADVANCED CONTROL SYSTEMS

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$\downarrow$

LPV model

Degradation model

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ADVANCED CONTROL SYSTEMS

Polytopic LPV model:

$$x(k+1) = A(\theta(k))x(k) + Bu(k)$$

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$$A(\theta(k)) = I + T_s \begin{bmatrix} k_1\theta_1(k) & 0 & -\frac{N_g}{J} & 0 \\ 0 & -\frac{1}{\tau_p} & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_g} & 0 \\ 0 & \theta_2(k) & 0 & \theta_3(k) \end{bmatrix} \qquad B = T_s \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{\tau_p} \\ \frac{1}{\tau_g} & 0 \\ 0 & 0 \end{bmatrix}.$$

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$A(\theta(k))$ selected based on observability check

Set-based Uncertainty Quantification for Prognostics

Estimation via ZKF

- Worst case encapsulation of uncertainties

ADVANCED CONTROL SYSTEMS

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- Additive uncertainties represented as symmetric interval sets.

ADVANCED CONTROL SYSTEMS

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$$x(k+1) = A(\theta(k))x(k) + Bu(k) + E_w w(k)$$
$$y(k) = Cx(k) + E_v v(k)$$

$$v(k) = [-\Delta v \ \Delta v] \qquad \Theta = [-\Delta\Theta \ \Delta\Theta] \qquad w(k) = [-\Delta w \Delta w]$$

ADVANCED CONTROL SYSTEMS

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$$v(k) = [-\Delta v \; \Delta v] \qquad \Theta = [-\Delta\Theta \; \Delta\Theta] \qquad w(k) = [-\Delta w \Delta w]$$

$$\hat{x}(k+1) = \sum_{i=1}^{2^{n_\theta}} (\mu_i(\theta(k)))\,(A_i(\theta(k))x(k) + Bu(k)) +$$
$$+ \mathbb{L}\,(\theta(k))\,(y(k) - \hat{y}(k))$$

ADVANCED CONTROL SYSTEMS

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$$\mathbb{L}\left(\theta\right) = \sum_{i=1}^{2^{n_\theta}} \mu_i\left(\theta\right)\mathbb{L}_i \qquad \sum_{i=1}^{2^{n_\theta}} \mu_i\left(\theta\right) = 1$$

ADVANCED CONTROL SYSTEMS

Assumption 1: The system matrices $A(\theta(k))$ and C are observable for any realization of $\theta(k)$.

ADVANCED CONTROL SYSTEMS

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- Solve the LMI minimization problem:

$$\begin{bmatrix} \gamma \mathbb{I}_n & \mathbb{I}_n \\ \mathbb{I}_n & \Upsilon \end{bmatrix} > 0, \qquad \begin{bmatrix} -\Upsilon & \Upsilon \mathbb{A}_i - \mathbb{W}^T C & \Upsilon \mathbb{H}^T & \mathbb{W}^T \\ \mathbb{A}_i^T \Upsilon - \mathbb{C}^T \mathbb{W} & -\Upsilon & 0 & 0 \\ \mathbb{H}\Upsilon & 0 & \mathbb{I}_{n_x} & 0 \\ \mathbb{W} & 0 & 0 & -\mathbb{R}^{-1} \end{bmatrix} < 0$$

$$\begin{bmatrix} -\Upsilon & \Upsilon \mathbb{A}_i - \mathbb{W}^T C & \Upsilon \mathbb{H}^T & \mathbb{W}^T \\ \mathbb{A}_i^T \Upsilon - \mathbb{C}^T \mathbb{W} & -\Upsilon & 0 & 0 \\ \mathbb{H}\Upsilon & 0 & \mathbb{I}_{n_x} & 0 \\ \mathbb{W} & 0 & 0 & -\mathbb{R}^{-1} \end{bmatrix} < 0$$

ADVANCED CONTROL SYSTEMS

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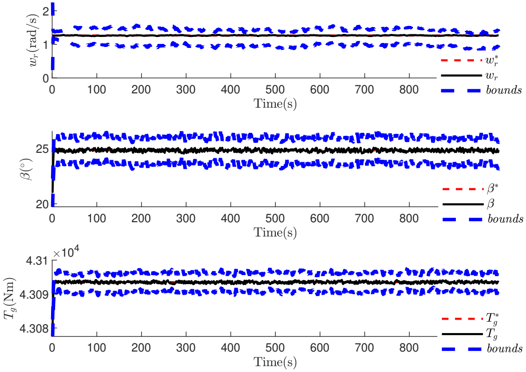

Figure: ZKF Estimation of states with bounds.

$$c_x(k+1) = c_p(k) + \mathbb{L}\left(y(k) - Cc_p(k)\right) \qquad R_x(k+1) = \left[\left(\mathbb{I} - \mathbb{L}C\right) R_p(k), -\mathbb{L}E_v\right]$$

Estimation via ZKF

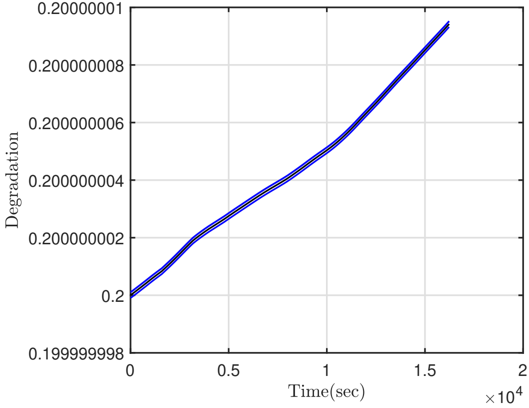

Figure: Estimated Monotonous degradation.

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Propagate estimated Zonotopes:

$\hat{\mathcal{X}}_{k_p} \subset \langle c_{x_{k_p}}, \; R_{x_{k_p}} \rangle$

(Inputs sourced through random sampling from an assumed known distribution of inputs.)

Propagation Positive invariant sets:

$[\zeta(k_p), \zeta(k_p+1) \ldots . \zeta(k_{EOL})]$

$$\zeta(k_{EOL}) \subseteq \bigcup_{k=k_p}^{k_{EOL}} \mathcal{R}_{k\tau}\left(\mathcal{R}_{[0,\ \tau]}(\hat{\mathcal{X}}_{k_p})\right)$$

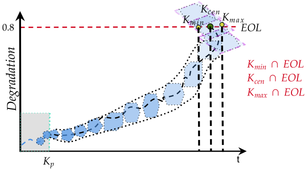

Figure: Set Propagation.

$$\Delta\mathbb{X}(k_p + j) \subseteq \bigoplus_{j=1}^{k_{EOL}} A(\theta(j))^j \Delta x(k_p) \oplus A(\theta(j))^{(j-1)}$$
$$B\Delta u(k_p + j - 1) \oplus E_w \Delta w(k_p + j - 1))$$

Reduction operator: $\langle c_k, \bar{H}_k \rangle \supseteq \langle c_k, H_k \rangle$

Set-based Uncertainty Quantification for Prognostics.

Results

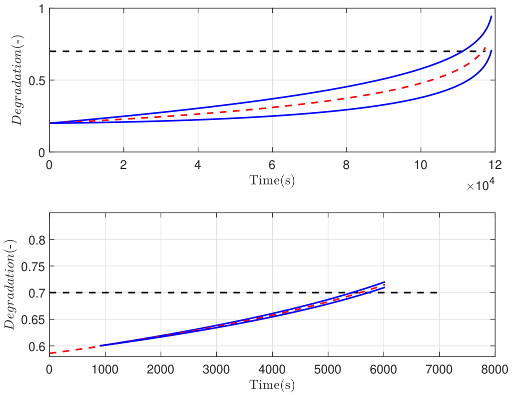

Figure: Propagation of degradation uncertainty set to EOL.

ADVANCED CONTROL SYSTEMS

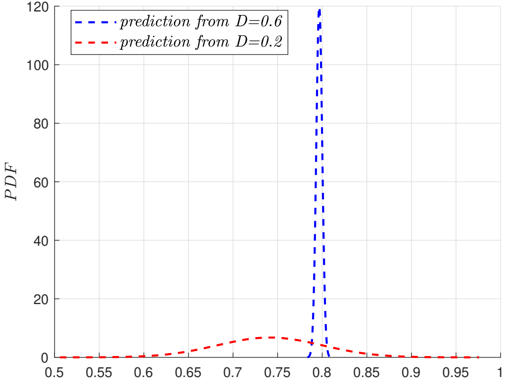

Figure: PDFs of degradation at the EOL.

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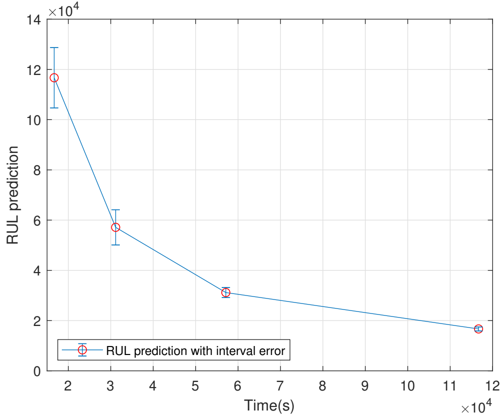

Figure: Remaining useful life Predictions.

ADVANCED CONTROL SYSTEMS

Outline

1 Introduction

2 Set-based Uncertainty Quantification For Prognostics.

3 Health-Aware Control Of a Wind Turbine

4 Data-Based Prognostics

5 Robust Reliability-Aware Control of a DWN

6 Concluding Remarks

Health Aware Control of a Wind Turbine

Motivation for its study:

- Exposed to harsh conditions.
- Damage before the expected economic lifetime
- Design a controller that manages component health
- Combine a degradation-independent model with a factor-based model
- The Designed controller must:
  - Computationally inexpensive
  - Account for discontinuity in degradation (Rainflow Counting)
  - Practically viable

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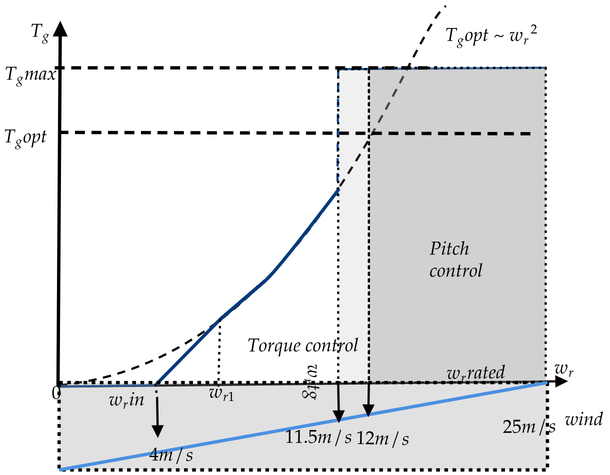
Wind turbine control.



