Sûreté de fonctionnement & Retour d'Expériences

Dependability and Feedback Data Collection

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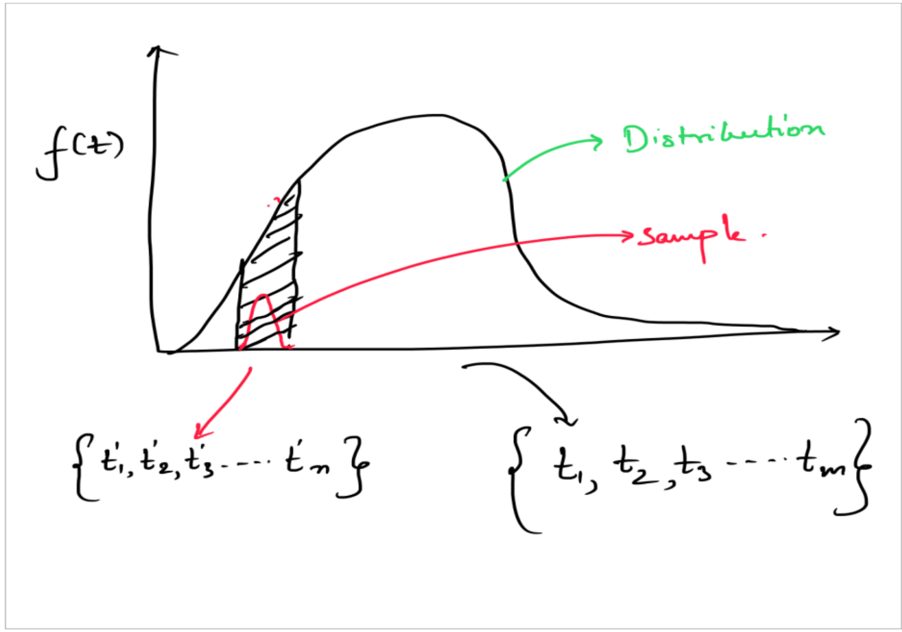
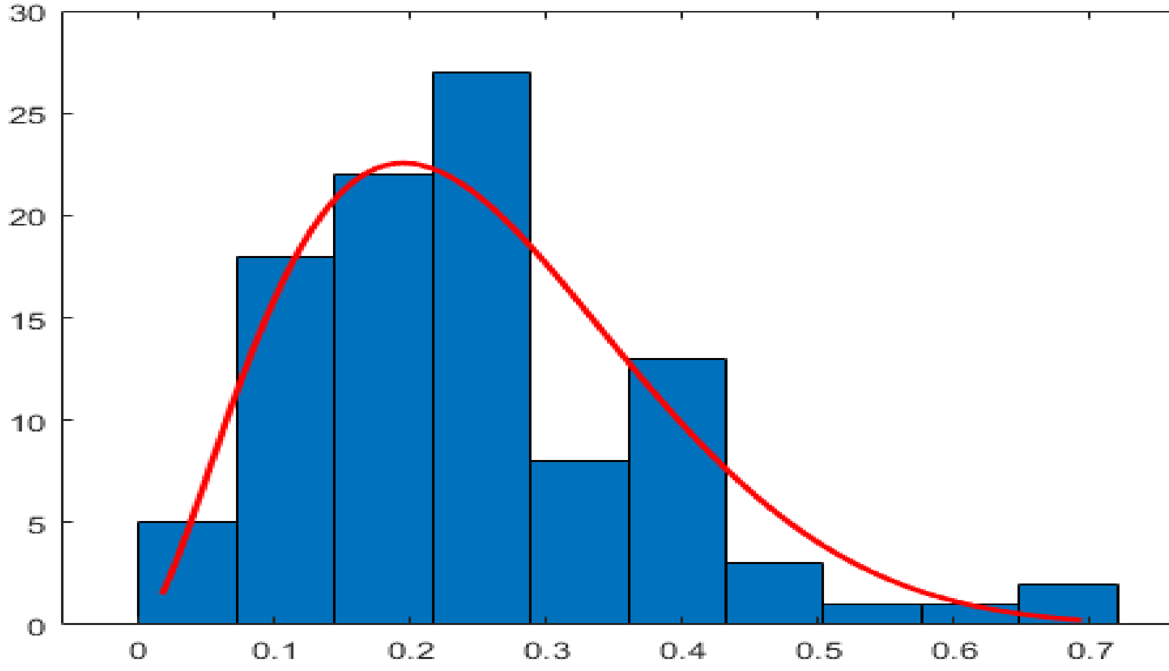
Identifying Failure distributions (Fitting failure distributions)

Objective : "Fit" theoretical distributions to random samples.

Advantages:

- Information beyond collected samples.
- Gives the probabilistic information about ***underlying*** failure process.
- Distribution enables complex analysis of failure processes.





