# Sûreté de fonctionnement & Retour d'Expériences

# Dependability and Feedback Data Collection

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# Data Collection & Empirical Methods

Course I and II → Basic reliability models applications.

Failure Data collection

↓

Data base construction

↓

Analysis of data,

↓

“Identify” failure distributions

↓

Estimate parameters of distributions

Fitting reliability distribution to failure data

**1st Approach**: *Non-parametric – distribution free method*

Identify distribution directly from data

2nd Approach: *Distribution fit*

Fit theoretical distribution (Weibull, normal, log-normal, exponential)

# Analysis of failure data

Generation or observation of failure data can be represented by: $t_{1,} t_{2,} t_{3,} \ldots, t_n$

$t_i$: represents time of failure of $i^{\text{th}}$ unit.

Each failure → *Independent* sample from same population.

Population → Distribution of all possible failure times

↓

$$f(t), R(t), F(t), \lambda(t)$$

***Objective:*** Determine the best failure distribution by *n* failure times from sample given.

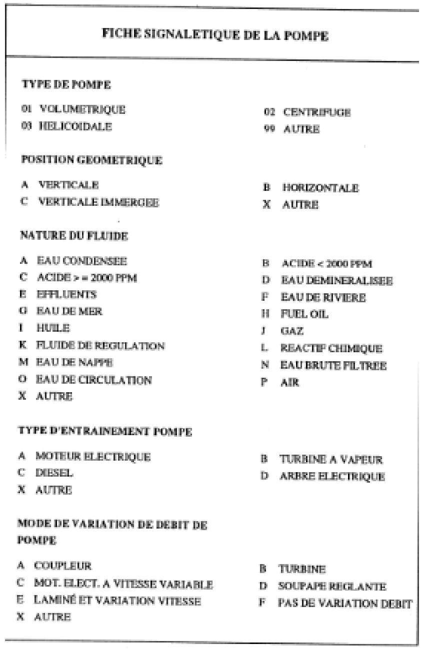
# Taxonomy of Data


Common failure data types:

**Operational failures** **vs** **Test-generated failures**

Field data collected (on ground)

Item operates in normal use



FICHE SIGNALETIQUE DE LA POMPE

TYPE DE POMPE

| | |
|---|---|
| 01 VOLUMETRIQUE | 02 CENTRIFUGE |
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POSITION GEOMETRIQUE

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| E EFFLUENTS | F EAU DE RIVIERE |
| G EAU DE MER | H FUEL OIL |
| I HUILE | J GAZ |
| K FLUIDE DE REGULATION | L REACTIF CHIMIQUE |
| M EAU DE NAPPE | N EAU BRUTE FILTREE |
| O EAU DE CIRCULATION | P AIR |
| X AUTRE | |

TYPE D'ENTRAINEMENT POMPE

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MODE DE VARIATION DE DEBIT DE POMPE

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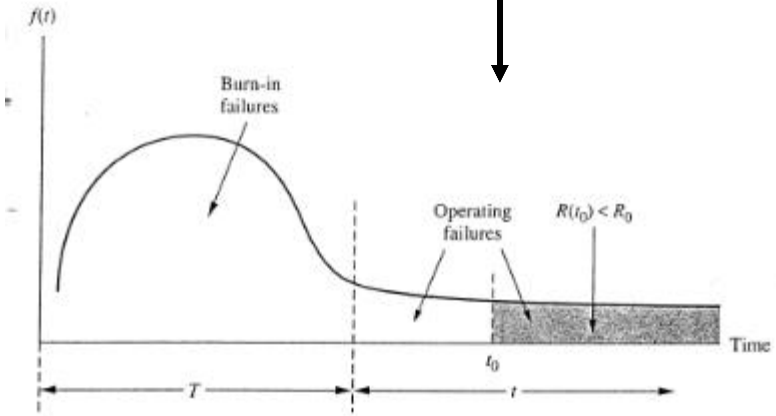
- Burn in tests : Specific duration usage
- Life test and Accelerated tests
  - Constant stress
  - Degradation injection
- Reliability Growth Tests



# Taxonomy of Data

Common failure data types:

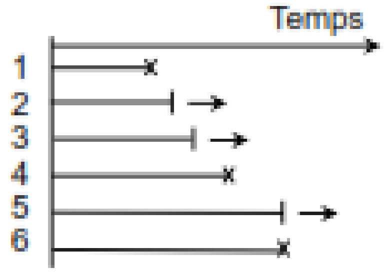
**Grouped data** **vs** **Ungrouped Data**

Failure Data in *time intervals*

- large groups
- depends on method of collecting,
- individual failures may not be recorded

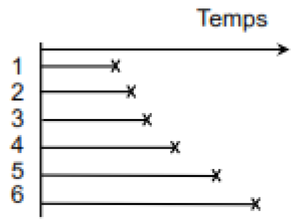
Failure Data at specific time

- Individual failures are recorded




**Complete Data** **vs** **Censored Data**

- Failure Data collected for all items/ tests till actual failure




- Data incomplete, test terminated or data collection finished before failure.
- Unit may be removed before failure due to other failure mode that occur in other components.

# Taxonomy of Data

## Censored Data

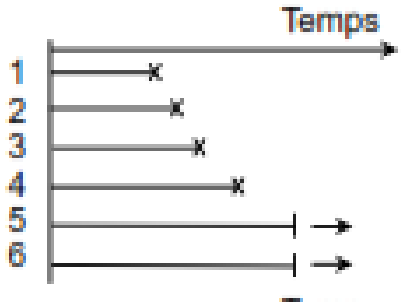
### Single Censored Data

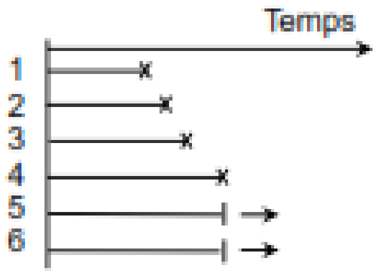
All items with same test time .
Test is terminated before item failure

- Censored on left : Failure times for some units occur *before* some specific time, test started after failure.

- Censored on right : failure times **after** some time

  - Type I : Test is terminated after a fixed length of time t*

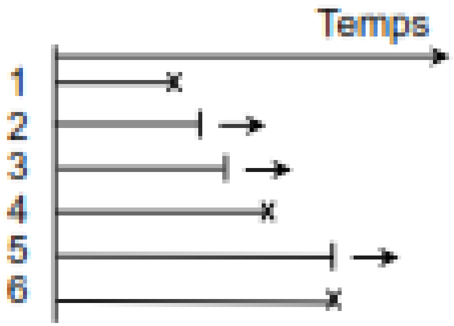


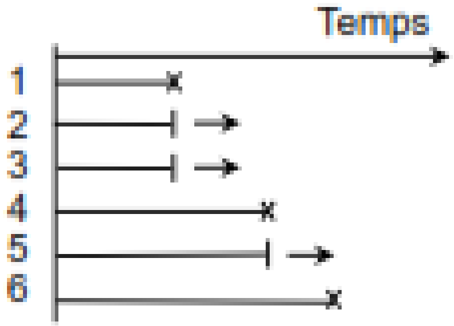




- Type II: testing is terminated after a specific number of failures '*r*' has occurred.

### Multiply Censor Data

Testing and operating times differ among censored units



# Empirical methods : Ungrouped Complete data

Non parametric , distribution free methods

Generation or observation of failure data can be represented by: $t_1, t_2, t_3, \ldots, t_n$

$t_i$: represents time of failure of $i^{th}$ unit.

Objective: derive from failure data :
$f(t), R(t), F(t), \lambda(t)$

- Consider '*n*' ordered failure times:

  $t_1, t_2, \ldots\ldots, t_n \quad ; t_i \leq t_{i+1}$

- Number of units surviving at time $t_i$ → *n-i*
- Reliability function estimation

$$\hat{R}(t_i) = \frac{n-i}{n} = 1 - \frac{i}{n}$$

$$\hat{F}(t_i) = 1 - \hat{R}(t_i) = \frac{i}{n}$$

**Improved Estimate** of cumulative failure distribution :

$$\hat{F}(t_i) = \frac{i}{n+1}$$

Reliability estimate :

$$\hat{R}(t_i) = 1 - \frac{i}{n+1} = \frac{n+1-i}{n+1}$$

- The equation provides plotting positions

  $\left(t_i, \hat{F}(t_i)\right)$

Skewed distribution → Median a good measure

$$\hat{F}(t_i) = \frac{i-0.3}{n+0.4}$$

# Empirical methods : Ungrouped Complete data

- Probability density function

$$\hat{f}(t) = -\frac{\hat{R}(t_{i+1}) - \hat{R}(t_i)}{t_{i+1} - t_i}$$

$$\hat{f}(t) = \frac{1}{(t_{i+1} - t_i) \times (n+1)}; t_i < t < t_{i+1}$$

$$\hat{\lambda}(t) = \frac{\hat{f}(t)}{\hat{R}(t)} = \frac{1}{(t_{i+1} - t_i) \times (n+1-i)}$$
$$= \frac{1}{(t_{i+1} - t_i) \times (n+1-i)}$$

MTTF estimate : obtained from sample mean

$$\widehat{MTTF} = \sum_{i=1}^{n} \frac{t_i}{n}$$

$$s^2 = \sum_{i=1}^{n} \frac{\left(t_i - \widehat{MTTF}\right)^2}{n-1} \wedge$$
$$= \frac{\Sigma_{i=1}^{n} t_i^2 - nMTTF^2}{n-1}$$

Approximate confidence interval of $100 \times (1-\alpha)$ for MTTF:

$$\widehat{MTTF} \pm t_{\alpha/2,n-1} \frac{s}{\sqrt[2]{n}}$$

From **student's t distribution**, for (*n*-*i*) degrees of freedom and desired Confidence Interval level

# Empirical methods : Ungrouped Complete data

- Example: 10 failure times in Hours are given, estimate the distribution $f(t), R(t), F(t), \lambda(t)$

18.9, 86.1, 15.4…….20.1 (10 failure times)