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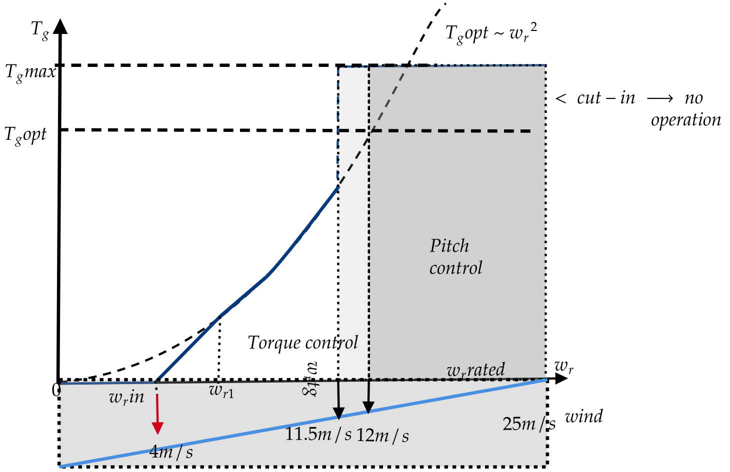

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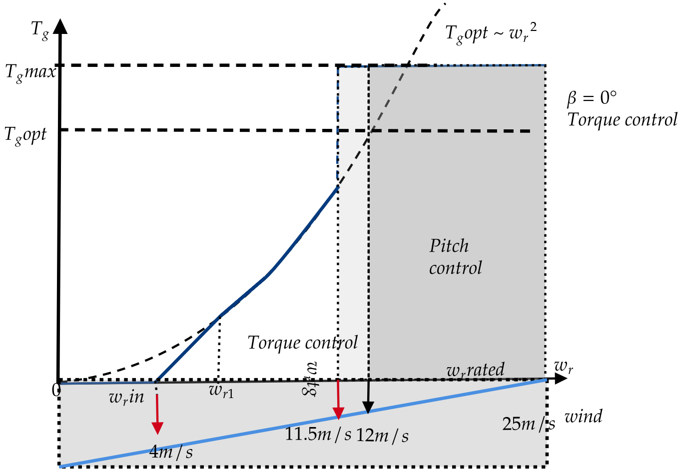

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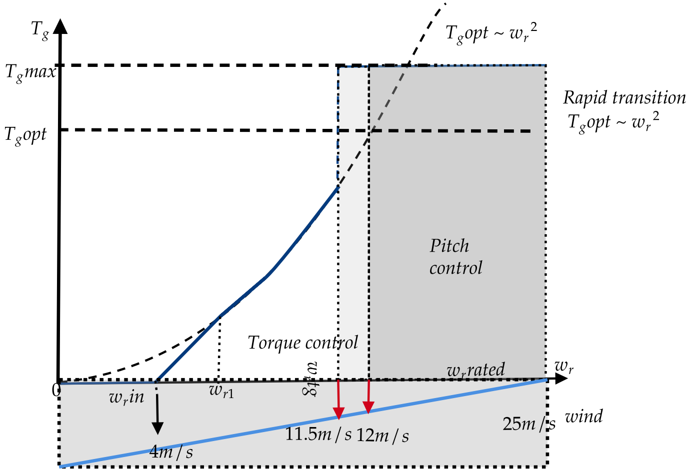

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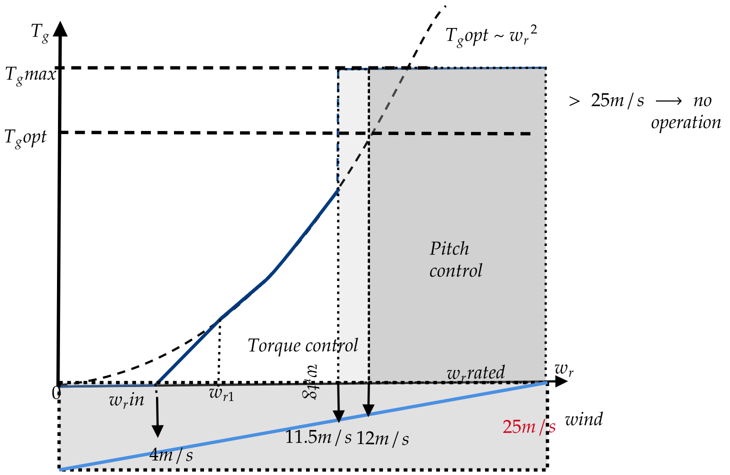

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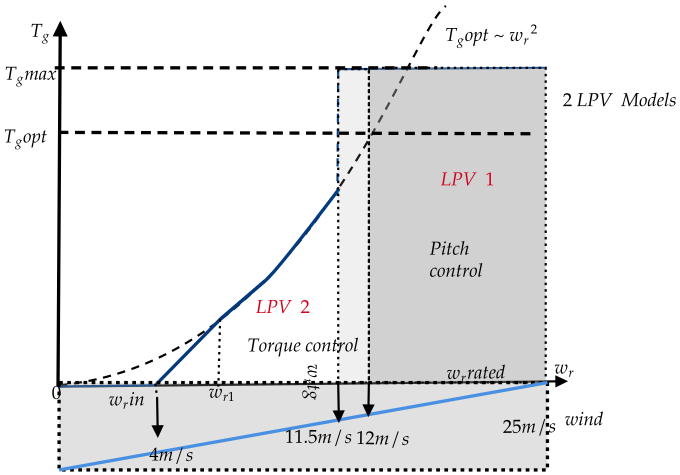

ADVANCED CONTROL SYSTEMS

Considering:

$$x(k+1) = Ax(k) + Bu(k) + B_w(\theta(k))V_w(k),$$

where $V_w(k) \in R^d$ is the disturbance from the wind.

ADVANCED CONTROL SYSTEMS

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LPV nonlinear parameter embedding:

$$B_w(\theta(k)) = \begin{bmatrix} 0_{4\times 1} \\ \frac{1}{Nm_B}\vartheta\theta_1(k) \\ \frac{1}{J_r}\vartheta R\theta_2(k) \\ 0_{3\times 1} \end{bmatrix}.$$

Varying parameters are $\theta_1(k) = C_t(\lambda(k), \beta(k))V_w(k)$ and $\theta_2(k) = C_p(\lambda(k), \beta(k))V_w(k)$.

* F. A. Inthamoussou et al. "LPV Wind Turbine Control With Anti-Windup Features Covering the Complete Wind Speed Range". In: *IEEE Transactions on Energy Conversion* 29.1 (2014), pp. 259–266

ADVANCED CONTROL SYSTEMS

Health Aware Control of a Wind Turbine

LPV Model 2.

Considering:

$$A = \begin{bmatrix} \frac{1}{J_r+J_g}(T_a(x_u, V_w) - T_g) & 0 \\ 0 & -\frac{1}{\tau_\beta} \end{bmatrix},$$

$$B_u = \begin{bmatrix} 0 \\ \frac{1}{\tau_\beta} \end{bmatrix}.$$

The matrix $A(\theta(k))$ is therefore:

$$A(\theta(k)) = \begin{bmatrix} \frac{1}{J_r+J_g}(\vartheta R\theta_2(k) - \frac{T_g}{\theta_3(k)}) & 0 \\ 0 & -\frac{1}{\tau_\beta} \end{bmatrix}.$$

$\theta_3(k) = w_r(k)$; $w_r(k) \neq 0$, where $x = [w_r,\ \beta]$.

* S. Georg. “Fault Diagnosis and Fault-Tolerant Control of ind Turbines”. PhD thesis. Universit$\ddot{a}t Rostock$, 2015

ADVANCED CONTROL SYSTEMS

Health Aware Control of a Wind Turbine

LPV Modelling

- Varying parameter, $\theta_i \in \left[\overline{\theta_i}, \underline{\theta_i}\right]$,
- candidate models are a linear combination of $n^v = 2^{n_0}$ vertices θ_i of a polytope.

For LPV 1:

$$B_w(\theta(k)) = \sum_{i=1}^{n_v} \alpha_i(k) B_w(\theta_i),$$

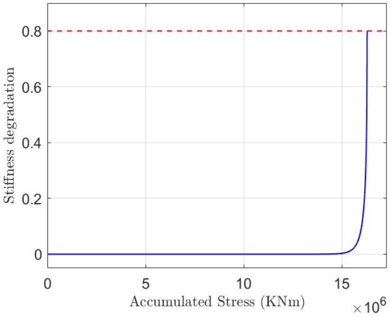
and LPV 2:

$$A(\theta(k)) = \sum_{i=1}^{n_v} \alpha_i(k) A(\theta_i),$$

where in both cases

$$\sum_{i=1}^{n_v} \alpha_i = 1, \, \alpha_i = [0, 1].$$

ADVANCED CONTROL SYSTEMS

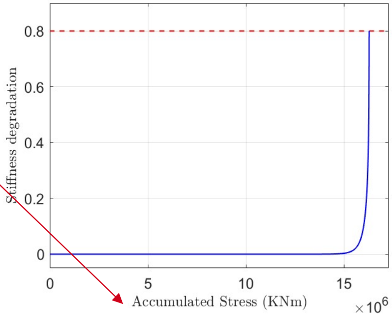


Stiffness degradation
0.8
0.6
0.4
0.2
0
0
5
10
15
Accumulated Stress (KNm)
$\times 10^6$

Health Aware Control of a Wind Turbine

Offline prognostics of a wind turbine blade

- *Stiffness degradation*
- *Flapwise root moment from OpenFAST*



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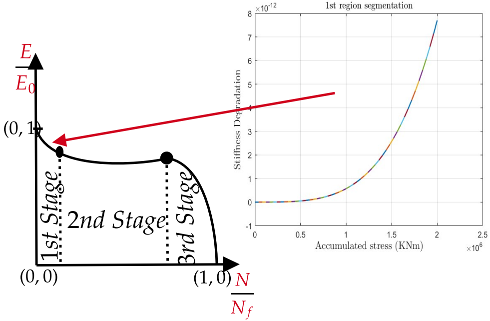
Health Aware Control of a Wind Turbine

Offline prognostics of a wind turbine blade

- *Stiffness degradation*
- *Flapwise root moment from OpenFAST*
- *Segment* 1_{st} *and* 3_{rd} *regions*