Identifying Failure distributions (Fitting failure distributions)

1. Identify the distribution
   - Weibull,
   - Exponential,
   - Poisson

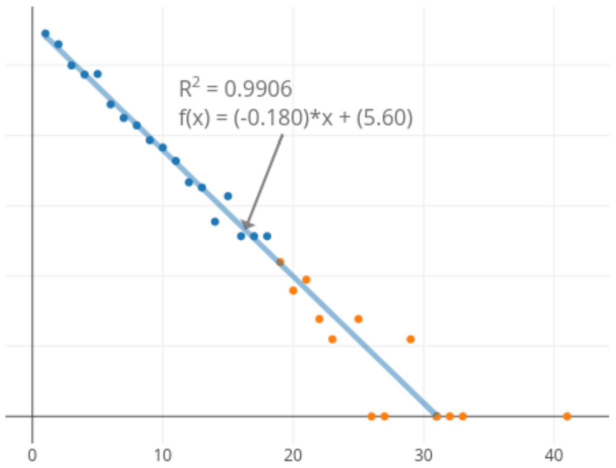
   …

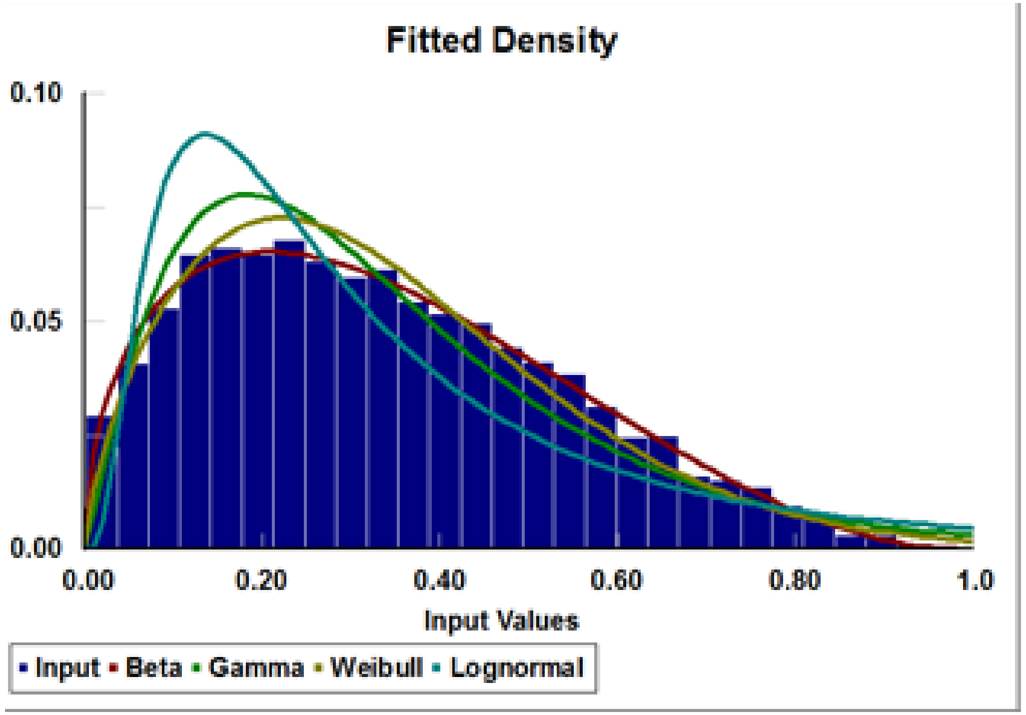
- **Prior knowledge of failure process**
- **Histogram of failure times**
- **Analyze Empirical failure rate**

2. Estimate the distribution parameters
   - find theoretical distribution parametrs

$$f(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\frac{(t-\mu)^2}{\sigma^2}\right]; -\infty < t < \infty$$

$$R(t) = \int_t^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\frac{(t'-\mu)^2}{\sigma^2}\right] dt'$$