Rank-order the data

| Time | Reliability | Density | Hazard rate |
| --- | --- | --- | --- |
| 0.0 | 1.00 | 0.0059 | 0.0059 |
| 15.4 | 0.909 | 0.0260 | 0.028 |
| 18.9 | 0.818 | | |
| 20.1 | 0.72 | | |
| . | | | |
| - | | | |
| - | | | |
| - | | | |
| 86.1 | | | |

$$\hat{R}(15 \cdot 4) = \frac{(10+1-1)}{11} = 0.9090$$

$$\hat{f}(t) = \frac{1}{(18 \cdot 9 - 15 \cdot 4) \cdot 11} = 0.0260 \quad for\ 15 \cdot 4 < t < 18 \cdot 9$$

$$\hat{\lambda}(t) = \frac{1}{(18 \cdot 9 - 15 \cdot 4) \cdot 10} = 0.0286$$

Approximate 90% confidence interval for MTTF:

$$\widehat{MTTF} = \frac{15 \cdot 4 + 18 \cdot 9 + \cdots}{10} = 40.31$$

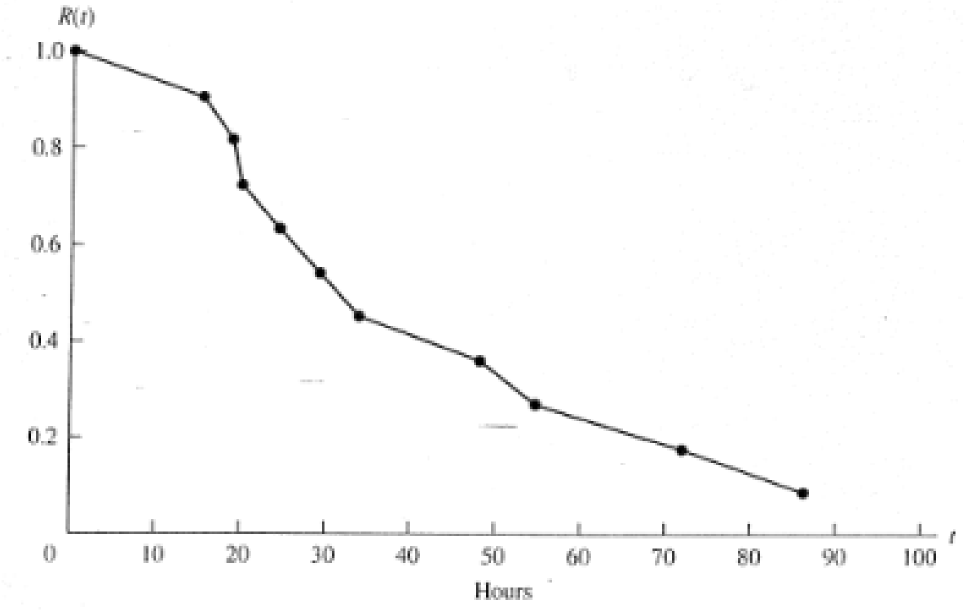
$$S^2 = \frac{15 \cdot 4^2 + 18 \cdot 9^2 + \cdots \ldots .}{9} = 585 \cdot 54$$

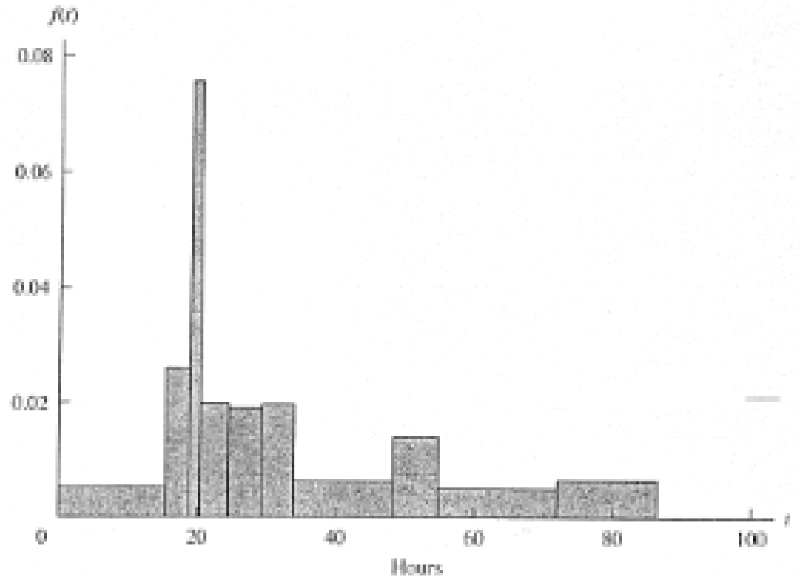
$$S = 24 \cdot 198$$

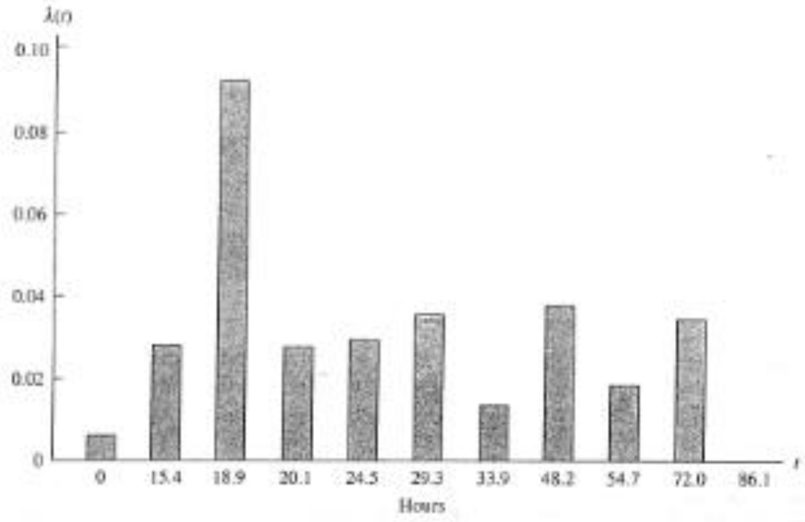
$$t_{0.05,9} = 1.833\ (See\ Students\ \mathrm{t}\ distribution\ chart)$$

$$40 \cdot 31 \pm 1 \cdot 833 \times \frac{(24 \cdot 198)}{\sqrt{10}}$$

$$CI\ interval = [26 \cdot 284, 54 \cdot 34]$$

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# Empirical methods : ~~Un~~ Grouped Complete data

- Failure data → grouped, time intervals, individual observations no longer available.
- Number of units survived at ordered times: $n_1, n_2, n_3, n_4, \ldots..n_k$
  $t_1, t_2, t_3, t_4, \ldots\ldots.t_k$

$$\hat{R}(t_i) = \frac{n_i}{n}; \qquad i = 1,2,\ldots\ldots k$$

$$\hat{f}(t) = -\frac{\hat{R}(t_{i+1}) - \hat{R}(t_i)}{t_{i+1} - ti} \quad for \quad t_i < t < t_{i+1}$$

$$= \frac{n_i - n_{i+1}}{(t_{i+1} - t_i) \cdot n}$$

$$\hat{\lambda}(t) = \frac{\hat{f}(t)}{\hat{R}(t)} = \frac{n_i - n_{i+1}}{(t_{i+1} - t_i) \cdot n_i}$$

$$\widehat{MTTF} = \sum_{i=0}^{k-1} \bar{t}_i \frac{n_i - n_{i+1}}{n}$$

$$s^2 = \sum_{i=0}^{k-1} \bar{t}_i^2 \frac{n_i - n_{i+1}}{n} - (\widehat{MTTF})^2$$

$$\bar{t_i} = \frac{t_i + t_{i+1}}{2};$$

$$t_0 = 0; n_0 = 0$$

$n_i - ni_{+1}$ *is the fraction that failed in interval* $i + 1$



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# Empirical methods : **Ungrouped Censored data**

- Assume *n* units are placed on test, *r* failures occurring (or less than n).
- Single censored data on right: (test time terminated before failure).
  - when test terminated → estimated reliability curve is truncated.
  - Mean , variance etc. cannot be obtained from previous formulas.
  - Theoretical distribution provides complete picture → Subject of next class !

Lifetime distribution of censored units same as non-censored units.

- Multiply censored data :
- $t_i$ represents a failure time, $t_i^+$ represents censored (removal) time, Considered together.

$$t_1, t_2, t_{3,}^+ \dots \dots t_i, t_{i+1}^+, \dots t_n$$

**Product Limit Estimator:**

- Estimation without censoring

$$\hat{R}(t_i) = 1 - \frac{i}{n+1} = \frac{n+1-i}{n+1}$$

$$\hat{R}(t_{i-1}) = \frac{n+2-i}{n+1}$$

$$\frac{\hat{R}(ti)}{\hat{R}(t_{i-1})} = \frac{n+1-i}{n+2-i}$$

$$\hat{R}(t_i) = \frac{n+1-i}{n+2-i} \cdot \hat{R}(t_{i-1})$$

If censoring (not failure) takes place, at time $t_i$ , the reliability should not change.

$$\hat{R}(t_i^+) = \hat{R}(t_{i-1})$$

$$\hat{R}(t_i) = \left(\frac{n+1-i}{n+2-i}\right)^{\delta_i} \hat{R}(t_{i-1})$$

$$\delta_i = \begin{cases} 1\, if\ failure\ at\ time\ ti \\ 0\ if censoring\ at\ time\ ti \end{cases}$$

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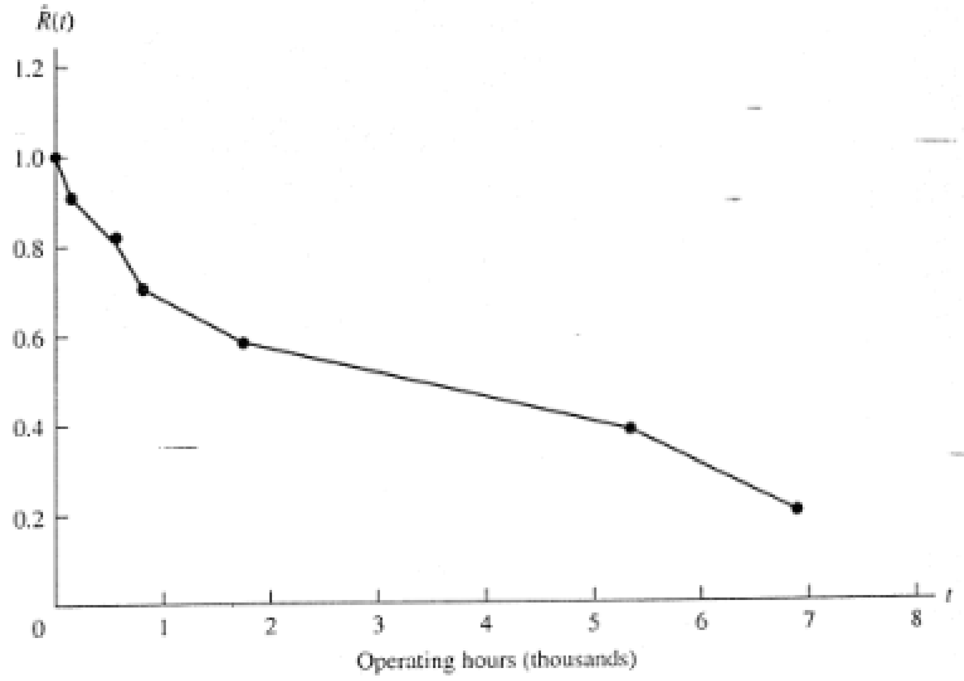
| $i$ | $t_i$ | $(11 - i)/(12 - i)$ | $\hat{R}(t_i)$ |
|---|---|---|---|
| 1 | 150 | 10/11 | $R(150) = (10/11)(1) = 0.9090$ |
| 2 | $340^+$ | 9/10 | |
| 3 | 560 | 8/9 | $R(560) = (8/9)(0.9090) = 0.8081$ |
| 4 | 800 | 7/8 | $R(800) = (7/8)(0.8081) = 0.7071$ |
| 5 | $1130^+$ | 6/7 | |
| 6 | 1720 | 5/6 | $R(1720) = (5/6)(0.7071) = 0.5892$ |
| 7 | $2470^+$ | 4/5 | |
| 8 | $4210^+$ | 3/4 | |
| 9 | 5230 | 2/3 | $R(5230) = (2/3)(0.5892) = 0.3928$ |
| 10 | 6890 | 1/2 | $R(6890) = (1/2)(0.3928) = 0.1964$ |