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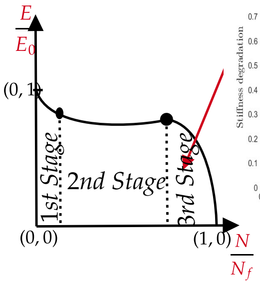

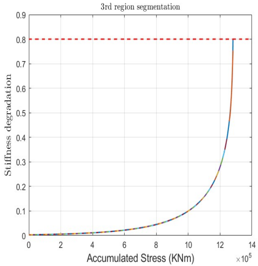

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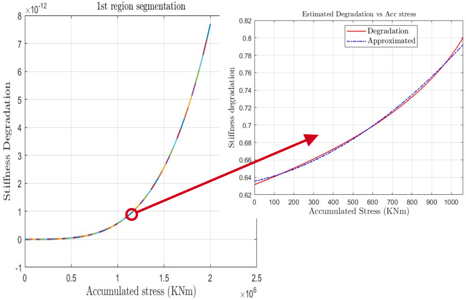
Health Aware Control of a Wind Turbine

Offline prognostics of a wind turbine blade

- *Stiffness degradation*
- *Flapwise root moment from OpenFAST*
- *Segment 1_{st} and 3_{rd} regions*
- *Polynomial fit*

$$\varphi \approx b_{2_j}\phi_{acc}(k)^2 + b_{1_j}\phi_{acc}(k) + b_{0j}$$



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Health Aware Control of a Wind Turbine

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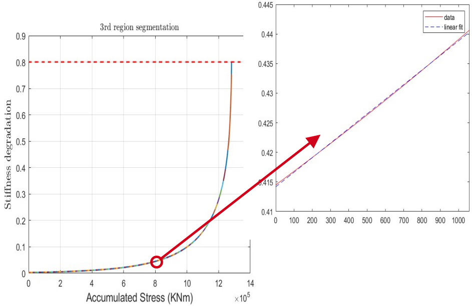

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Health Aware Control of a Wind Turbine

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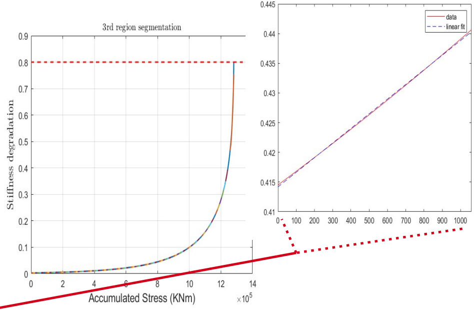
$$\varphi \approx b_{2_j}\phi_{acc}(k)^2 + b_{1_j}\phi_{acc}(k) + b_{0j}$$

- $Deg(k) = \{\, \varphi(k)_j \text{ for } \phi_{acc(min)} \le \phi_{acc} \le \phi_{acc(max)}$



$$\phi = a_1\beta(t) + a_2V_w(t) \longrightarrow \text{Factor-based} \,\|\, b_i, W_j\text{(weights for each segment)}$$

ADVANCED CONTROL SYSTEMS

- Estimate effective load cycles in stress
- Sequential and nonlinear (not suitable for control)
- Externalize (PORC)
- Stress magnitude and duration allocated in prediction horizon.
- Assuming a pre-processed data of stress (moving window).

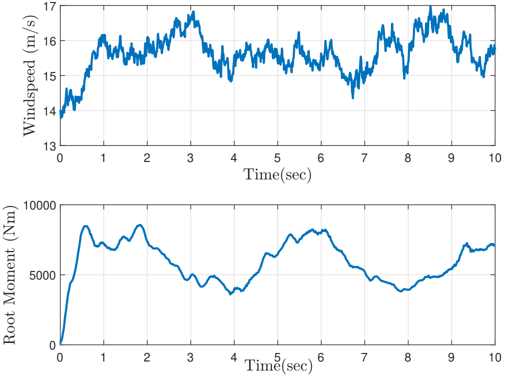


Figure: Input wind and respective Blade root moment

* S. Löw, D. Obradovic, and C. Bottasso. "Model predictive control of wind turbine fatigue via online rainflow-counting on stress history and prediction". In: *Journal of Physics Confernce Series* 118 (Sept. 2020)

ADVANCED CONTROL SYSTEMS

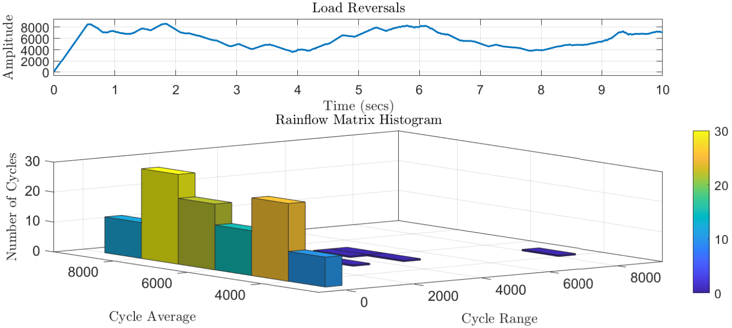
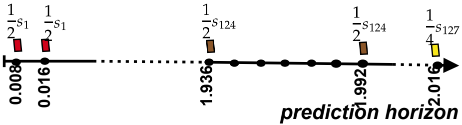


Figure: Information from rainflow counting.

Health Aware Control of a Wind Turbine

Rainflow counting in control

cycle #	Count	Range	Mean	Start	End
1	1	7	8534.5	0.008	0.016
2	1	2	7028	0.024	0.032
3	1	29	6880.5	0.04	0.048
4	1	1	6467.5	0.056	0.064
5	1	1	6455.5	0.072	0.08
⋮	⋮	⋮	⋮	⋮	
124	1	307	7583.5	1.936	1.992
125	1	1698	7622	1.68	1.896
126	0.5	8467.2	4328.4	0	1.64
127	0.5	8467.2	4328.4	1.64	2.016



ADVANCED CONTROL SYSTEMS

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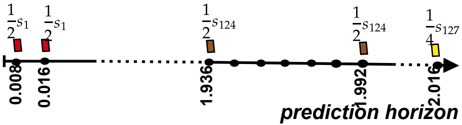

ADVANCED CONTROL SYSTEMS

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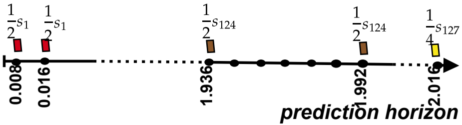

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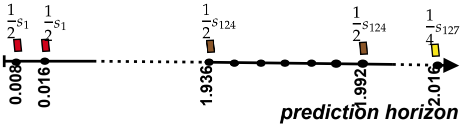
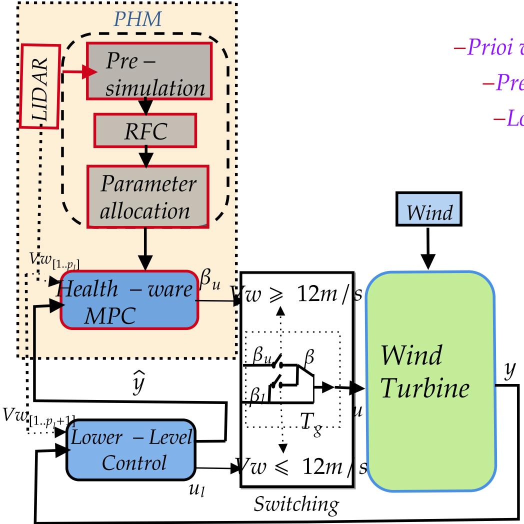


Health Aware Control of Wind Turbine

Control setup



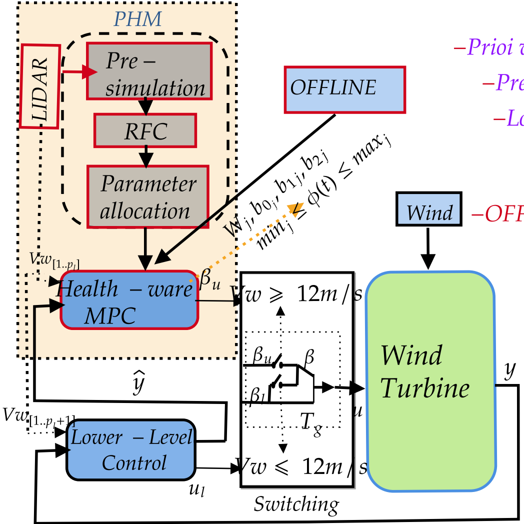
–Prioi window Wind information Known

–Presimulation on OpenFAST for stress

–Load reversal counted and allocated

ADVANCED CONTROL SYSTEMS

ADVANCED CONTROL SYSTEMS



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Health Aware Control of a Wind Turbine

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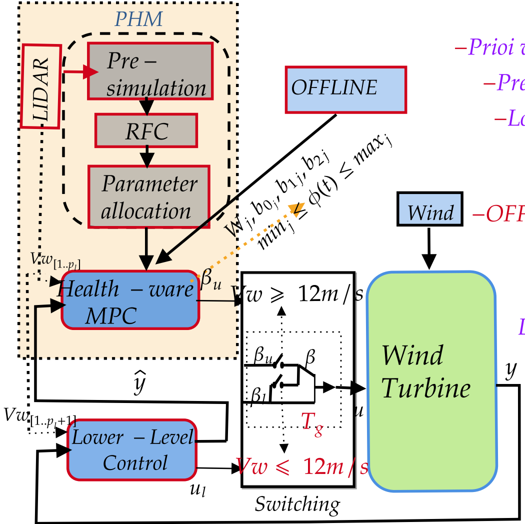


–*Prioi window Wind information Known*

–*Presimulation on OpenFAST for stress*

–*Load reversal counted and allocated*

–*OFFLINE pronostics for local degredation fxn & weights*

Lower – Level MPC – Control below cut – in

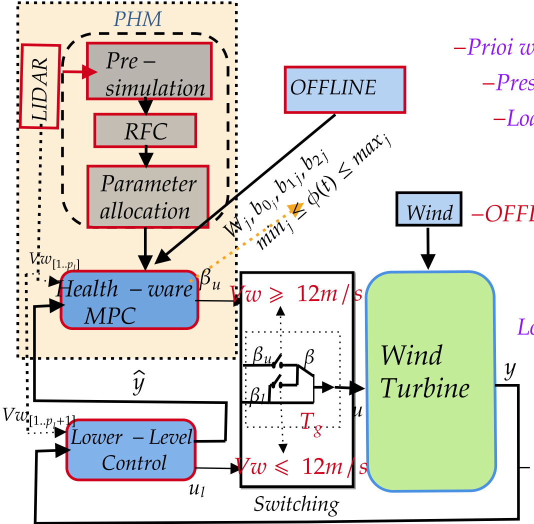
– *Torque control*

– *Bilinearity*

ADVANCED CONTROL SYSTEMS

Health Aware Control of a Wind Turbine

Control setup



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Lower – Level MPC – Control below cut – in