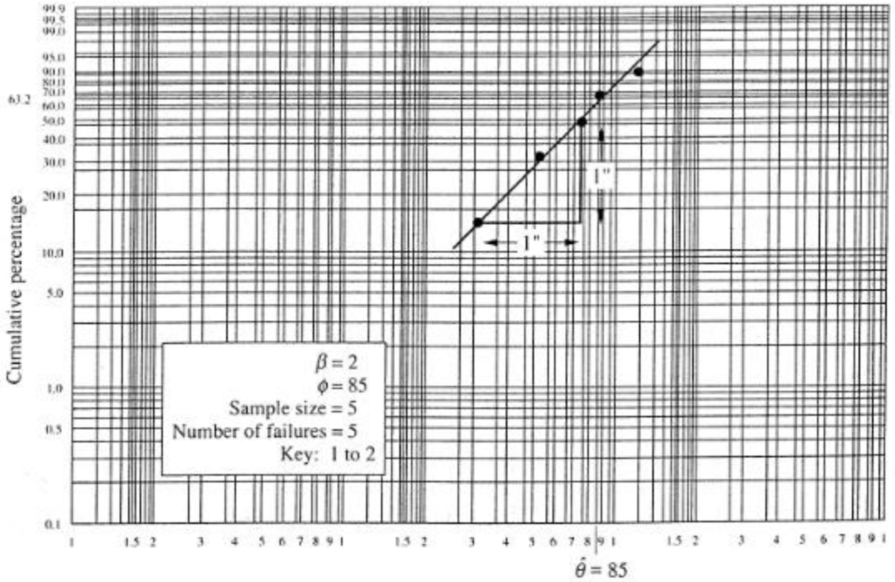


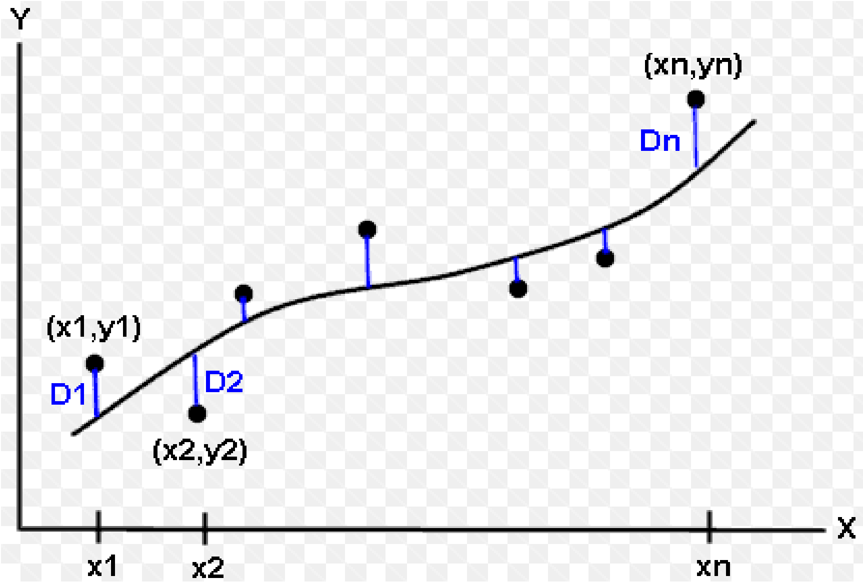
3. Perform goodness of fit test.

Probability plots and Least Square

- fit a set of data to distribution

$$\left(t_i, F(t_i)\right); i = 1,2,0 \ldots\ldots n$$



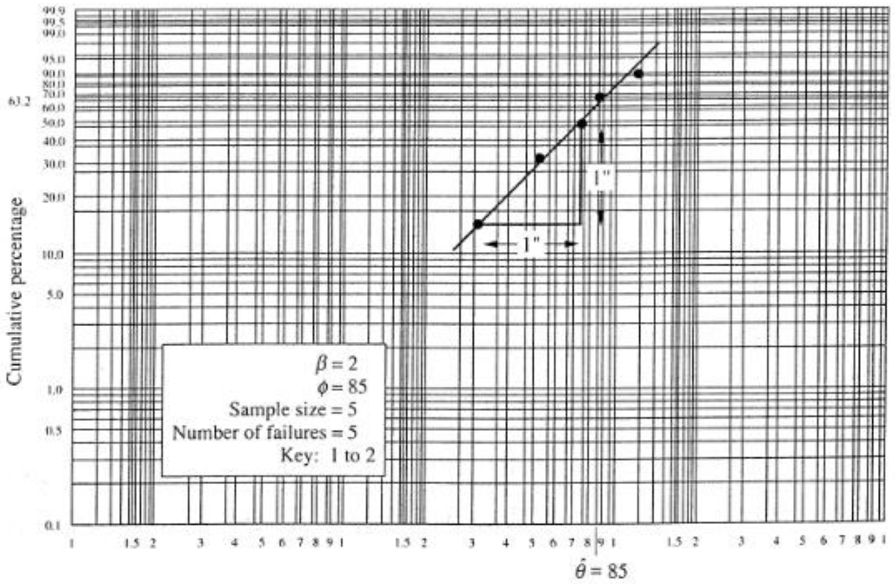


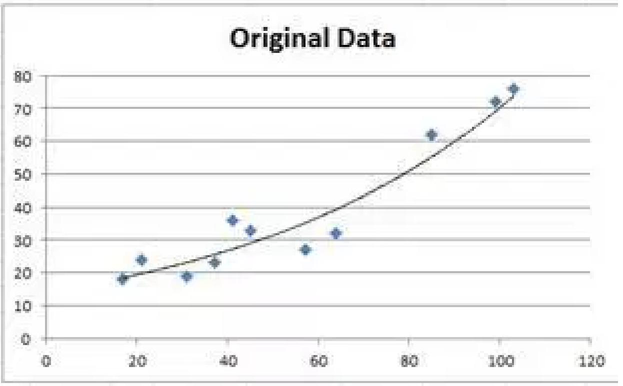
Probability plots and Least Square

- Fit a set of data to distribution

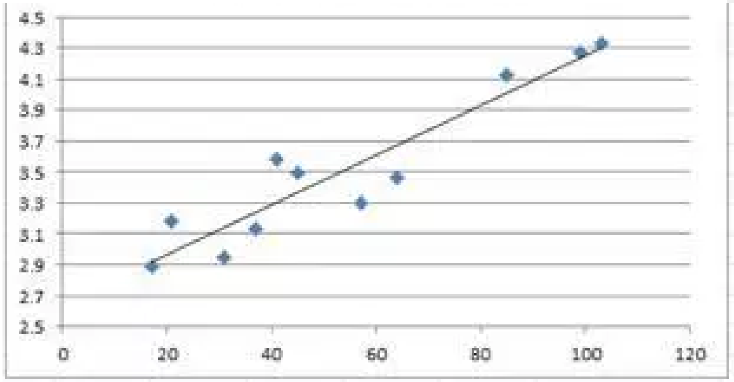
$$(t_i, F(t_i)); i = 1,2,0 \ldots\ldots n$$