# Empirical methods : **Ungrouped Censored data Rank Adjustment Method (Johnson)**

- Basée sur l'ajustement de l'ordre de rang des données de défaillances.
- Ajuster le rang des i$^{ème}$ données de la i$^{ème}$ défaillance,

pour tenir compte du temps de censure qui se produit avant la i$^{ème}$ défaillance.

Une unité de censeur a une certaine probabilité de défaillance avant ou après la défaillance suivante.
ainsi, il peut influencer le classement des défaillances subséquentes.

Example:
Le premier échec a un rang (1),
mais le second échec pourrait avoir un rang (2) s’il tombe en panne avant 160,
ou un rang (3) s'il échoue après 160Hr,
donc le rang de la seconde unité doit être ajusté.

- $n$: nombre total d'unités à risque

$$Rank\ Increment = \frac{(n+1) - i_{t_{i-1}}}{1 + number\ of\ units\ after\ present\ censored\ unit}$$

- $i_{t_{i-1}}$ ordre de rang du temps de défaillance $i$-1.
- l'incrément de rang reste le même jusqu'à ce que la prochaine censure se produise.
- L'incrément de rang est recalculé pour la prochaine défaillance suivant une unité censurée.
- le rang ajusté devient :

$$i_{t_i} = i_{t_{i-1}} + rank\ increment$$

- Fiabilité estimeé:

$$R(t_i) = 1 - \frac{i_{t_i} - 0.3}{n + 0.4}$$

# Empirical methods : **Grouped Censored data**

- Grouped Censor data : Construct Life tables
- Elaborate the survival experience of items in each group
- Group Failure and censor times :
- Number of groups : $k+1$

$[t_{i-1}, t_i) \quad for \;\; i = 1,2, \ldots . k+1$

$t_0=0, \quad t_{k+1}=\infty$

- The intervals **do not need to** be equal width!!

$F_i =$ Number of failures in $i^{th}$ interval

$C_i =$ Number of removals (censored) in $i^{th}$ interval

$H_i =$ Number at risk at time $t_i$

$= H_i - F_{i-1} - C_{i-1}$

$H'_i = Hi - \frac{C_i}{2}$ Adjusted Number at risk , Assuming that censored times occur uniformly over the interval

$\frac{F_i}{H'_i}$ Conditional probability of ***failure*** in $i^{th}$ interval , given survival till time $t_{i-1}$

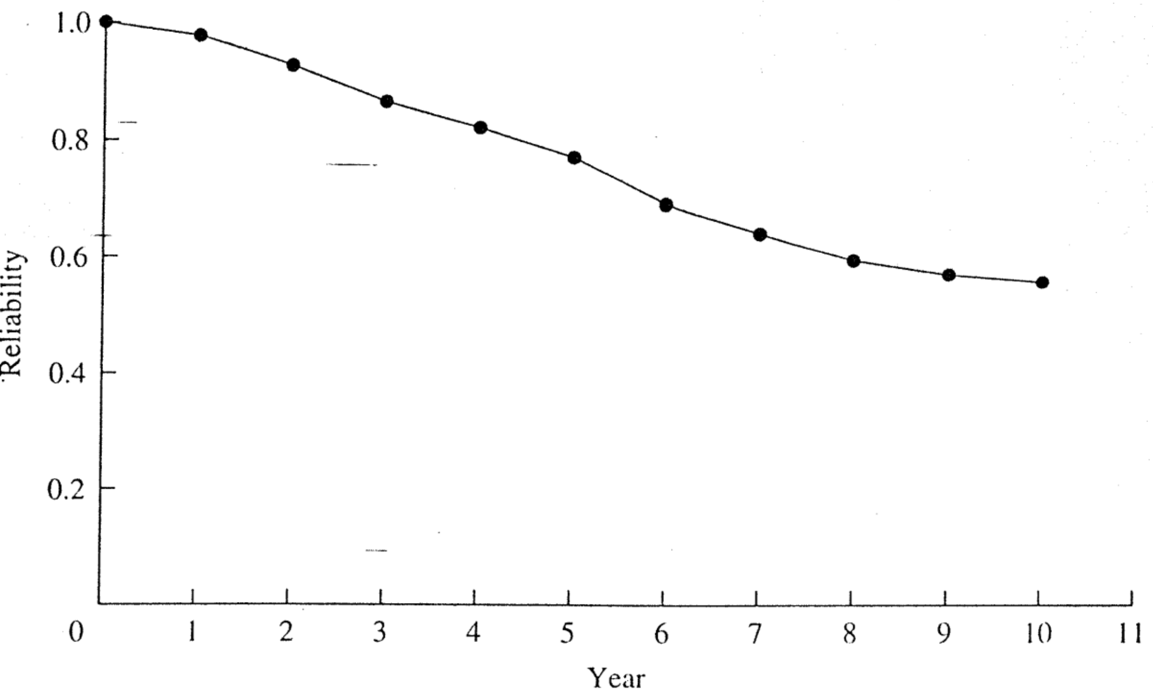
$p_i = 1 - \frac{F_i}{H'_i}$ Conditional probability of ***survival*** in $i^{th}$ interval , given survival till time $t_{i-1}$

Reliability of surviving after $i^{th}$ interval = Pr {surviving till $t_i$ given it has survived till $t_{i-1}$} X Pr {survived till $t_{i-1}$}

$$\widehat{R}_i = \left[1 - \frac{F_i}{H'_i}\right] \widehat{R_{i-1}}$$

| Year | Number of failures | Number of removals |
|---|---|---|
| 1981 | 5 | 0 |
| 1982 | 10 | 1 |
| 1983 | 12 | 5 |
| 1984 | 8 | 2 |
| 1985 | 10 | 0 |
| 1986 | 15 | 6 |
| 1987 | 9 | 3 |
| 1988 | 8 | 1 |
| 1989 | 4 | 0 |
| 1990 | 3 | 1 |

| Year | $F_i$ | $C_i$ | $H_i$ | $H'_i$ | $p_i$ | $R_i$ |
|---|---|---|---|---|---|---|
| 1 | 5 | 0 | 200 | 200 | 0.975 | 0.975 |
| 2 | 10 | 1 | 195 | 194.5 | 0.949 | 0.925 |
| 3 | 12 | 5 | 184 | 181.5 | 0.934 | 0.864 |
| 4 | 8 | 2 | 167 | 166 | 0.952 | 0.822 |
| 5 | 10 | 0 | 157 | 157 | 0.936 | 0.770 |
| 6 | 15 | 6 | 147 | 144 | 0.896 | 0.690 |
| 7 | 9 | 3 | 126 | 124.5 | 0.928 | 0.640 |
| 8 | 8 | 1 | 114 | 113.5 | 0.930 | 0.595 |
| 9 | 4 | 0 | 105 | 105 | 0.962 | 0.572 |
| 10 | 3 | 1 | 101 | 100.5 | 0.970 | 0.555 |



# Empirical methods :Static Life Estimation

Reliability estimate required for single specified point in time $t_0$.
$n$ units put on test for time $t_0$ → number of failures $r$ recorded.

$$\hat{R}(t_0) = 1 - \frac{r}{n}$$

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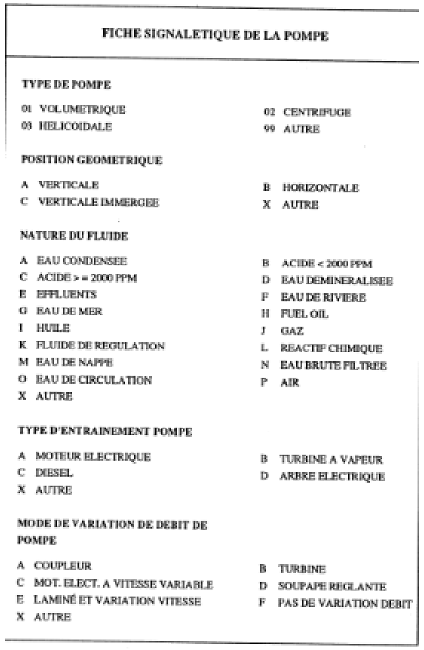

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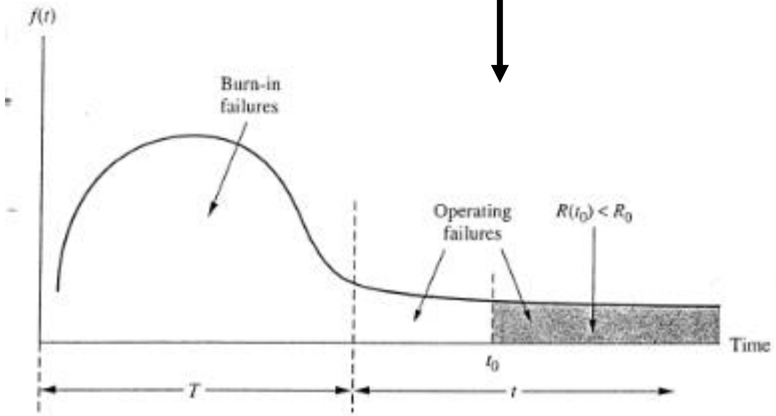