– Torque control

– Bilinearity

Upper – Level MPC – HM control

Health Aware Control of a Wind Turbine

Control setup-upper Level

$$\mathcal{L}_u(k, u(k), x(k), \Psi(k)) = W_j Deg(k) - W_1 P(k) + W_2 \Delta u(k)$$

ADVANCED CONTROL SYSTEMS

Health Aware Control of a Wind Turbine

Control setup-upper Level

$$\mathcal{L}_u(k, u(k), x(k), \Psi(k)) = W_j Deg(k) - W_1 P(k) + W_2 \Delta u(k)$$

$$\min_{\beta_h(1\ldots N_p), \mathbf{x_u}(1\ldots N_p), Deg(1\ldots N_p)} \sum_{i=0}^{N_p-1} \mathcal{L}_u(k, u(k), x_u(k), \Psi(k))$$

subject to

$$x_u(i+1|k) = A(\theta(k)) x_u(i|k) + B_u \beta_h(i|k)$$

$$Deg(i+1|k) = Deg(i|k) + (R_f(i).f(\phi(k), \Lambda(k))).T_u,$$

$$P(i+1|k) = n_g T_{g_{opt}} w_g(i|k),$$

$$P(i|k) \leq P_{max},$$

$$\beta_{min} \leq \beta_h(i|k) \leq \beta_{max},$$

$$[w_{r_{min}}, \beta_{min}]^\top \leq x_u(i+1|k) \leq [w_{r_{max}}, \beta_{max}]^\top,$$

$$\Delta\beta_h(i+1|k) = \beta_h(i+1|k) - \beta_h(i|k).$$

Health Aware Control of a Wind Turbine

Control setup-Lower Level

$$\mathcal{L}_l(k, u_l(k), x(k)) = \varphi_1(w_r(k) - w_r^*)^2 + \varphi_2(T_g(k) - T_g^*)^2 ..$$

$$... + \varphi_3(\beta(k) - \beta^*)^2 + \varphi_4 \Delta u_l$$

ADVANCED CONTROL SYSTEMS

Control setup-Lower Level

$$\mathcal{L}_l(k, u_l(k), x(k)) = \varphi_1(w_r(k) - w_r^*)^2 + \varphi_2(T_g(k) - T_g^*)^2..$$

$$... + \varphi_3(\beta(k) - \beta^*)^2 + \varphi_4 \Delta u_l$$

$$\min_{\mathbf{u}_l(1 \ldots N_p), \hat{\mathbf{y}}(1 \ldots N_p)} \sum_{i=0}^{N_p-1} \mathcal{L}_l(k, u_l(k), x(k))$$

subject to

$$x(i+1|k) = Ax(i|k) + Bu_l(i|k) + B_w(\theta_{pi})V_w(i|k),$$

$$u_l(i|k) \subseteq \mathcal{U},$$

$$x(i+1|k) \subseteq \mathbb{X},$$

$$\Delta u_l(i+1|k) = u_l(i+1|k) - u_l(i|k).$$

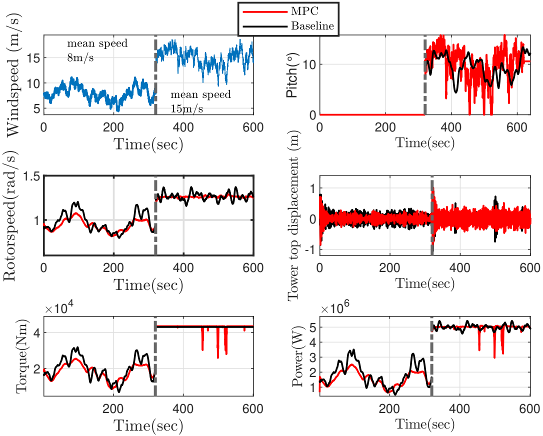

Figure: The lower level MPC in the two regions of control.

Health Aware Control of a Wind Turbine

Results

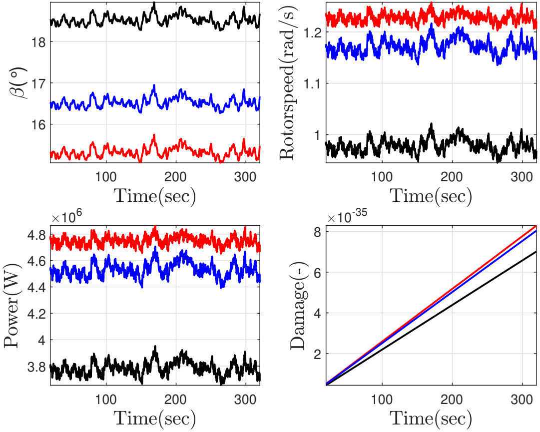

Figure: Operation of the upper-level control.

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W_j	Power(MW)	Acc. Stress
$W_{j,1}$	5	4.11×10^{10}
$W_{j,2}$	4.54	3.74×10^{10}
$W_{j,3}$	4.164	3.43×10^{10}
$W_{j,4}$	3.97	3.26×10^{10}

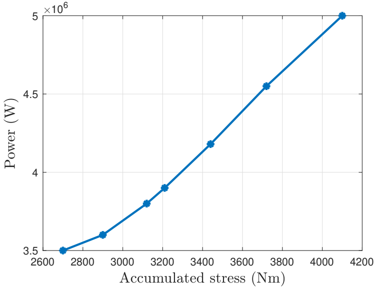

Figure: Pareto front Power vs Accumulated stress.

Outline

ADVANCED CONTROL SYSTEMS

Data-Based Prognostics

Motivation of study:

- Most data-based are black-box models (lack interpretability)

ADVANCED CONTROL SYSTEMS

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ADVANCED CONTROL SYSTEMS

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- ***1.** Develop an interpretable model of degradation
  - EEFIG model is employed
  - Degradation of a power semiconductor (IGBT)

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- ***1.** Develop an interpretable model of degradation
  - EEFIG model is employed
  - Degradation of a power semiconductor (IGBT)
- ***2.** Contribute to set-based uncertainty quantification in data-based prognostics
  - Interval uncertainty set description via interval predictor estimation

* B. Khoury et al. "Data-driven Prognostics based on Evolving Fuzzy Degradation Models for Power Semiconductor Devices". In: *Proceedings of the 7th European Conference of the Prognostics and Health Management Society 2022.* PHM Society, 241 Woodland Drive, State College, PA 16803., June 2022

ADVANCED CONTROL SYSTEMS

Data-Based Prognostics

Evolving Ellipsoidal Fuzzy Information Granules Modelling.

Introduction

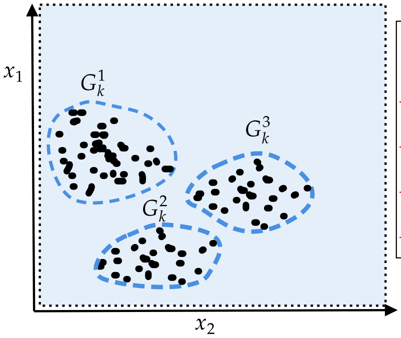
Set-based Uncertainty Quantification For Prognostics.

Health-Aware Control Of a Wind Turbine

Data-Based Prognostics

Robust Reliability-Aware Control of a DWN

Concluding Remarks



– *Granulatation in hyperelipsoids – E*

– *Granule has membership degree – G*

– *Acquire granule models*

– *Weighted average sum*

– *Takagi – sugeno Fuzzy inference – F*

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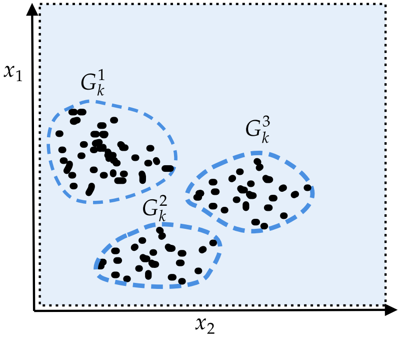
Data-Based Prognostics

Evolving Ellipsoidal Fuzzy Information Granules Modelling



Granule prototype.

$$\mathcal{P}_k^i = \left(\underline{\mu}_k^i, \mu_k^i, \overline{\mu}_k^i, \Sigma_k^i\right)$$

Data-Based Prognostics

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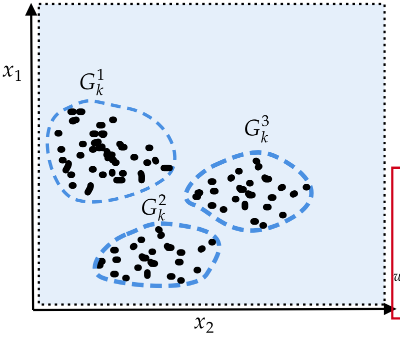

Granule prototype.

$$\mathcal{P}_k^i = \left(\underline{\mu}_k^i, \mu_k^i, \overline{\mu}_k^i, \Sigma_k^i\right)$$

Membership func.

$$G_k^i = \left(\mathbb{R}^{n_z}, g_k^i\right);\ g_k^i : \mathbb{R}^{n_z} \to [0,1]$$

$$g_k^i(z_k) = \frac{w_k^i(z_k)}{\sum_{i=1}^{N} w_k^i(z_k)}$$

$$w_k^i(z_k) = exp\left\{-\left[\left(z_k - \mu_k^i\right)^T \left(\Delta_k^i\right)^- \left(z_k - \mu_k^i\right)\right]^{\frac{1}{2}}\right\}$$

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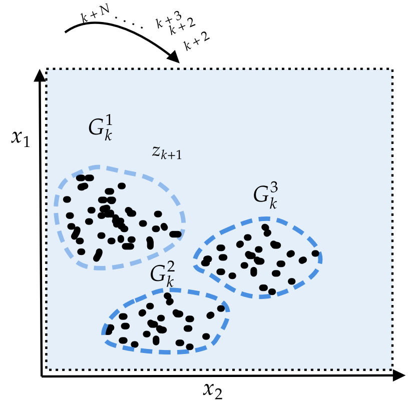