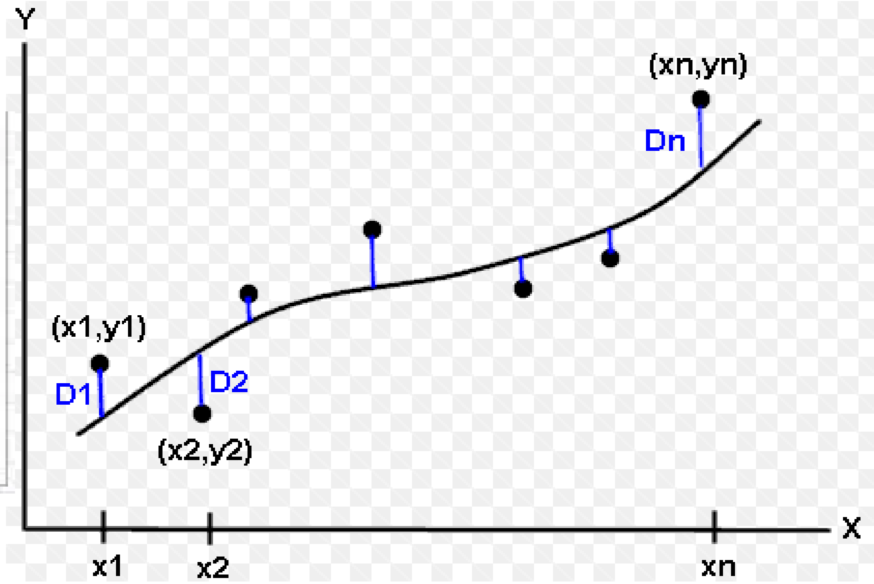
- Modify (transform) vertical and horizontal scale.





Logarithmic Transformation





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Ordinary Least Square (OLS) based regression

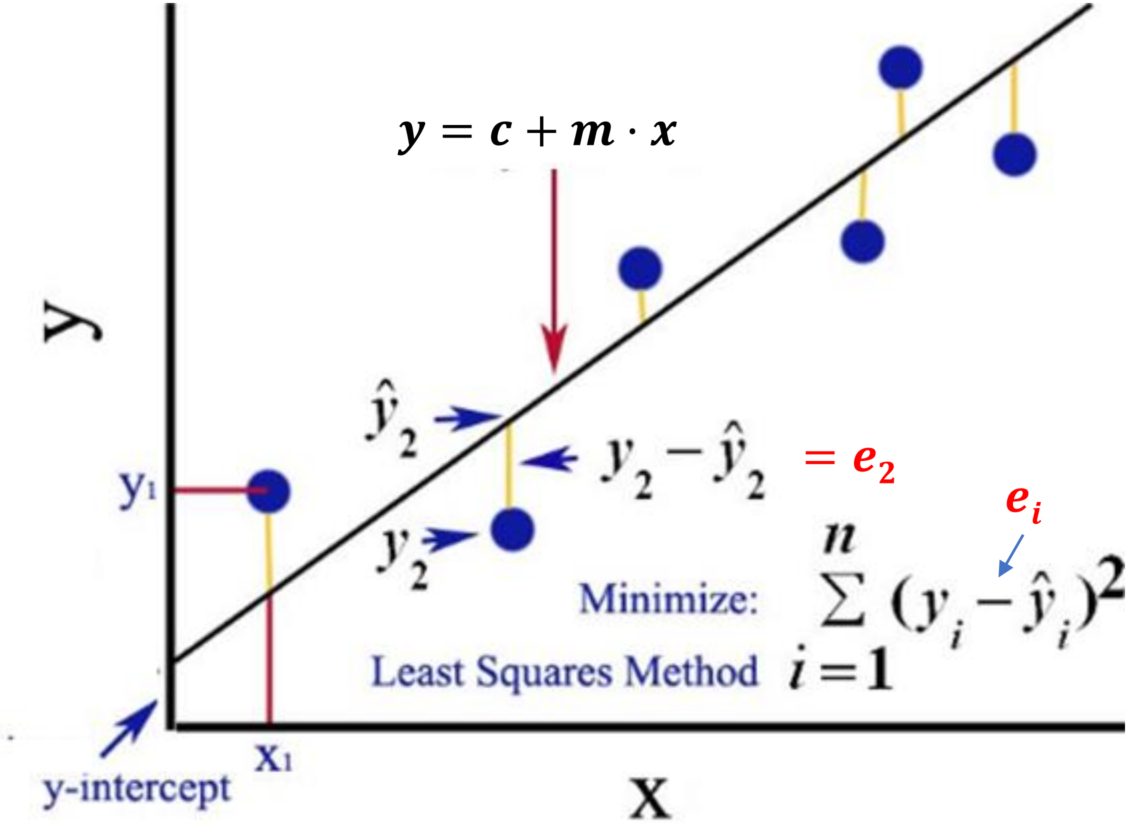
$(x_i, y_i); \quad i = 1,2,3 \dots n$

- Error term $\quad e_i = y_i - (\widehat{c + mx_i})$

- Objective : Minimize the sum of square of errors

$$e_1 + e_2 + e_3 \dots e_n$$

$$\sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n} \left(y_i - (\widehat{c + mx_i})\right)^2$$



$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\widehat{m} = \frac{\sum_{i=1}^{n}(x_i - \bar{x}) \times (Y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\hat{c} = \bar{y} - \widehat{m}\bar{x}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