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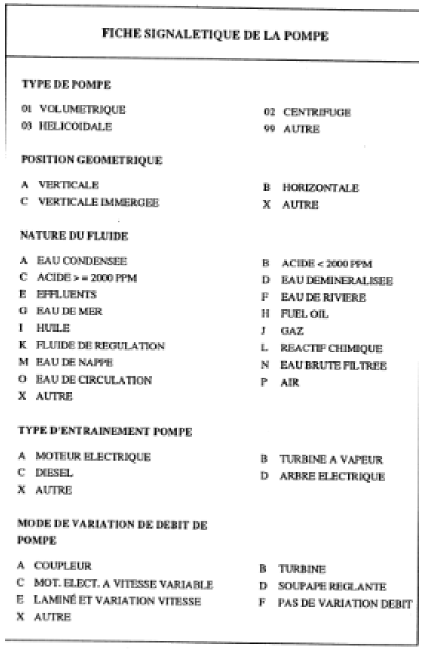

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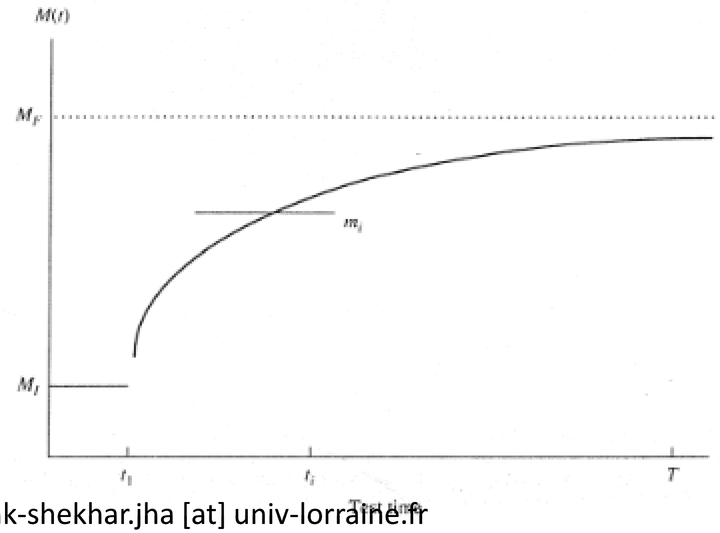


# Reliability Testing

Reliability Life Testing

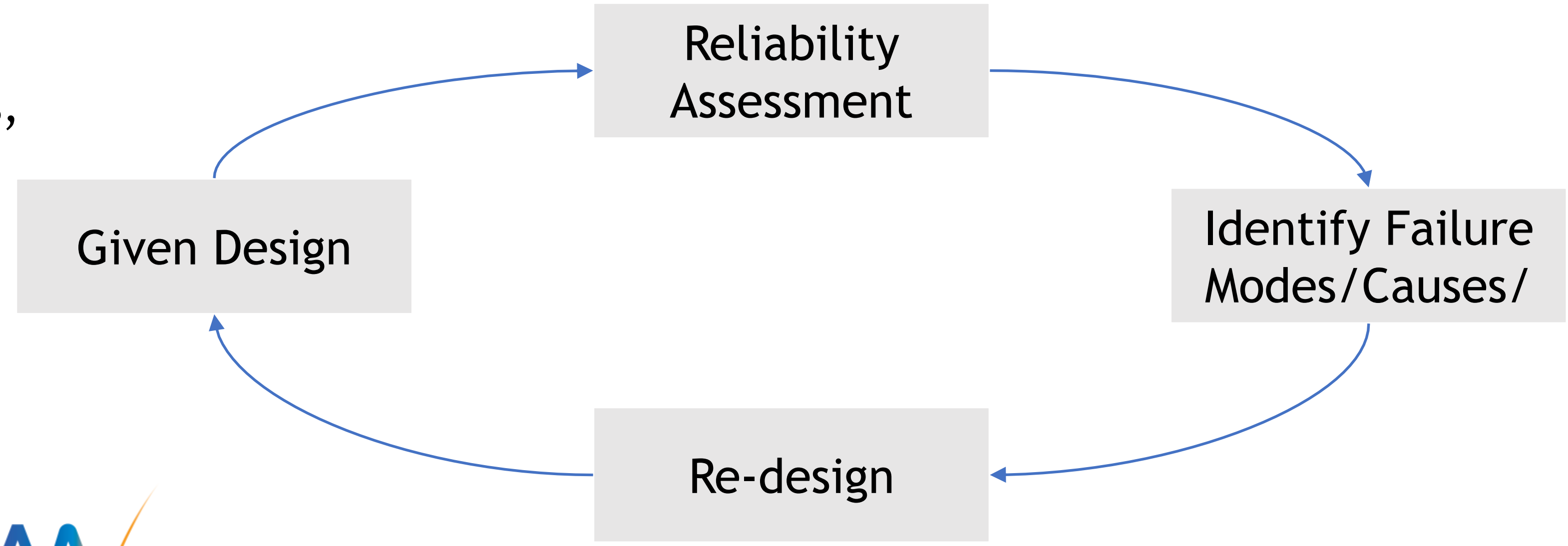
- A new product design →functional, operational tests to determine satisfaction of performance and reliability requirements.

- Reliability qualification test →measure performance and reliability under :
  - various operational conditions,
  - functional requirements.

Reliability Growth testing:

Elimination of Failure causes,

growth in reliability



# Reliability Testing

Reliability Life Testing

Objective : Failure data collection → Assess reliability and safety goals are met?

**Failure data collection , Analysis (Non-parametric methods, fit distributions**

Common types of reliability testing :

- Burn-in and screen testing:
  - eliminate infant mortality tests.
- Acceptance & Qualification test:
  - demonstration of reliability goals are met, or within acceptable standards
- Sequential tests:
  - to assess reliability goal is satisfied or not.
- Accelerated life tests:
  - Shorten the length (period) of tests.
  - Accelerating stress conditions,

# Reliability Testing

Factors to consider before any reliability test conducted:

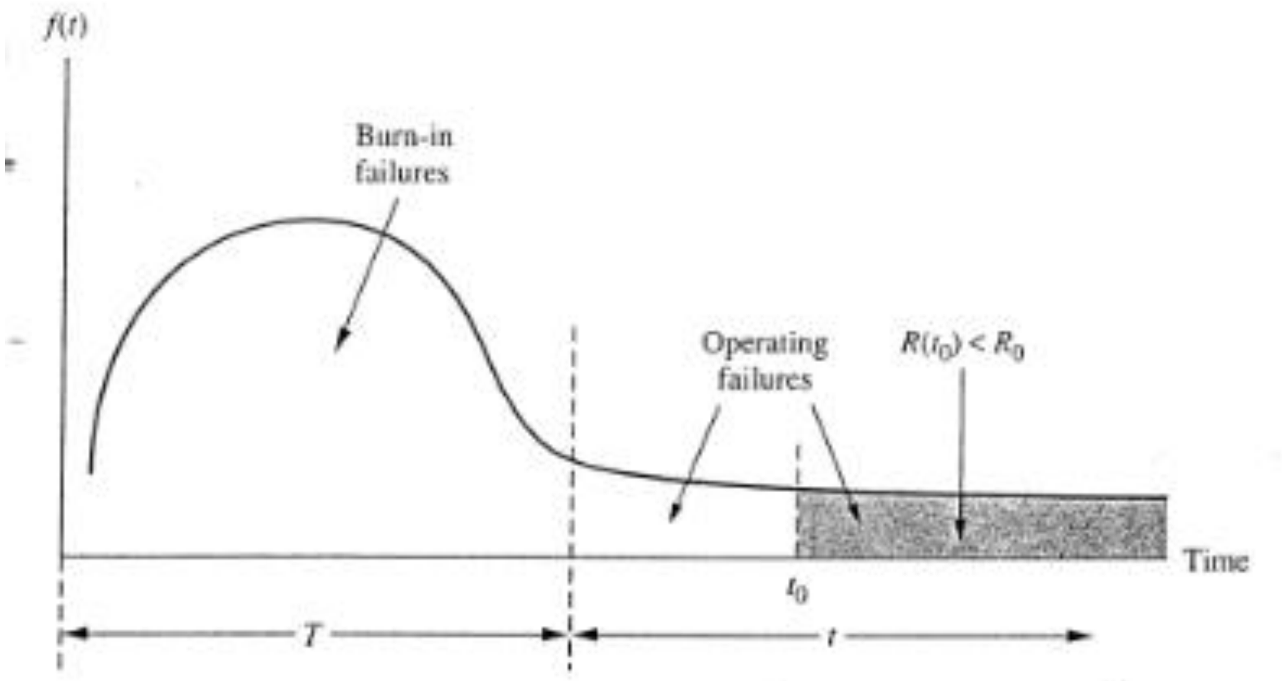
- Objective of test (fix desired reliability, design life )
- Definition of failure (threshold, symptoms)
- Type of test
- Operating and Environmental conditions (normal, accelerated)
- Duration of test
- Number of units to test

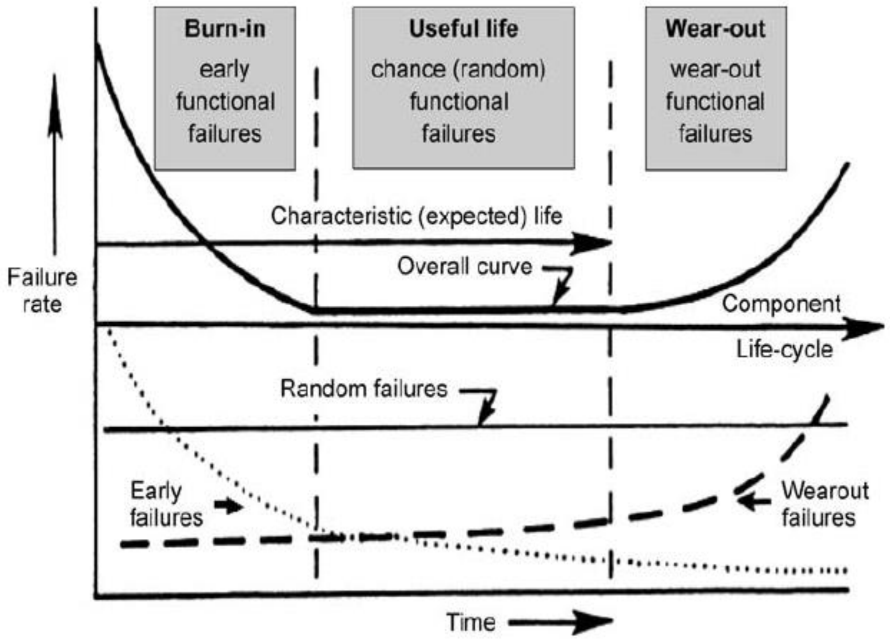
# Reliability Testing

1. Burn-in and screen testing:

- Analysis of conditional reliability (survive till time (initial) $T_0$)
- increase the mean residual life
- Decrease infant mortality , eliminate early failures, increase MTTF.
- Identify failure cause for infant mortality , redesign etc.