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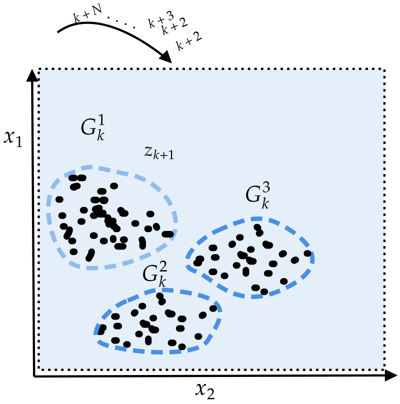

Admissability test

square mehalanobis distance

$$d\left(z_k, \mu_k^i\right) < \nu$$

EFFIG performance index

$$\mathbb{Q}_k^i > \mathbb{Q}_{k-}^i$$

ADVANCED CONTROL SYSTEMS

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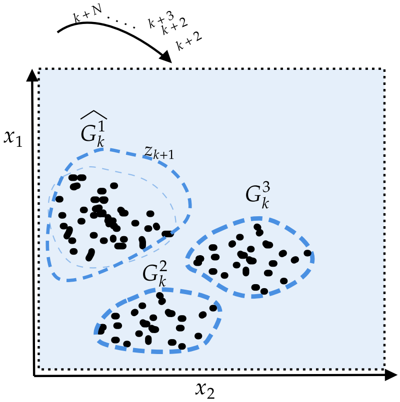

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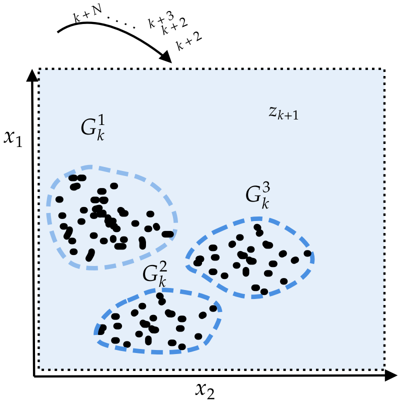

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Data-Based Prognostics

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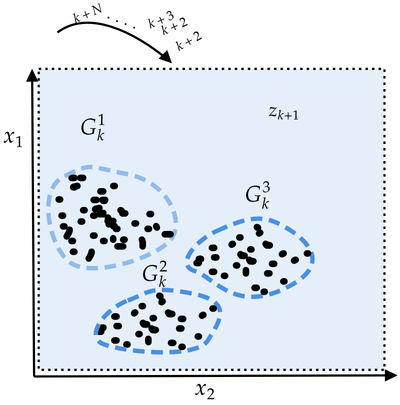

Admissability test ✕

square mehalanobis distance

$$d\left(z_k, \mu_k^i\right) < \nu$$

EFFIG performance index

$$\mathbb{Q}_k^i > \mathbb{Q}_{k-}^i$$

* *Anomaly* *

ADVANCED CONTROL SYSTEMS

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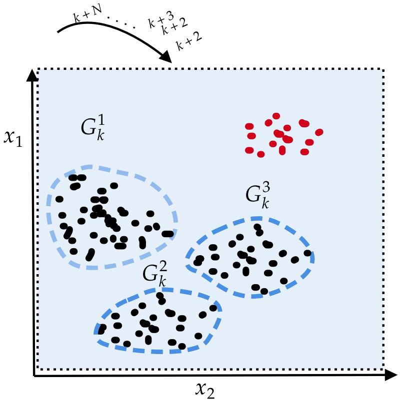
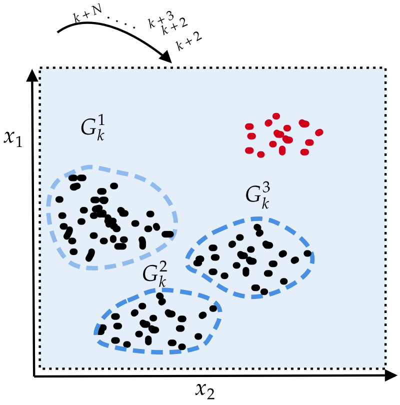


Creation of new granule

- *Concurrently a tracker,* $\mathcal{T}_k$
- $\mathcal{T}_k\left(\mu_k^{\mathcal{T}}, \Sigma_k^{\mathcal{T}}\right)$



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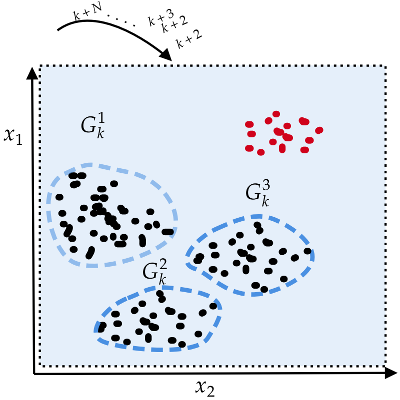

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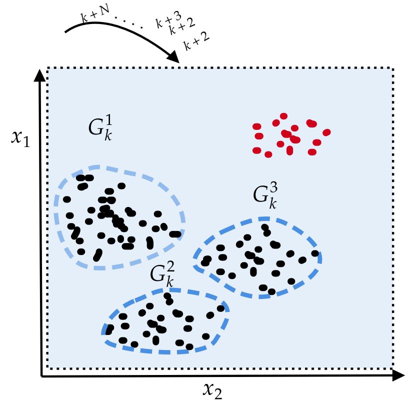

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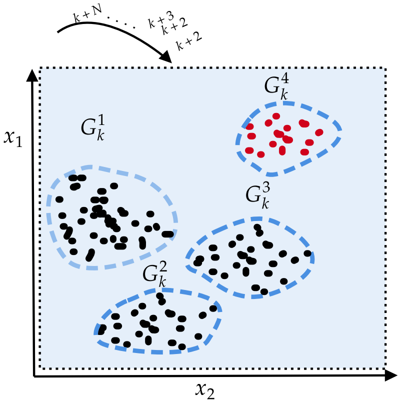

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$$G_k^4 = \left(\mathbb{R}^{n_z}, g_k^4\right);\ g_k^4 : \mathbb{R}^{n_z} \to [0, 1]$$

ADVANCED CONTROL SYSTEMS

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ADVANCED CONTROL SYSTEMS

Data-Based Prognostics

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Autoregressive consequent

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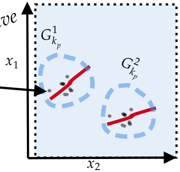

ADVANCED CONTROL SYSTEMS

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Health index

$$y_{k_p} = \sum_{i=1}^{n_c} g^i_{k_p} \, \Theta^T_{k_p} [y_{k_{p-1}} \; y_{k_{p-2}} \; \ldots\ldots \; y_{k_p \, -l}]^T$$

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one – step ahead

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ADVANCED CONTROL SYSTEMS

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Data-Based Prognostics

Application-IGBT and its degradation

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- IGBT combines:
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- NASA Ames research center's IGBT degradation dataset

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Data-Based Prognostics

IGBT Data set

- Run to failure test on 4 IGBTs
- Subjected to aggressive electrical cycles
  - DC square voltage $[0v\ 4v]$
  - Under control temp. $[320^\circ c\ 330^\circ c]$
- Run until failure (latch-up or thermal runway)
- V_{ce}, V_G, I_c
- $V_{ce} \rightarrow$ Failure precursor.

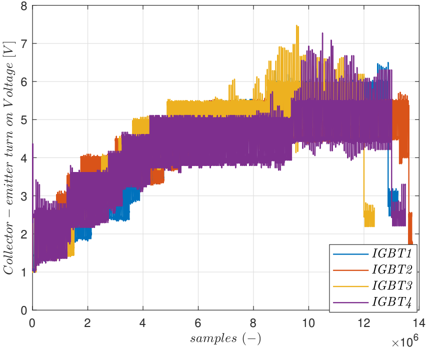

Figure: V_{ce} from run to failure test.

ADVANCED CONTROL SYSTEMS

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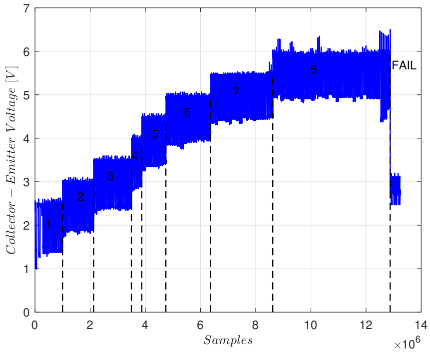

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ADVANCED CONTROL SYSTEMS

- Pseudo-representative features are extracted

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Data-Based Prognostics

Feature extraction

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  - Antecedent $\rightarrow$ Maintains data granularity (original information)
  - Consequent $\rightarrow$ Best for prognostics
- Denoising by a moving average

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- **Consequent:**

$$\text{Suitability} = \begin{bmatrix} \text{Monotonicity} \\ \text{Trendability} \\ \text{Prognosability} \end{bmatrix}^{\top} \begin{bmatrix} 1 \\ 0.976 \\ 1 \end{bmatrix}$$

cumulative SD of trig function of the dataset used

Feature	Formula
SD of asinh(X)	$\sigma\left(\log\left[x_i + (x_i^2+1)^{\frac{1}{2}}\right]\right.$
SD of atan(X)	$\sigma\left(\frac{i}{2}\log\left(\frac{i+x_i}{i-x_i}\right)\right)$

- **Antecedent** $\rightarrow$ Energy and RMS

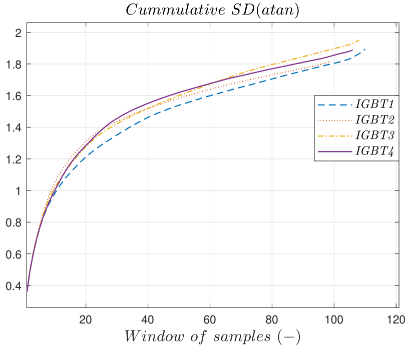

Figure: C-SD(atan).

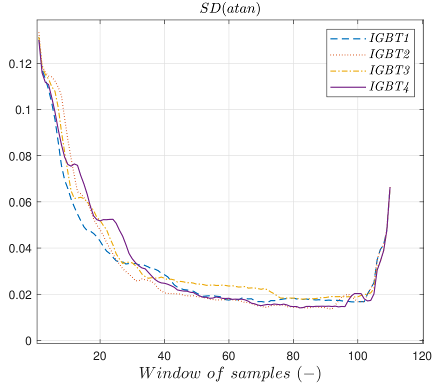

Figure: SD atan.