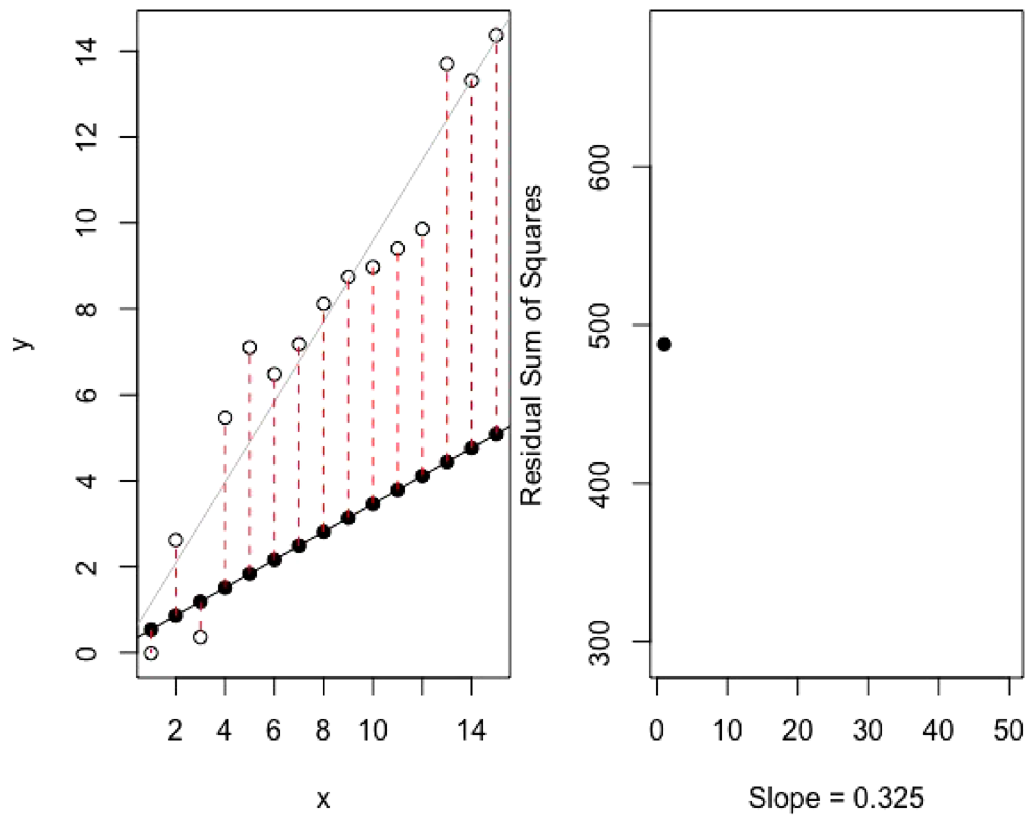
least-squares-regression_en.html

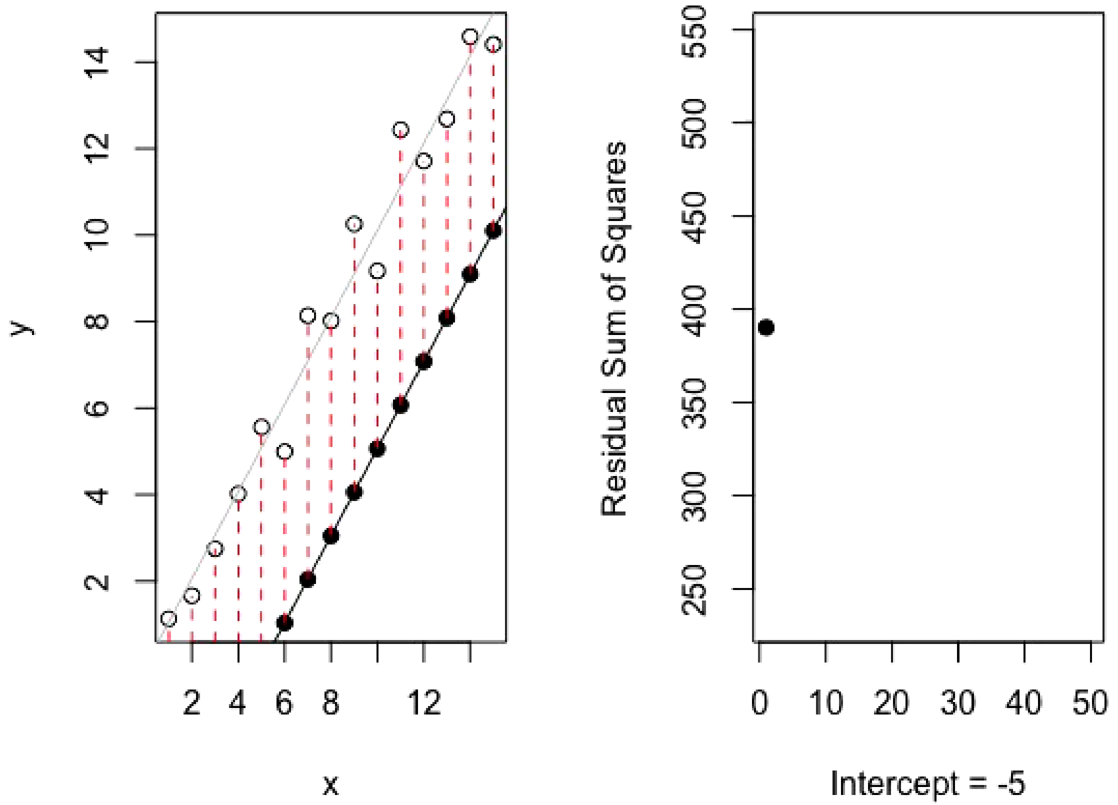
Ordinary Least Square (OLS) based regression

- Index of fit : *r*

$$r^2 = \frac{\sum_{i=1}^{n}(y_i - \widehat{m}x_i - \hat{c})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