Economic when failure rate is DFR!!
(Decreasing Failure rate Model)





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## Burn-in and screen testing

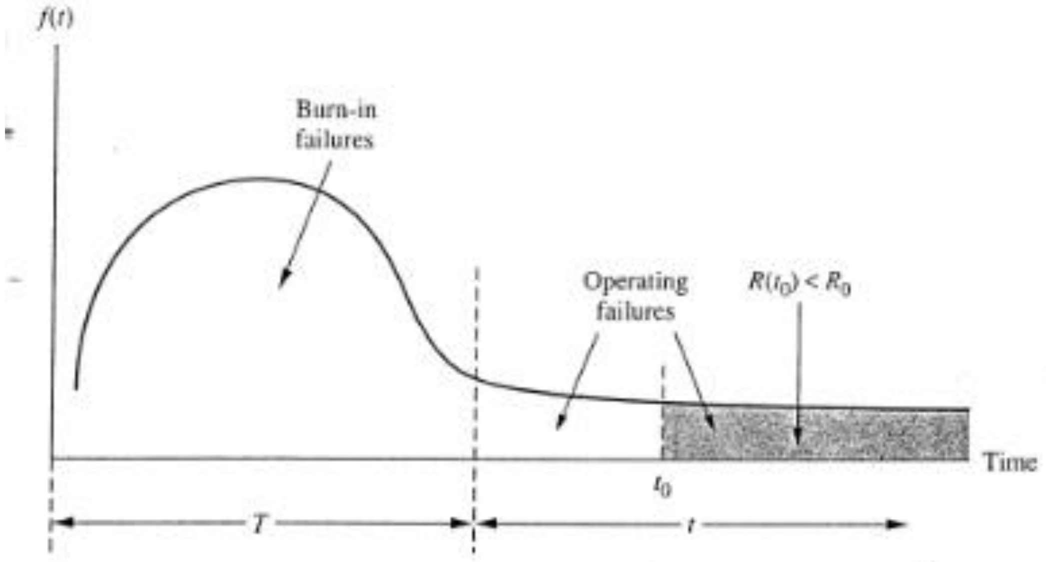
- Desired Reliability at time $t_0$ : $R_0$

with $R(t_0) < R_0$

- $R(t) \rightarrow DFR$
- A burn-in period $T$ is desired with $R(t_0|T) = R_0$

$$R(t\,|\,T_0) = \frac{e^{-(t+T_0/\alpha)^\beta}}{e^{-(T_0/\alpha)^\beta}} = \exp\left[-\left(\frac{t+T_0}{\alpha}\right)^\beta + \left(\frac{T_0}{\alpha}\right)\right]$$

$$\Rightarrow \exp\left[-\left(\frac{t+T_0}{\alpha}\right)^\beta\right] - R_0 \exp\left[-\left(\frac{T}{\alpha}\right)^\beta\right] = 0$$



# Reliability Testing

2. Acceptance Tests:

- Used for demonstrating reliability acceptance (quality satisfaction)

- Reliability satisfaction

- Pre-defined sample size, or sequential process.

- Usually, specified, environmental operating conditions.

- Based on sequential probability ratio (Wald, 1947).

# Reliability Testing

3. Sequential Tests:

- Used for demonstrating reliability acceptance (quality satisfaction)
- NOT for estimating failure rate or reliability parameters!

- Based on sequential probability ratio (Wald, 1947).

Method : Accept or Reject a specified Hypothesis

- Reliability parameter (MTTF, failure rate, failure probability..etc. ) : $\phi$
- Specification : $\phi_0$
- Unacceptable value : $\phi_1$

Statistical Hypothesis Test :

- test static computed with each sequential data

- Till, Null hypothesis is accepted or rejected.

$$t_1, t_2, t_3, \dots . t_r$$
$$y_r = h(t_1, t_2, \dots t_r)$$

$$H_0 : \phi = \phi_0$$
$$H_1 : \phi = \phi_1 > \phi_0$$

# Reliability Testing

**4 . Accelerated-life testing**

- Amount of time available for testing less than expected failure/fault time of component.
- Must accelerate the aging process to determine/identity design weakness in a given time.

- Common ways:
  - Increase number of units on test.
    - increases rate of failure
    - statistical test accomplished fast.
  - Accelerated Cycling
    - Product usage, loading conditions, cyclic usage accelerated.
    - Example: Motor usage, PEMFC,
  - Increase the stress levels that generate failures.
    - loading, demand levels, stress, force levels accelerated,
    - humidity /temperature levels of machines, radial force on bearing,

# Reliability Testing

**4 . Accelerated-life testing**

Accelerated Cycling (simplistic case)

Assumptions :

- no new failure modes are introduced due to cycling
- failures occur due to cycling only

Since, number of cycles to failure is same for both normal and accelerated conditions

Under accelerated cycling:

- only characteristic life changes,
- Weibull distribution retains its shape

numbers of cycles per unit time under normal operating conditions: $x_n$
number of cycles per unit time under accelerated conditions: $x_s$,
time to failure under $x_n$ cycles per unit time : $t_n$,
time to failure under $x_s$ cycles per unit time: $t_s$

$$x_n t_n = x_s t_s$$

$$t_s = \frac{x_n}{x_s} t_n$$

$$R_n(t_n) = R_s(t_s) = R_s\left(\frac{x_n}{x_s} t_n\right)$$

$$R_n(t_n) = \exp\left[-\left(\frac{t_n}{\alpha_n}\right)^{\beta_n}\right]$$

$$= \exp\left[-\left(\frac{t_s}{\alpha_s}\right)^{\beta_s}\right]$$

$$= \exp\left[-\left(\frac{x_n}{x_s}\frac{t_n}{\alpha_s}\right)^{\beta_s}\right]$$

$$\beta_s = \beta_n = \beta$$

$$\alpha_n = \frac{x_s}{x_n} \alpha_s$$

# Reliability Testing

## Degradation Models

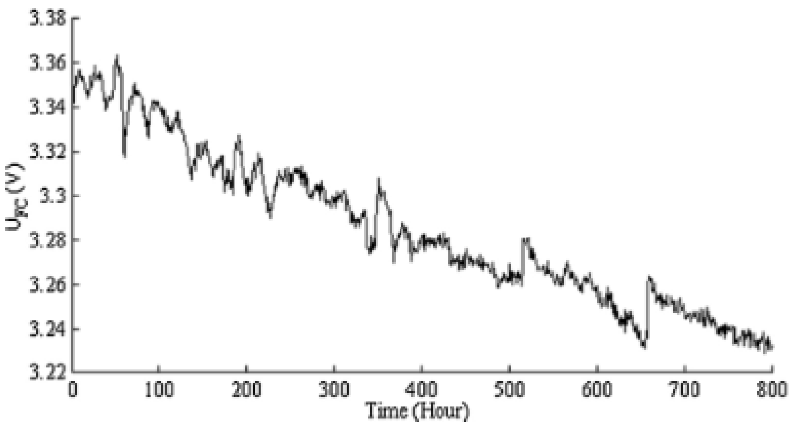
- Physics of Failure Models
  - Based on physics of degradation
    - Crack propagation, fatigue propagation on blades (Paris Models) $\frac{da}{dN} = C\Delta K^m$
    - Accelerated temperature levels : Arrhenius Model
- Data Driven Models : Fit failure data → time based function (deterioration of health as function of time)
  - Indication of system health based upon a feature, sets of features.
    - Ex: Motor ageing : Change in resistance of coil,
    - Rolling Bearing : surface wear, damage in inner/outer cages.
    - Fuel Cell : Change in resistance of membrane,

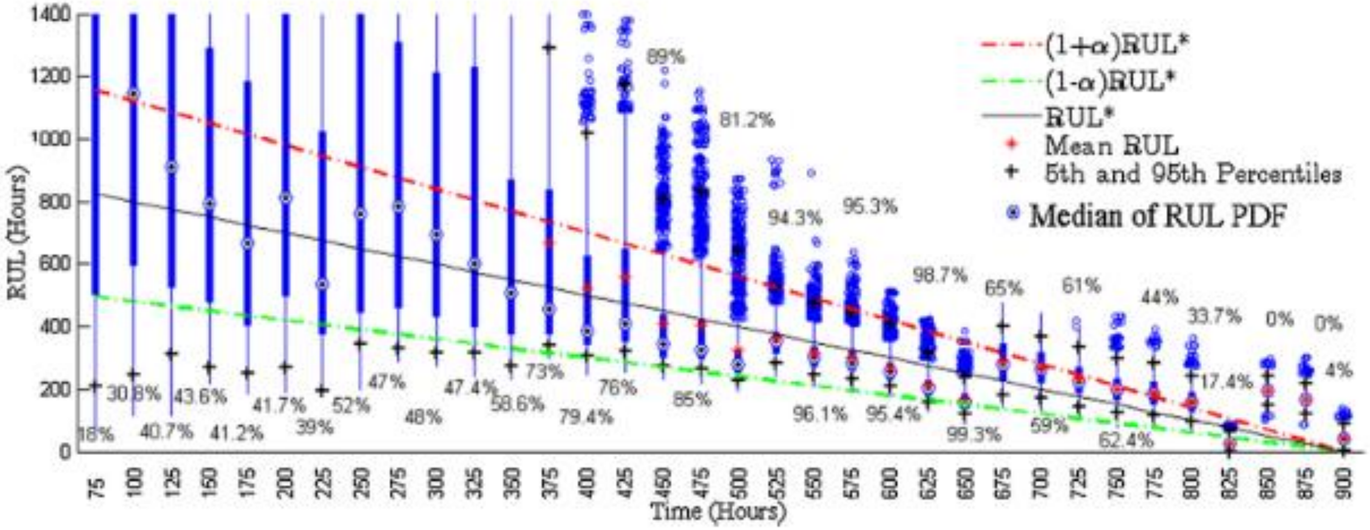
- Feature Extraction → Feature Selection → Data Fitting →Parameter Estimation of model