C-SD(atan) selected due to superior suitability index

Data-Based Prognostics

Premise feature

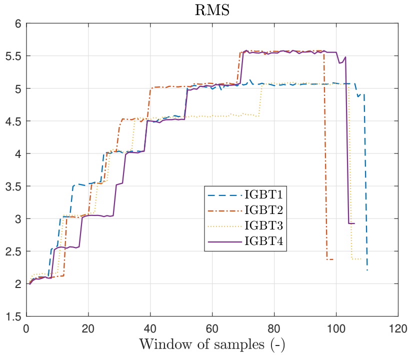

Figure: RMS of the data set.

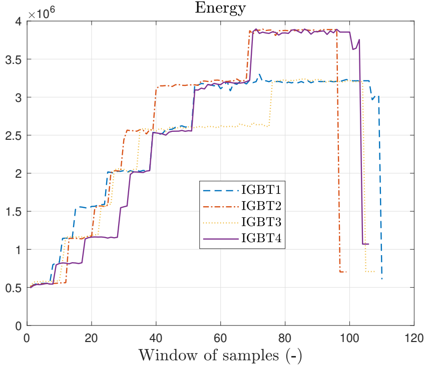

Figure: Energy feature from the data set.

RMS selected due to similarity and also based on tests

- Grid search for optimal hyper-parameters
- $l = (L, \tau, \eta, \varphi, \zeta)$ is the vector of hyper-parameters, $\mathcal{L} = [2, 5] \times [2, 5] \times [0.96, 1] \times [2, 6] \times [2, 6]$

$$\ell(D) = \underset{l}{\text{argmin}} \sum_{k=1}^{\text{EOL}_D} k\text{ra}_k(D, l) \quad \text{s.t.} \quad l \in \mathcal{L}$$

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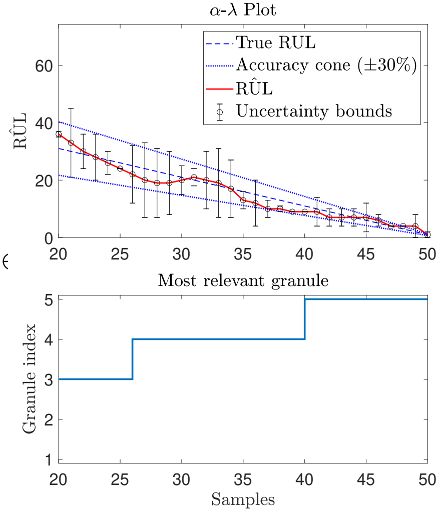
prediction for the 2nd IGBT with parameters obtained for the test dataset with data from the 4th IGBT.



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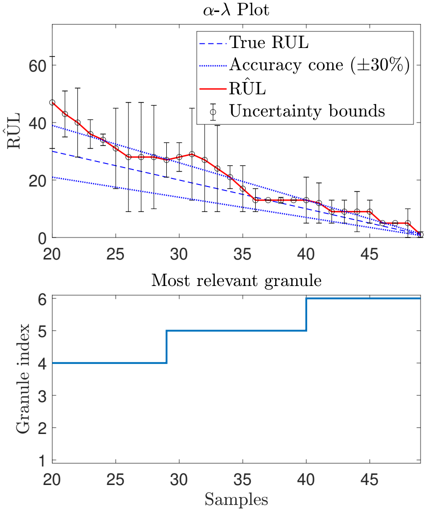
RUL prediction for the 3rd IGBT with parameters obtained for the test dataset with data from the 2nd IGBT.



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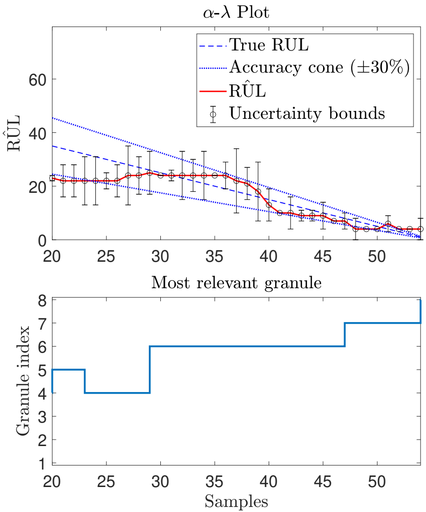
RUL prediction for the 2nd IGBT with parameters obtained for the test dataset with data from the 1st IGBT.



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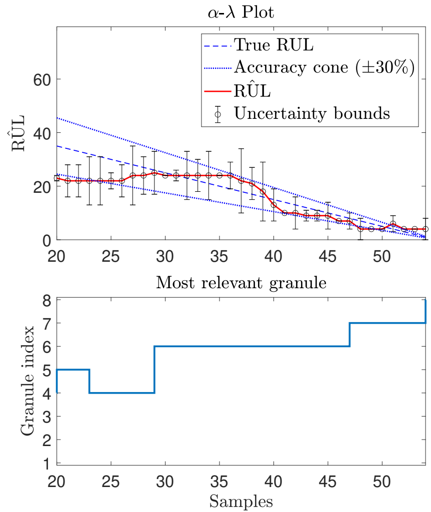

Data-Based Prognostics

Experiment and results

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Desirable results from data sets 1,2 and 4, however 3 posed challenges

Table:Mape$_{20}$ results

Tuning dataset (columns) / *UUT dataset* (rows)

	1	2	3	4
1	20.35	31.59	59.65	48.32
2	23.03	15.19	34.40	15.29
3	67.41	76.05	70.84	74.9
4	40.78	31.46	37.66	28.23

$$MAPE_k = \frac{100}{EOl - k + 1} \sum_{i=k+1}^{EOL} || \frac{RUL_i - \widehat{RUL_i}}{RUL}$$

ADVANCED CONTROL SYSTEMS

Data-Based Prognostics

Set based uncertainty quantification

- ***Based on*** $i \in \mathbb{N}_{\leq n_c}$ ***granules in granulation at prediction*** k_p:
- ***Uncertainty*** $\Delta y = [\underline{y}\ \ \overline{y}];\ \hat{y} = y^c + \Delta y$

Health index

$$y_{k_p} = \sum_{i=1}^{n_c} g^i_{k_p} \Theta^T_{k_p} [y_{k_{p-1}}\ y_{k_{p-2}}\ \ldots\ldots\ y_{k_{p-l}}]^T + w$$

$$w \in W;\ w \leq \sigma$$

$$[y_{k_{p-1}}\ y_{k_{p-2}}\ \ldots\ldots\ y_{k_{p-l}}]^T = \phi$$

ADVANCED CONTROL SYSTEMS

Data-Based Prognostics

Set based uncertainty quantification

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$$\min f(\lambda)$$

$$s.t\ \ \lambda \geq \frac{|y_{k_p} - \widehat{y}_{k_p}| - \sigma}{\phi \Delta y_0}$$

$$\lambda\phi\ \Delta y_0 \geq |\widehat{y_{k_p}} - \widehat{y}_{k_p}| - \sigma$$

$$f(\lambda) = \sum_{k=1}^{N} width\ \Delta y_0\ = 2\lambda \sum_{k=1}^{N} \phi\ \Delta y_0$$

ADVANCED CONTROL SYSTEMS

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One step ahead

$$\Delta y_{k_{p+1}|k_p} = \sum_{i=1}^{n_c} g_{k_p}^i \lambda \Theta_{k_p}^T [y_{k_{p-1}}\ y_{k_{p-2}}\ \ldots\ y_{k_{p-l}}]^T + \Delta\epsilon_k$$

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Multistep ahead

$$\Delta y_{k_{p+N}|k_p}(\Delta y(1, \ldots\ldots k_p - 1)) \to {}^*$$

Note autoregressive

Predict ΔRUL

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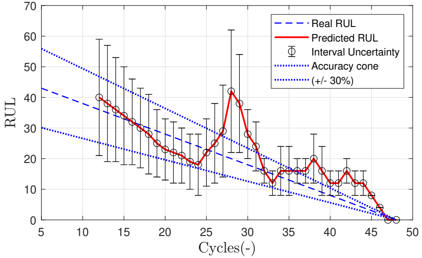
Data-Based Prognostics

Set based uncertainty quantification-Results



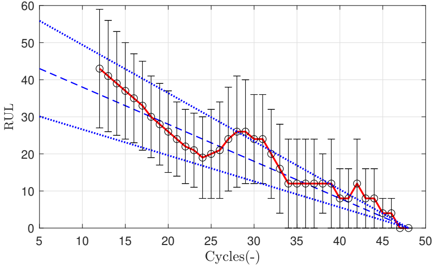


Figure: : RUL prediction and uncertainty set description for the IGBT2 with parameters obtained for the test dataset with data from the IGBT2.

Figure: :RUL uncertainty set description for IGBT1 with parameters obtained for the test dataset with data from the IGBT2.

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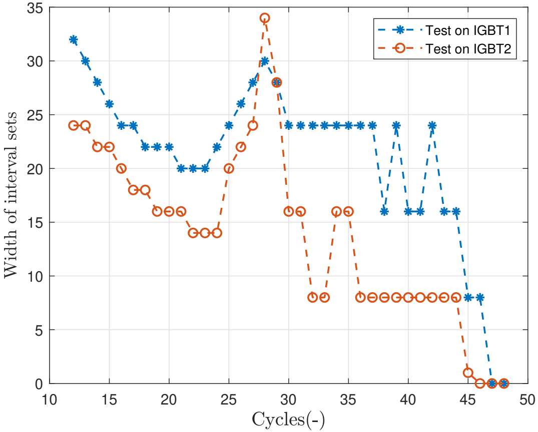


Figure: Width of uncertainty set at each RUL prediction cycle.