- *0 < r <1* , r =1 (indicates perfect fit)





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Exponential Plots

$$\left(t_i, \hat{F}(t_i)\right); i = 1,2,0 \ldots\ldots n$$

$$\hat{F}(t_i) = \frac{i - 0.3}{n + 0.4}$$

CDF : $F(t) = 1 - e^{-\lambda t}$

Reliability function: $R(t) \Rightarrow 1 - \hat{F}(t_i) = e^{-\lambda t}$

Transformation of the real data:

$$y_i = \ln\left(\frac{1}{\left(1 - \hat{F}(t_i)\right)}\right)$$

$$x_i = t_i$$

Logarithm transformation → $\ln\left(1 - \hat{F}(t_i)\right) = \ln\left(e^{-\lambda t}\right)$

$$or, \ln\left(\frac{1}{1 - \hat{F}(t_i)}\right) = \lambda t$$

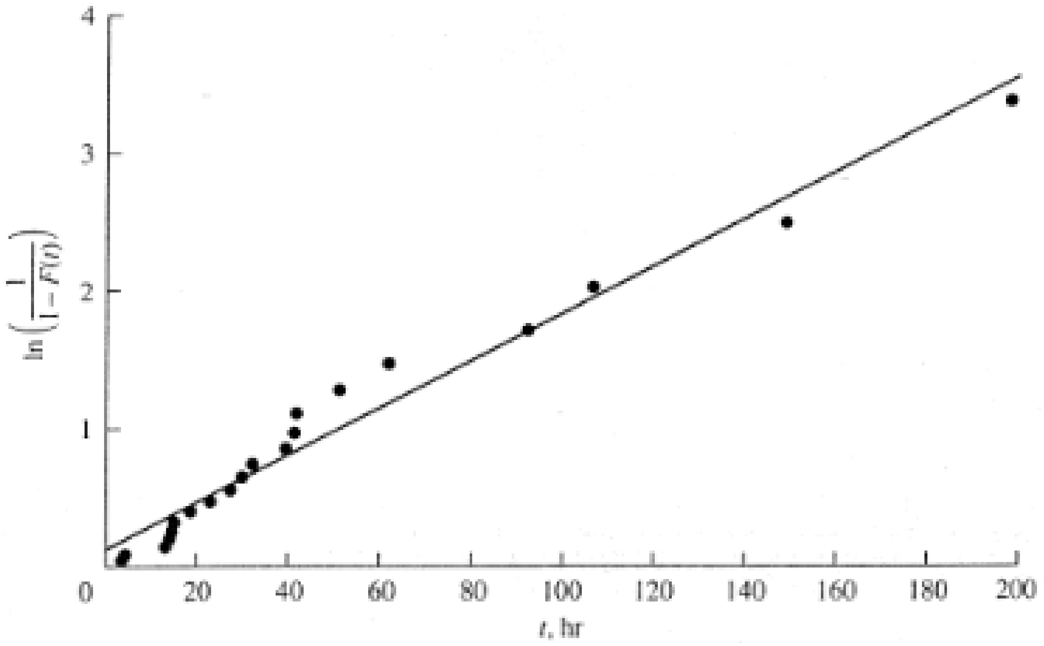
$$y_i = m \ x_i + c$$

$$\hat{\lambda} = \frac{\sum_{i=1}^{i=n}(x_i - \bar{x}) \times (y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Intercept : c=0

So,

$$\hat{\lambda} = \frac{\sum_{i=1}^{i=n}(x_i)(y_i)}{\sum_{i=1}^{n}(x_i)^2}$$



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Weibull Plots

- CDF : $F(t) = 1 - e^{-(t/\alpha)^{\beta}}$

- Logarithmic Transformations:

$$\ln\left(\frac{1}{1-F(t)}\right) = \left(\frac{t}{\alpha}\right)^{\beta}$$

$$\ln\ln\left(\frac{1}{1-F(t)}\right) = \beta\ln t - \beta\ln\alpha$$

- Plot:

$$x_i = \ln t_i$$

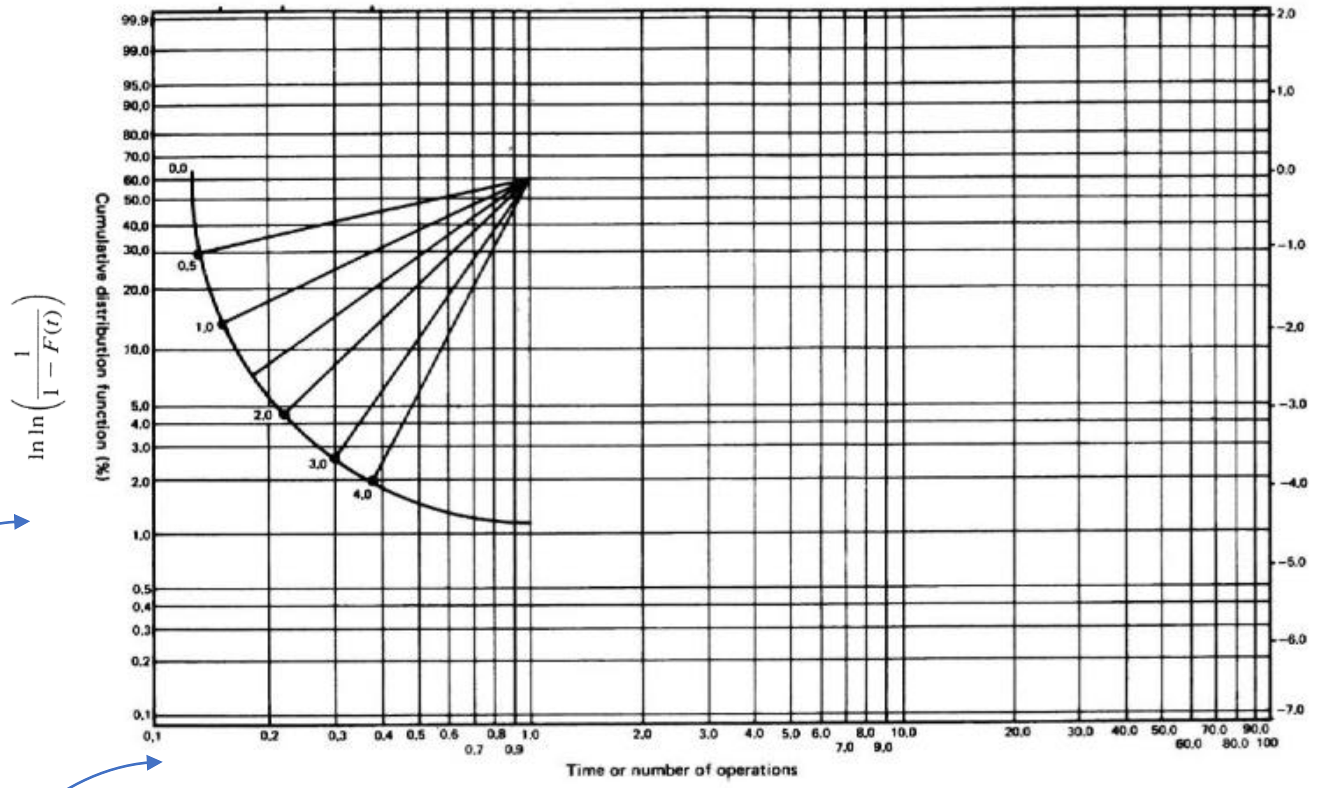
$$y_i = \ln\ln\left(\frac{1}{1-\hat{F}(t_i)}\right)$$

$$\left(\ln t_i, \quad \ln\ln\left[\frac{1}{1-\hat{F}(t_i)}\right]\right)$$

Weibull Graphing:

$$\left(t_i, \hat{F}(t_i)\right); i = 1,2,0 \ldots\ldots n$$

$$\hat{F}(t_i) = \frac{i-0.3}{n+0.4}$$



Weibull Plots: Least Square Approach

- CDF :

$$F(t) = 1 - e^{-(t/\alpha)^{\beta}}$$

- Logarithmic

Transformations:

$$\ln\left(\frac{1}{1-F(t)}\right) = \left(\frac{t}{\alpha}\right)^{\beta}$$

$$\ln\ln\left(\frac{1}{1-F(t)}\right) = \beta \ln t - \beta \ln \alpha$$

$$y_i = \ln\ln\left(\frac{1}{(1-F(t_i))}\right)$$

$$x_i = \ln t_i$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\hat{c} = \bar{y} - m\bar{x}$$

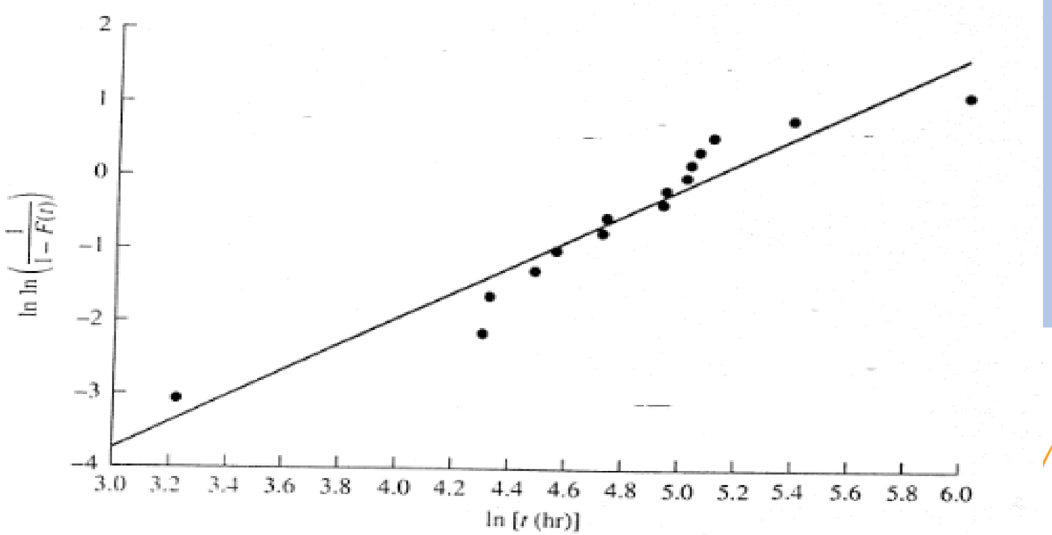
$$\hat{m} = \hat{\beta}$$

$$\hat{c} = \bar{y} - \hat{\beta}\bar{x}$$

$$\hat{\alpha} = e^{-\hat{c}/\hat{\beta}}$$

$$\hat{\beta} = \frac{\sum_{i=1}^{i=n}(x_i - \bar{x}) \times (y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\hat{\alpha} = e^{-\hat{c}/\hat{\beta}}$$