→ Health /Life Prediction

# Reliability Testing

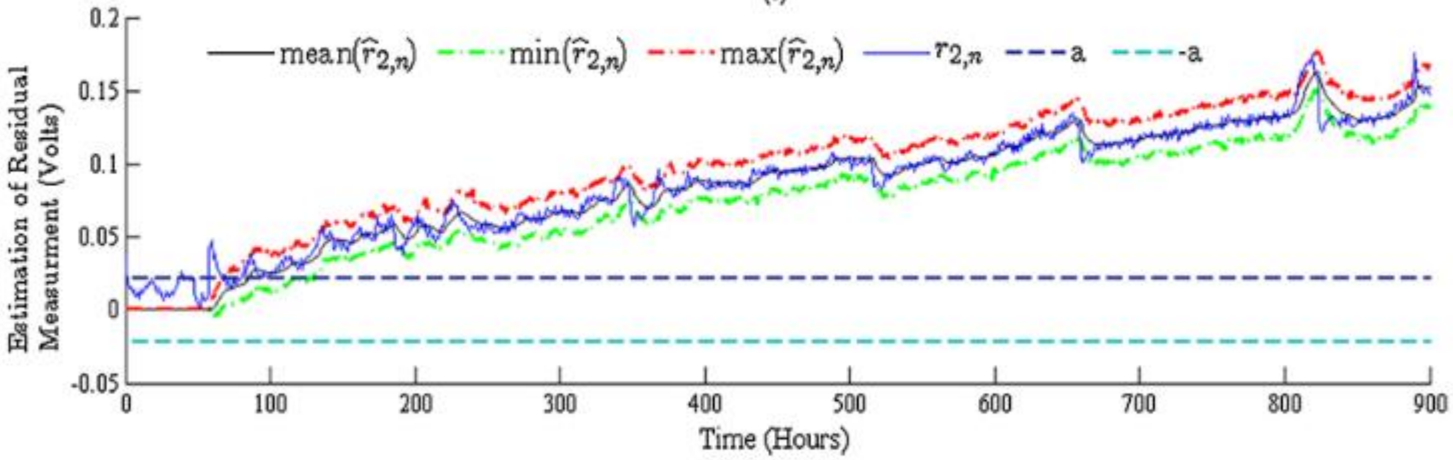
Degradation models : Fuel Cell (PEM)




Time (Hours)

(c)

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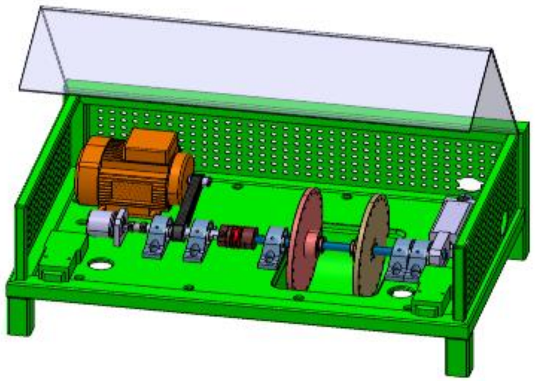
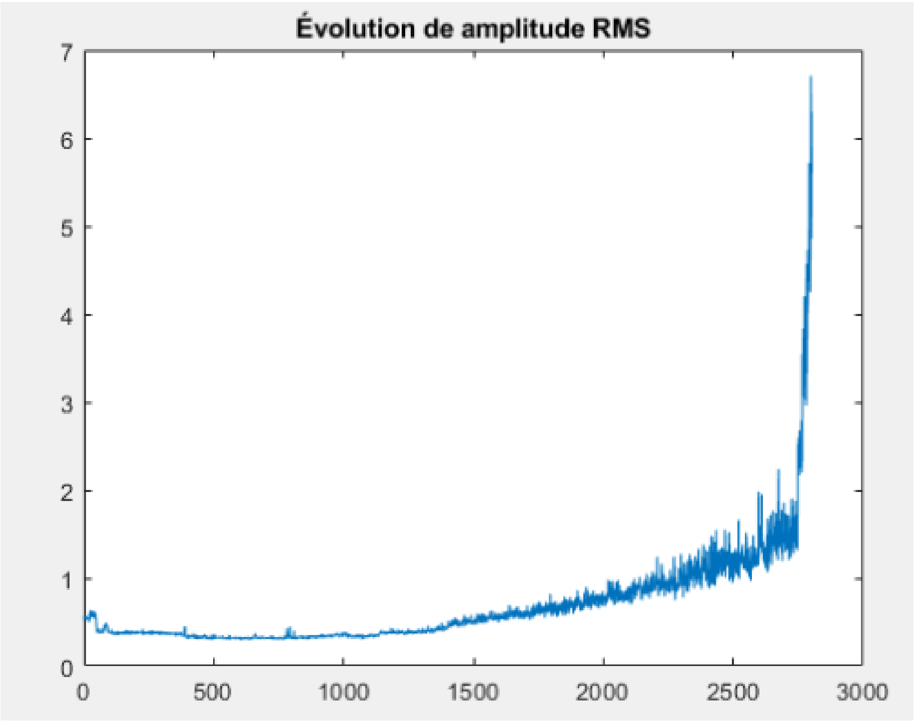
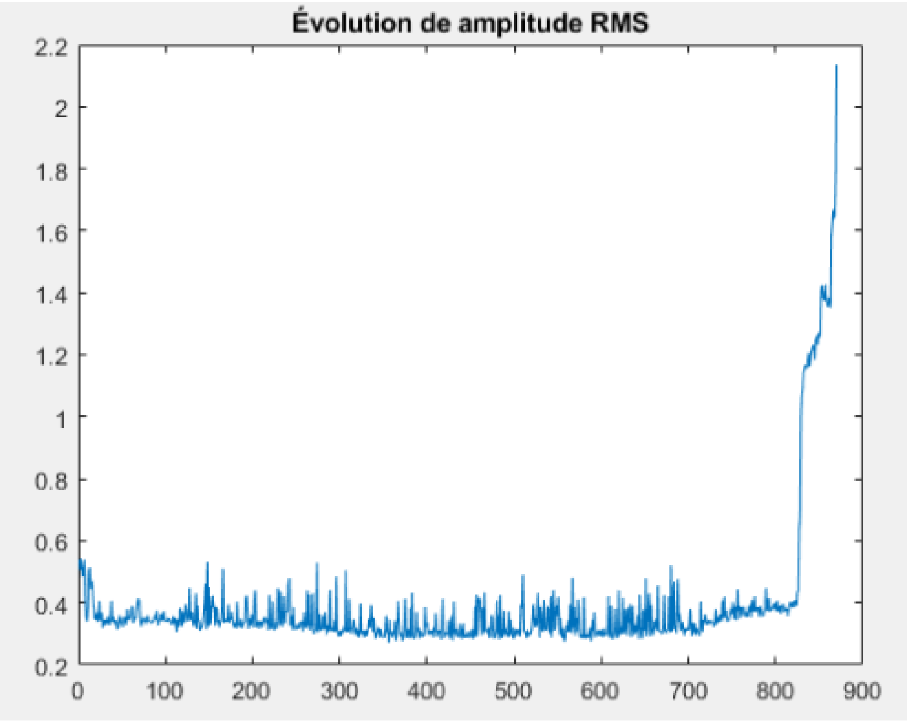
# Reliability Testing