Outline

ADVANCED CONTROL SYSTEMS

Robust Reliability-Aware Control of a DWN

Motivation of this study:

- Employ EMPC for DWN ensuring reliability.
- Important for critical infrastructure
  - Involves uncertainty in demand (human behaviour)
  - RobustEMPC avoids intractability and ensures stability
  - Uncertainty described as zonotopic sets
- Reliability of an interconnected system
  - Reliability model $\rightarrow$ Factor-based model
  - WDN model$\rightarrow$Degradation independent model
  - Account for uncertainty (*practical)

Robust Reliability-Aware Control of a DWN

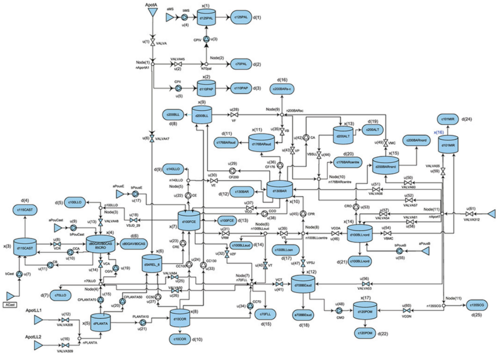
Application-Drinking water network

$$x(k+1) = Ax_k + B_u u_k + B_d d_k,$$
$$0 = E_u u_k + E_d d_k.$$

- u_k is in linear variety
  - Affine parametrization
  - $u_k = \tilde{P}\tilde{M}_1 \hat{u}_k + \tilde{P}\tilde{M}_2 d_k$. (Gauss-Jordan elimination)
  - Reduce decision variables and aid in set construction

$$x(k+1) = Ax(k) + \hat{B}\hat{u}(k) + \hat{B}_d d(k).$$

$$\hat{B} = B\tilde{P}\tilde{M}_1 \qquad \hat{B}_d = B\tilde{P}\tilde{M}_2 + B_d.$$



ADVANCED CONTROL SYSTEMS

Ensure: For all $\Sigma = f(k, x_k, u_k, d_k)$

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- Recursive feasibility
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Assuming $\Delta d_k \subseteq \mathbb{D}_k$ is unknown but bounded

- The sets $\Delta x_k \subseteq \delta X$ and $\Delta u_k \subseteq \delta U$ constructed in RPI sets (tube).

ADVANCED CONTROL SYSTEMS

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- $\tilde{X} = \{\tilde{X}_0, \tilde{X}_1, ..., \tilde{X}_N\}, \forall\ \tilde{X}_k = x_k \oplus \delta\mathbb{X}_k$ and an accompanying control tube $\tilde{U} = \{\tilde{U}_0, \tilde{U}_1, ..., \tilde{U}_N\}, \forall\ \tilde{U}_k = \hat{u}_k \oplus \delta\mathbb{U}_k$
- $\delta\mathbb{D}(k) \triangleq [0]^{n_d} \oplus H_{dk} B^{n_d}$.

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Robust Reliability-Aware Control of a DWN

Robust MPC preliminaries

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- $\delta\mathbb{D}(k) \triangleq [0]^{n_d} \oplus H_{dk} B^{n_d}$.
  - Appropriately to ensure that $\delta\mathbb{X}_k \subset interior(\mathbb{X})$ and $\delta\mathbb{U}_k \subset interior(\mathbb{U})$

* D. Mayne, M. Seron, and S. V. Raković. "Robust model predictive control of constrained linear system with bounded disturbances". In: *Automatica* 41 (Feb. 2005), pp. 219–224

ADVANCED CONTROL SYSTEMS

Robust Reliability-Aware Control of a DWN

Online computation of zonotopic reachable sets

A feedback LQR controller K is designed to :

- Minimize spread of trajectories
- Asymptotic stability to a predefined terminal set

$$J_{[\tilde{\hat{u}}_0,..,\tilde{\hat{u}}_\infty)} = \sum_{i=0}^{\infty} (\tilde{x}_k - x_k)^T Q (\tilde{x}_k - x_k) + \tilde{\hat{u}}_k^T R \tilde{\hat{u}}_k$$

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Decomposing: $\tilde{x}_k = x_k + \Delta x_k$

- Uncertain dynamic part:
  $\Delta x_{k+1} \triangleq (A + \hat{B}K)\Delta x_k + \hat{B}_d \Delta d_k,$
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Assuming that initial deviation, $\Delta x(0) = 0$

- $\delta\mathbb{X}_{k+i} \subseteq \bigoplus_{j=1}^{i} (A + \hat{B}K)^{i-j} \hat{B}_d \delta\mathbb{D}(i)$

ADVANCED CONTROL SYSTEMS

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- $\delta \mathbb{U}_{k+i} \subseteq \bigoplus_{j=1}^{i} [\tilde{P}\tilde{M}_1 K \Psi_{[1,i]K+i}, \tilde{P}\tilde{M}_2 H_{dk+i}] B^{2n_d}.$

Used for Robust constraints satisfaction

- Robust stability and recursive feasibility
  Terminal mRPI set constructed:
- An approximate of the exact $\tilde{\Omega}$:
  $\Omega_\infty \triangleq \bigoplus_{j=0}^{\infty}(A+\hat{B}K)^j\hat{B}_d\delta\mathbb{D}$,
  $\Omega_\infty \subseteq \tilde{\Omega}$; $(A+\hat{B}K)=\hat{A}$ and $\hat{B}_d\delta\mathbb{D}\subseteq\mathcal{W}$
  $\hat{A}$ is strictly stable
- Guaranteed convergence under infinite Minkowski sum*
  $\bigoplus_{j=0}^{\infty}(\hat{A})^j\mathcal{W} \subseteq (1-\alpha)^{-1}\bigoplus_{j=0}^{k-1}(\hat{A})^j\mathcal{W}$
  Truncated, Geometric series

* S. V. Raković et al. “Invariant approximations of the minimal robust positively Invariant set”. In: *Automatic Control, IEEE Transactions on* 50 (Mar. 2005), pp. 406–410

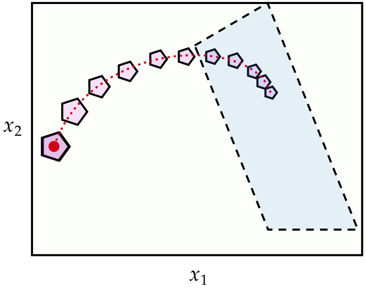

Figure: Transition through tubes to mRPI set

ADVANCED CONTROL SYSTEMS

- Reliability modeling is stochastic and complex
- Markov chain $\rightarrow$ Combinatorial explosions
- Stochastic petri-nets $\rightarrow$ Monte Carlo simulations (computationally demanding)
- Bayesian network modelling is used

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Bayesian network model depends on:

- Structure of the graph
- Conditional dependencies between the nodes related with the arcs
- $R_i(k) = R_{0,i} e^{\left(-T_s \sum_{s=0}^{k} \lambda_i(u(s))\right)}$ where; $\lambda_i(t) = \lambda_i^0 e^{(\beta_i u_i(t))}$*

* F. Karimi Pour, V. Puig, and G. Cembrano. "Economic Health-Aware LPV-MPC Based on System Reliability Assessment for Water Transport Network". In: *Energies* 12 (Aug. 2019), p. 3015

ADVANCED CONTROL SYSTEMS

$$B_N = (P, A_B, N_B)$$

- Direct dependencies of nodes with $P(n_i)$
- Joint probability under Conditional probability assumptions
  $P_r(n_i, n_2, ...n_N) = P_r(n_1) \prod_{i=2}^{N} P_r(n_i | P_a(n_i))$

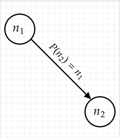

Figure: Dependencies between nodes and parents

Broad Reliability Modelling

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$$P_r(n_i, n_2, ...n_N) = P_r(n_1) \prod_{i=2}^{N} P_r(n_i | P_a(n_i))$$

- However static
  - Introduce temporal dependencies through dynamic BN

$$P_r(X_i(k+1)) = (A | X_i(k) = A) = R_{0,i} e^{(-T_s \sum_{s=0}^{k} \lambda_i(u,k))}$$

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DWN Reliability modelling:

- Represented in a DAG actuators as node and pipes as arcs
- A resultant Series-parallel architecture

$R_s(k) = 1 - \prod_{j=1}^{s} (1 - \prod_{i \in P_j} R_i(k))$, and thus:

$$\log(R_s(k+1)) = \log(R_s(k)) + \sum_{i \in P_j}^{s} \vartheta_j(k) \sum_{i \in P_j} \log R_i(k)$$

$\downarrow$

NONLINEAR

$$\mathcal{L}(k,\widehat{u},x) = W_1\mathfrak{J}_s(k) + W_1\mathfrak{J}_E(k) + W_1\mathfrak{J}_{\Delta\widehat{u}}(k) - W_1\mathfrak{J}_R(k)$$

safety in tanks

$$\mathfrak{J}_s = \|\varepsilon(k)\|^2$$
$$s.t$$
$$x(k) = x_s - \varepsilon(k)$$

ADVANCED CONTROL SYSTEMS

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Cost

$$\mathfrak{J}_E = (\alpha_1 + \alpha_2(k))\widehat{u}(k)$$

$$\mathcal{L}(k,\widehat{u},x) = W_1\mathfrak{J}_s(k) + W_1\mathfrak{J}_E(k) + W_1\mathfrak{J}_{\Delta\widehat{u}}(k) - W_1\mathfrak{J}_R(k)$$

Slew rate

$$\boxed{\mathfrak{J}_{\Delta\widehat{u}} = \|\widehat{u}(k)\|^2}$$

ADVANCED CONTROL SYSTEMS

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Reliability

$$\boxed{\mathfrak{J}_R = x_r}$$

ADVANCED CONTROL SYSTEMS

Robust Reliability-Aware Control of a DWN

Reliability-aware MPC

$$\mathcal{L}(k,\widehat{u},x) = W_1\mathfrak{J}_s(k) + W_1\mathfrak{J}_E(k) + W_1\mathfrak{J}_{\Delta\widehat{u}}(k) - W_1\mathfrak{J}_R(k)$$

$$\min_{\widehat{u},\, x(k),\, x_r} \sum_{i=0}^{Np-1} \mathcal{L}(k,\widehat{u},x(k),x_r(k))$$

subject to

$$x(i+1|k) = Ax(i|k) + \widehat{B}\widehat{u}(i|k) + \widehat{B_d}d(i|k)$$

$$\widehat{u}(i|k) \subseteq U(i|k) \ominus_{\boxdot} \delta U(i|k),$$

$$x(i|k) \subseteq X(i|k) \ominus_{\boxdot} \delta X(i|k),$$

$$x(N_p-1) \subseteq \widetilde{\Omega}$$

$$x_r(i+1|k) = A_r(\theta(k))x_r(i|k) + B_r(\theta(k))\widehat{u}(i|k)$$

$$x_r(i|k) \subseteq (0,1]$$

Robust Reliability-Aware Control of a DWN

Reliability-aware MPC

$$\mathcal{L}(k,\widehat{u},x) = W_1\mathfrak{J}_s(k) + W_1\mathfrak{J}_E(k) + W_1\mathfrak{J}_{\Delta\widehat{u}}(k) - W_1\mathfrak{J}_R(k)$$

$$\min_{\widehat{u},\,x(k),\,x_r} \sum_{i=0}^{Np-1} \mathcal{L}(k,\widehat{u},x(k),x_r(k))$$