Weibull Graphing:

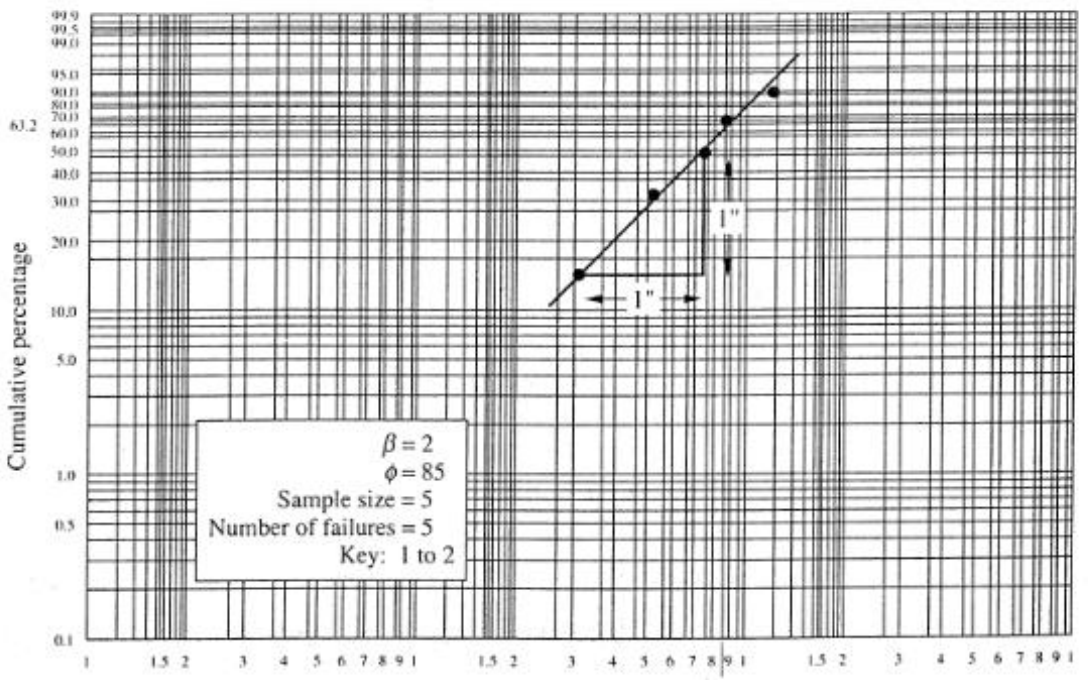
- α can be obtained from graphing.

$F(\alpha) = 0.632$

- β can be obtained from slope of straight line.

- or, $$\frac{\ln\ln\left(\frac{1}{1-F(t)}\right)}{\ln t - \ln\alpha} = \beta$$

- Multiple estimates of β → average estimate.



What happens when Weibull plotting is curve , not straight line?

Normal Distribution : Least Square Approach

- Normal distribution:

$$F(t) = \phi\left(\frac{t-\mu}{\sigma}\right) = \phi(z)$$

- Inverse function (linear in t) :

$$z_i = \phi^{-1} \times \left(F(t)\right) = \frac{t_i - \mu}{\sigma} = \frac{t_i}{\sigma} - \frac{\mu}{\sigma}$$

z_i: Look in Tables

-

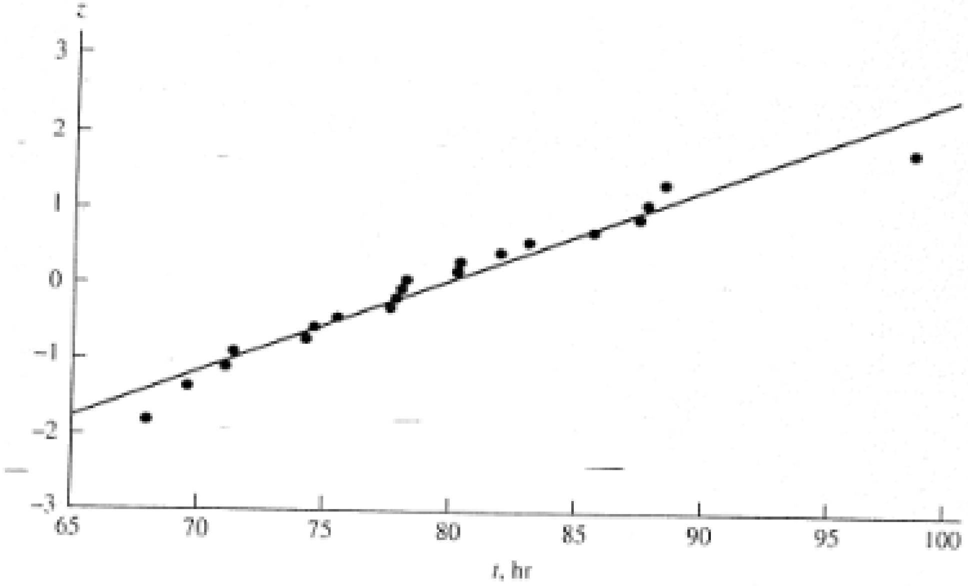
$$\left(t_i, \hat{F}(t_i)\right)$$

$$x_i = t_i;$$

$$y_i = z_i = \phi^{-1}[F(t_i)]$$

- Least Square Estimates:

$$\hat{\sigma} = \frac{1}{m}; \hat{\mu} = -c\hat{\sigma} = -\frac{c}{m}$$



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