Degradation Model (Time Domain) : Rolling Bearing under Radial force

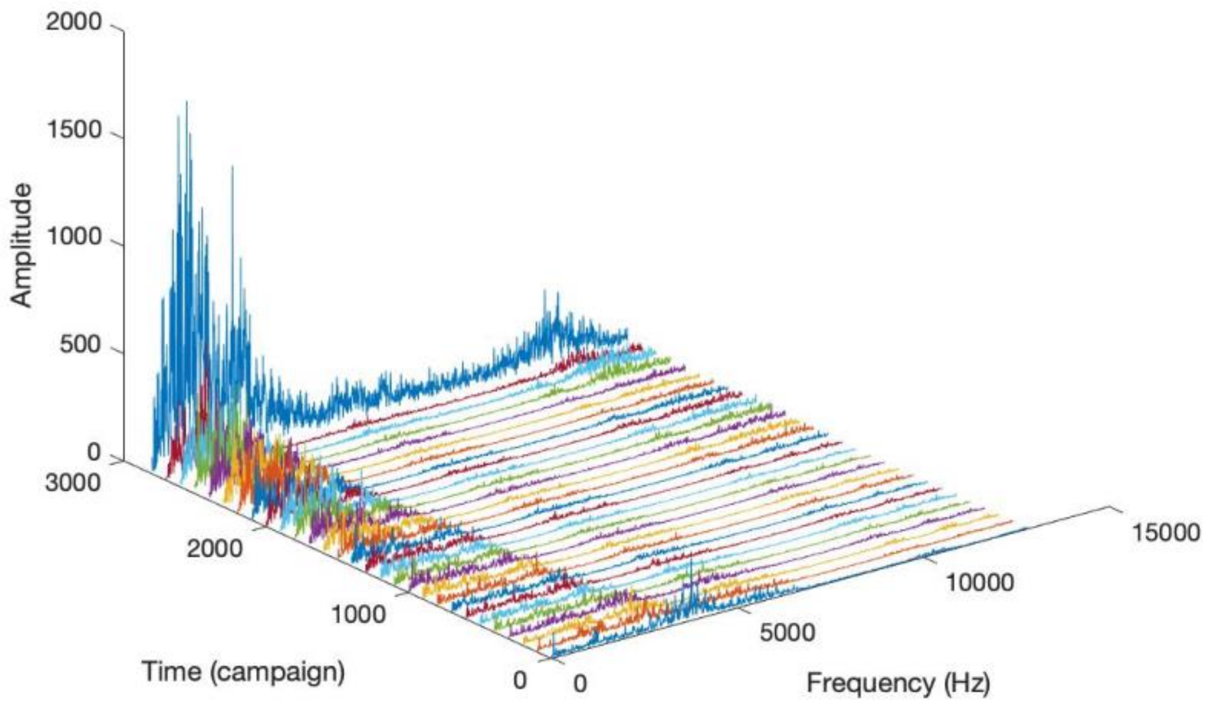




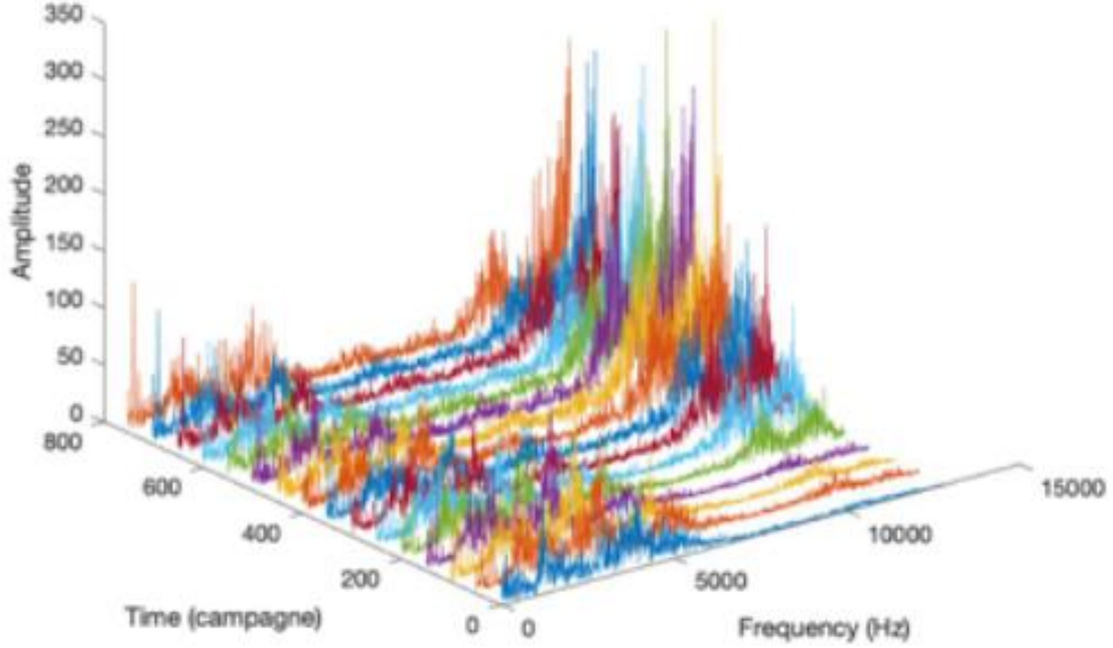



# Reliability Testing

Degradation Modelling (Frequency FFT) : Rolling Bearing under Radial force



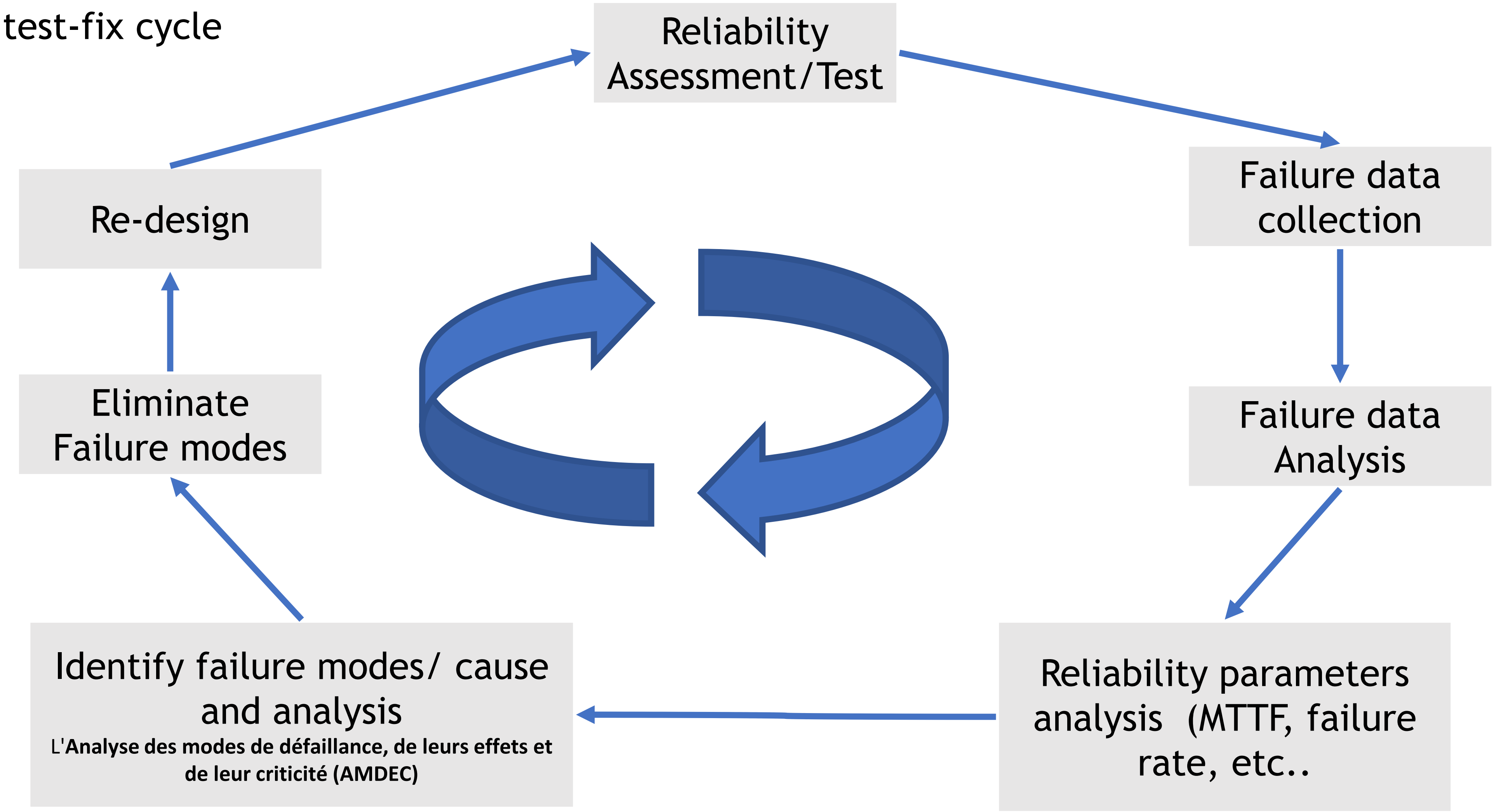

1600 N force




4000 N force

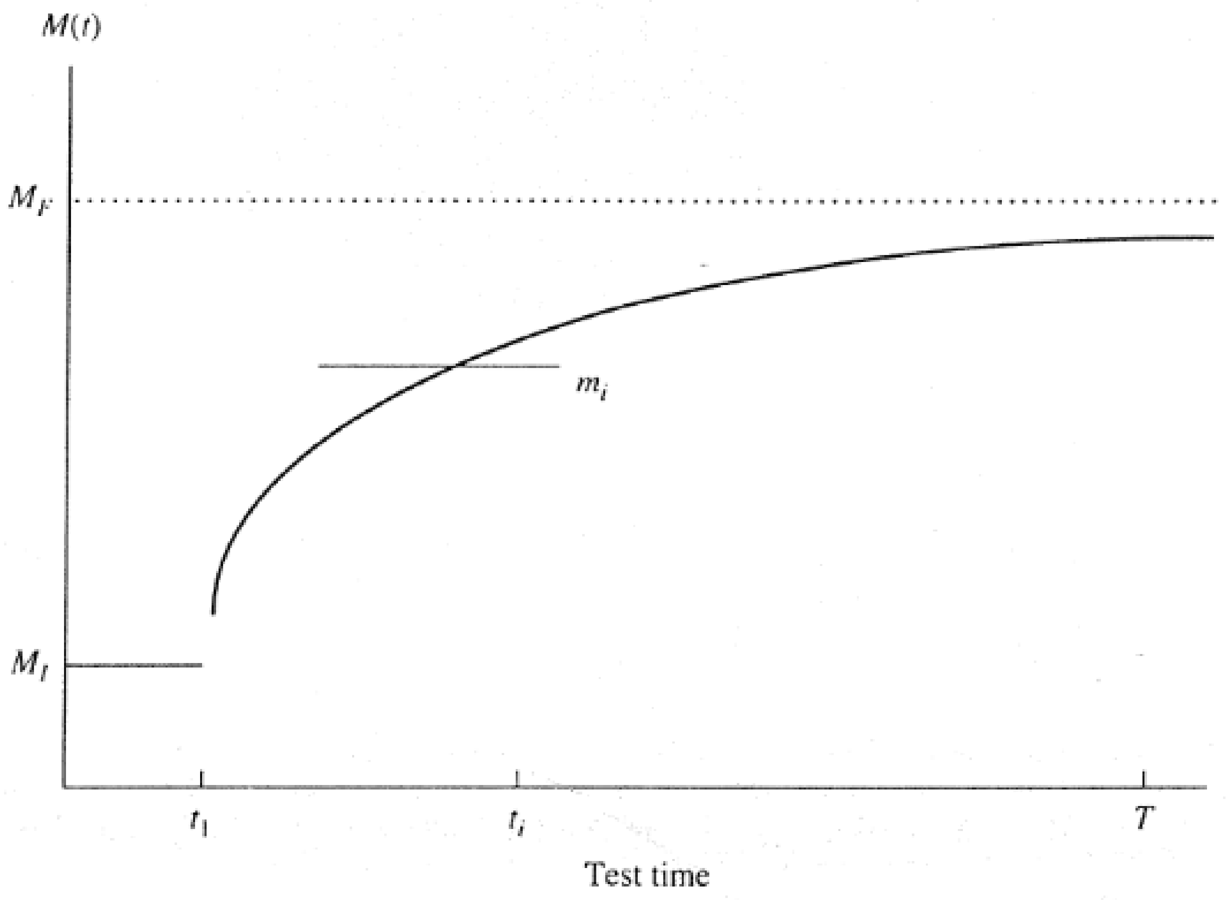
# Reliability Growth Testing

- Test-fix-test-fix cycle



# Reliability Growth Testing

- Idealized Reliability Curve



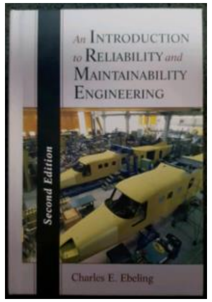



TABLE A.1

**Standardized normal probabilities:** $\Phi(z) = \int_{-\infty}^{z}(1/\sqrt{2\pi})e^{-y^2/2}\,dy$

| z | Φ(z) | 1 − Φ(z) | z | Φ(z) | 1 − Φ(z) | z | Φ(z) | 1 − Φ(z) |
|---|---|---|---|---|---|---|---|---|
| −4.00000 | 0.00003 | 0.99997 | −3.51000 | 0.00022 | 0.99978 | −3.02000 | 0.00126 | 0.99874 |
| −3.99000 | 0.00003 | 0.99997 | −3.50000 | 0.00023 | 0.99977 | −3.01000 | 0.00131 | 0.99869 |
| −3.98000 | 0.00003 | 0.99997 | −3.49000 | 0.00024 | 0.99976 | −3.00000 | 0.00135 | 0.99865 |
| −3.97000 | 0.00004 | 0.99996 | −3.48000 | 0.00025 | 0.99975 | −2.99000 | 0.00139 | 0.99861 |
| −3.96000 | 0.00004 | 0.99996 | −3.47000 | 0.00026 | 0.99974 | −2.98000 | 0.00144 | 0.99856 |
| −3.95000 | 0.00004 | 0.99996 | −3.46000 | 0.00027 | 0.99973 | −2.97000 | 0.00149 | 0.99851 |
| −3.94000 | 0.00004 | 0.99996 | −3.45000 | 0.00028 | 0.99972 | −2.96000 | 0.00154 | 0.99846 |
| −3.93000 | 0.00004 | 0.99996 | −3.44000 | 0.00029 | 0.99971 | −2.95000 | 0.00159 | 0.99841 |
| −3.92000 | 0.00004 | 0.99996 | −3.43000 | 0.00030 | 0.99970 | −2.94000 | 0.00164 | 0.99836 |
| −3.91000 | 0.00005 | 0.99995 | −3.42000 | 0.00031 | 0.99969 | −2.93000 | 0.00169 | 0.99831 |
| −3.90000 | 0.00005 | 0.99995 | −3.41000 | 0.00032 | 0.99968 | −2.92000 | 0.00175 | 0.99825 |
| −3.89000 | 0.00005 | 0.99995 | −3.40000 | 0.00034 | 0.99966 | −2.91000 | 0.00181 | 0.99819 |
| −3.88000 | 0.00005 | 0.99995 | −3.39000 | 0.00035 | 0.99965 | −2.90000 | 0.00187 | 0.99813 |
| −3.87000 | 0.00005 | 0.99995 | −3.38000 | 0.00036 | 0.99964 | −2.89000 | 0.00193 | 0.99807 |
| −3.86000 | 0.00006 | 0.99994 | −3.37000 | 0.00038 | 0.99962 | −2.88000 | 0.00199 | 0.99801 |
| −3.85000 | 0.00006 | 0.99994 | −3.36000 | 0.00039 | 0.99961 | −2.87000 | 0.00205 | 0.99795 |
| −3.84000 | 0.00006 | 0.99994 | −3.35000 | 0.00040 | 0.99960 | −2.86000 | 0.00212 | 0.99788 |
| −3.83000 | 0.00006 | 0.99994 | −3.34000 | 0.00042 | 0.99958 | −2.85000 | 0.00219 | 0.99781 |
| −3.82000 | 0.00007 | 0.99993 | −3.33000 | 0.00043 | 0.99957 | −2.84000 | 0.00226 | 0.99774 |
| −3.81000 | 0.00007 | 0.99993 | −3.32000 | 0.00045 | 0.99955 | −2.83000 | 0.00233 | 0.99767 |
| −3.80000 | 0.00007 | 0.99993 | −3.31000 | 0.00047 | 0.99953 | −2.82000 | 0.00240 | 0.99760 |
| −3.79000 | 0.00008 | 0.99992 | −3.30000 | 0.00048 | 0.99952 | −2.81000 | 0.00248 | 0.99752 |
| −3.78000 | 0.00008 | 0.99992 | −3.29000 | 0.00050 | 0.99950 | −2.80000 | 0.00255 | 0.99745 |
| −3.77000 | 0.00008 | 0.99992 | −3.28000 | 0.00052 | 0.99948 | −2.79000 | 0.00264 | 0.99736 |
| −3.76000 | 0.00008 | 0.99992 | −3.27000 | 0.00054 | 0.99946 | −2.78000 | 0.00272 | 0.99728 |
| −3.75000 | 0.00009 | 0.99991 | −3.26000 | 0.00056 | 0.99944 | −2.77000 | 0.00280 | 0.99720 |
| −3.74000 | 0.00009 | 0.99991 | −3.25000 | 0.00058 | 0.99942 | −2.76000 | 0.00289 | 0.99711 |
| −3.73000 | 0.00009 | 0.99991 | −3.24000 | 0.00060 | 0.99940 | −2.75000 | 0.00298 | 0.99702 |
| −3.72000 | 0.00010 | 0.99990 | −3.23000 | 0.00062 | 0.99938 | −2.74000 | 0.00307 | 0.99693 |