subject to

$$x(i+1|k) = Ax(i|k) + \widehat{B}\widehat{u}(i|k) + \widehat{B_d}d(i|k)$$

Robust constraint satisfaction

$$\boxed{\begin{aligned} \widehat{u}(i|k) &\subseteq U(i|k) \ominus \boxdot\, \delta U(i|k), \\ x(i|k) &\subseteq X(i|k) \ominus \boxdot\, \delta X(i|k), \end{aligned}}$$

$$x(N_p-1) \subseteq \widetilde{\Omega}$$

$$x_r(i+1|k) = A_r(\theta(k))x_r(i|k) + B_r(\theta(k))\widehat{u}(i|k)$$

$$x_r(i|k) \subseteq (0,1]$$

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$$\mathcal{L}(k,\widehat{u},x) = W_1\mathfrak{J}_s(k) + W_1\mathfrak{J}_E(k) + W_1\mathfrak{J}_{\Delta\widehat{u}}(k) - W_1\mathfrak{J}_R(k)$$

$$\min_{\widehat{u},\, x(k),\, x_r} \sum_{i=0}^{Np-1} \mathcal{L}(k,\widehat{u},x(k),x_r(k))$$

subject to

$$x(i+1|k) = Ax(i|k) + \widehat{B}\widehat{u}(i|k) + \widehat{B_d}d(i|k)$$

$$\widehat{u}(i|k) \subseteq U(i|k) \ominus\boxdot\, \delta U(i|k),$$

$$x(i|k) \subseteq X(i|k) \ominus\boxdot\, \delta X(i|k),$$

$$\boxed{x(N_p-1) \subseteq \widetilde{\Omega}}$$

$$x_r(i+1|k) = A_r(\theta(k))x_r(i|k) + B_r(\theta(k))\widehat{u}(i|k)$$

$$x_r(i|k) \subseteq (0,1]$$

Robust stability
&
Recursive feasibility

Robust Reliability-Aware Control of a DWN

Reliability-aware MPC

Introduction

Set-based Uncertainty Quantification For Prognostics.

Health-Aware Control Of a Wind Turbine

Data-Based Prognostics

Robust Reliability-Aware Control of a DWN

Concluding Remarks

$$\mathcal{L}(k,\widehat{u},x) = W_1\mathfrak{J}_s(k) + W_1\mathfrak{J}_E(k) + W_1\mathfrak{J}_{\Delta\widehat{u}}(k) - W_1\mathfrak{J}_R(k)$$

$$\min_{\widehat{u},\, x(k),\, x_r} \sum_{i=0}^{Np-1} \mathcal{L}(k,\widehat{u},x(k),x_r(k))$$

subject to

Reliability constraints

$$x(i+1|k) = Ax(i|k) + \widehat{B}\widehat{u}(i|k) + \widehat{B_d}d(i|k)$$

$$\widehat{u}(i|k) \subseteq U(i|k) \ominus\boxdot \delta U(i|k),$$

$$x(i|k) \subseteq X(i|k) \ominus\boxdot \delta X(i|k),$$

$$x(N_p-1) \subseteq \widetilde{\Omega}$$

$$\boxed{x_r(i+1|k) = A_r(\theta(k))x_r(i|k) + B_r(\theta(k))\widehat{u}(i|k)}$$

$$x_r(i|k) \subseteq (0,1]$$

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Robust Reliability-Aware Control of a DWN

Experimental Setup and Results

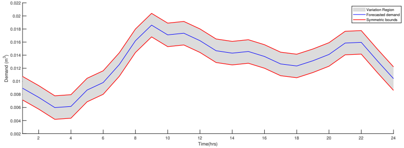

Figure: A 24 hrs demand profile of node C129PAL with symmetric bounded uncertainty.

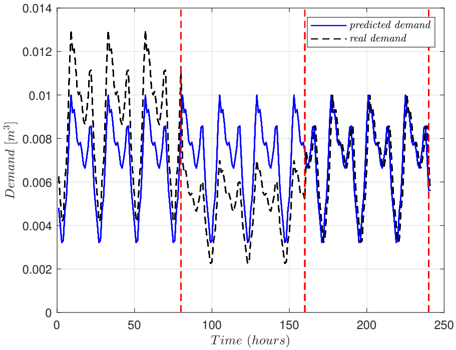

Figure: 80-hour test scenarios for robust control considering demand node c125PAL.

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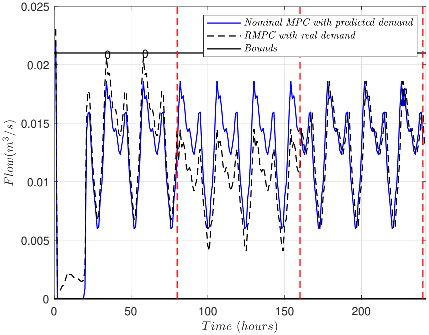

Figure: Control action of bMS.

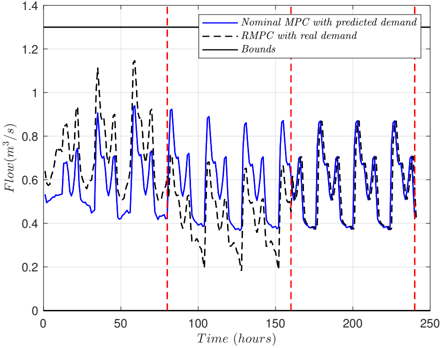

Figure: Control action of CPIV.

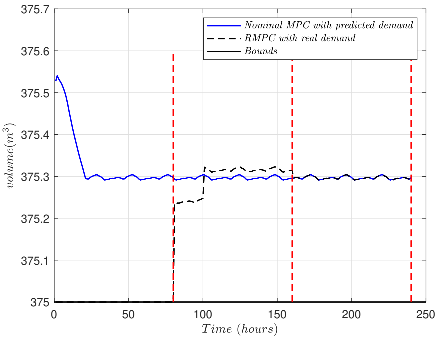

Figure: Level of Tank d125PAL during robust control test scenarios.

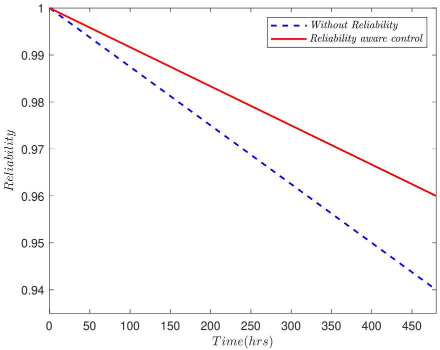

Figure: Network reliability under different controller configurations.

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Outline

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Conclusion and Contributions

[1] Set based prognostics quantification for prognostics

Model-based prognostics for wind turbine blade is undertaken,

- Direct degradation incorporation into wind turbine model
- Applied in ZKF for set estimation.
- Reachability analysis for uncertainty propagation.

*∗ A novel means of quantifying uncertainty in prognostics (nascent).

[2] HW control of a wind turbine

With information from [1],

- Segmentation of degradation path for stage models
- Accounts for discontinuity in stress identification.
- Allocated weights per segment.

*∗ Controller operates in a practical fashion.

Conclusion and Contributions

[3] Data-based prognostics

Data-based prognostics with interpretability properties are designed,

- EEFIG is applied to the IGBT data set
- Competitive rules are predicted
- Interpretability capabilities.
- Set-based quantification is duly applied.

$^{*}\ast$ Novel tool for interpretation and set-based quantification.

[4] Robust Reliability-Aware control of DWN

Reliability Aware controller is designed,

- Improves reliability at a cost
- Designed to be robust

$^{*}\ast$ Controller operates in a practical fashion.

Future Works

[1] Set based prognostics quantification for prognostics

- Test with statistical methods

[2] HW control of a wind turbine

- Robustify the control
- efficient way of including degradation function

[3] Data-based prognostics

- Test uncertainty quantification on complex data-based methods
- Compare the set based with statistical methods.
- use other set-representation.

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Journal articles:

- khoury, B., Bessa, I., Puig, V., Nejjari, F., and Palhares, R.M. (2022).”Reliability–aware zonotopic tube–based model predictive control of a drinking water network”. In: *Systems & Control Letters*. Int. J. Appl. Math. Comput. Sci., 32(2), 197–211. doi: 10.34768/amcs-2022-0015
- F. Nejjari, Khoury B., V. Puig and Ocompos S.. ”Economic Linear Parameter Varying Model Predictive Control of the Aeration System of a Wastewater Treatment Plant”. In: *Sensors*.

Conference proceedings:

- khoury, B., Bessa, I., Puig, V., Nejjari, F., and Palhares, R.M. (2022). ”Data-driven prognostics based on evolving fuzzy degradation models for power semiconductor devices”. In: *Proceedings of the 7th European Conference of the Prognostics and Health Management Society 2022.*
- Khoury, B., Bessa, I., Nejjari, F., and Puig, V. (2022). ” set-based uncertainty quantification of evolving fuzzy models for data-driven prognostics.”. In: *15th International Conference on Diagnostics of Processes and Systems.*

ADVANCED CONTROL SYSTEMS

Conclusion and Contributions

Publications

- Khoury, B., Puig, V., and Nejjari, F. (2022). ” A set-based prognostics approach for wind turbine blade health monitoring.”. In: *IFAC-PapersOnLine, 55, 402–407.*
- Khoury, B., Nejjari, F., and Puig, V. (2022). ” Model-based prognosis approach using a zonotopic kalman filter with application to a wind turbine..”. In: *In Proceedings of the 5th European Conference of the Prognostics and Health Management Society (PHME 2020)).*
- Khoury, B., Thuller, J., Didier T. and Puig, V. (2020). ”Assessing a Statistical and a Set-based Approach for Remaining UsefulLife Prediction ”. In: *2023 Mediterranean Conference on Control and Automation.* submittted
- Khoury, B., Nejjari, F., and Puig, V. (2020). ”Health-aware LPV model predictive control of wind turbines ”. In: *IFAC-PapersOnLine, 53(2), 826–831.*
- Khoury, B., Nejjari, F., and Puig, V. . (2023). ”Health-aware linear parameter varying model predictive control of wind turbines”. In: *2023 Mediterranean Conference on Control and Automation.*
- Khoury, B., Nejjari, F., and Puig, V. . (2020). ”Robust economic model predictive control of water transport networks”. In: *8th Mediterranean Conference on Control and Automation, MED 2020, Saint-Rapha¨el, France.*

Thanks for your attention!