| −3.71000 | 0.00010 | 0.99990 | −3.22000 | 0.00064 | 0.99936 | −2.73000 | 0.00317 | 0.99683 |
| −3.70000 | 0.00011 | 0.99989 | −3.21000 | 0.00066 | 0.99934 | −2.72000 | 0.00326 | 0.99674 |
| −3.69000 | 0.00011 | 0.99989 | −3.20000 | 0.00069 | 0.99931 | −2.71000 | 0.00336 | 0.99664 |
| −3.68000 | 0.00012 | 0.99988 | −3.19000 | 0.00071 | 0.99929 | −2.70000 | 0.00347 | 0.99653 |
| −3.67000 | 0.00012 | 0.99988 | −3.18000 | 0.00074 | 0.99926 | −2.69000 | 0.00357 | 0.99643 |
| −3.66000 | 0.00013 | 0.99987 | −3.17000 | 0.00076 | 0.99924 | −2.68000 | 0.00368 | 0.99632 |
| −3.65000 | 0.00013 | 0.99987 | −3.16000 | 0.00079 | 0.99921 | −2.67000 | 0.00379 | 0.99621 |
| −3.64000 | 0.00014 | 0.99986 | −3.15000 | 0.00082 | 0.99918 | −2.66000 | 0.00391 | 0.99609 |
| −3.63000 | 0.00014 | 0.99986 | −3.14000 | 0.00084 | 0.99916 | −2.65000 | 0.00402 | 0.99598 |
| −3.62000 | 0.00015 | 0.99985 | −3.13000 | 0.00087 | 0.99913 | −2.64000 | 0.00415 | 0.99585 |
| −3.61000 | 0.00015 | 0.99985 | −3.12000 | 0.00090 | 0.99910 | −2.63000 | 0.00427 | 0.99573 |
| −3.60000 | 0.00016 | 0.99984 | −3.11000 | 0.00094 | 0.99906 | −2.62000 | 0.00440 | 0.99560 |
| −3.59000 | 0.00016 | 0.99984 | −3.10000 | 0.00097 | 0.99903 | −2.61000 | 0.00453 | 0.99547 |
| −3.58000 | 0.00017 | 0.99983 | −3.09000 | 0.00100 | 0.99900 | −2.60000 | 0.00466 | 0.99534 |
| −3.57000 | 0.00018 | 0.99982 | −3.08000 | 0.00103 | 0.99897 | −2.59000 | 0.00480 | 0.99520 |
| −3.56000 | 0.00019 | 0.99981 | −3.07000 | 0.00107 | 0.99893 | −2.58000 | 0.00494 | 0.99506 |
| −3.55000 | 0.00019 | 0.99981 | −3.06000 | 0.00111 | 0.99889 | −2.57000 | 0.00508 | 0.99492 |
| −3.54000 | 0.00020 | 0.99980 | −3.05000 | 0.00114 | 0.99886 | −2.56000 | 0.00523 | 0.99477 |
| −3.53000 | 0.00021 | 0.99979 | −3.04000 | 0.00118 | 0.99882 | −2.55000 | 0.00539 | 0.99461 |
| −3.52000 | 0.00022 | 0.99978 | −3.03000 | 0.00122 | 0.99878 | −2.54000 | 0.00554 | 0.99446 |

| z | Φ(z) | 1 − Φ(z) | z | Φ(z) | 1 − Φ(z) | z | Φ(z) | 1 − Φ(z) |
|---|---|---|---|---|---|---|---|---|
| −2.53000 | 0.00570 | 0.99430 | −2.03000 | 0.02118 | 0.97882 | −1.53000 | 0.06301 | 0.93699 |
| −2.52000 | 0.00587 | 0.99413 | −2.02000 | 0.02169 | 0.97831 | −1.52000 | 0.06426 | 0.93574 |
| −2.51000 | 0.00604 | 0.99396 | −2.01000 | 0.02222 | 0.97778 | −1.51000 | 0.06552 | 0.93448 |
| −2.50000 | 0.00621 | 0.99379 | −2.00000 | 0.02275 | 0.97725 | −1.50000 | 0.06681 | 0.93319 |
| −2.49000 | 0.00639 | 0.99361 | −1.99000 | 0.02330 | 0.97670 | −1.49000 | 0.06811 | 0.93189 |
| −2.48000 | 0.00657 | 0.99343 | −1.98000 | 0.02385 | 0.97615 | −1.48000 | 0.06944 | 0.93056 |
| −2.47000 | 0.00676 | 0.99324 | −1.97000 | 0.02442 | 0.97558 | −1.47000 | 0.07078 | 0.92922 |
| −2.46000 | 0.00695 | 0.99305 | −1.96000 | 0.02500 | 0.97500 | −1.46000 | 0.07214 | 0.92786 |
| −2.45000 | 0.00714 | 0.99286 | −1.95000 | 0.02559 | 0.97441 | −1.45000 | 0.07353 | 0.92647 |
| −2.44000 | 0.00734 | 0.99266 | −1.94000 | 0.02619 | 0.97381 | −1.44000 | 0.07493 | 0.92507 |
| −2.43000 | 0.00755 | 0.99245 | −1.93000 | 0.02680 | 0.97320 | −1.43000 | 0.07636 | 0.92364 |
| −2.42000 | 0.00776 | 0.99224 | −1.92000 | 0.02743 | 0.97257 | −1.42000 | 0.07780 | 0.92220 |
| −2.41000 | 0.00798 | 0.99202 | −1.91000 | 0.02807 | 0.97193 | −1.41000 | 0.07927 | 0.92073 |
| −2.40000 | 0.00820 | 0.99180 | −1.90000 | 0.02872 | 0.97128 | −1.40000 | 0.08076 | 0.91924 |
| −2.39000 | 0.00842 | 0.99158 | −1.89000 | 0.02938 | 0.97062 | −1.39000 | 0.08226 | 0.91774 |
| −2.38000 | 0.00866 | 0.99134 | −1.88000 | 0.03005 | 0.96995 | −1.38000 | 0.08379 | 0.91621 |
| −2.37000 | 0.00889 | 0.99111 | −1.87000 | 0.03074 | 0.96926 | −1.37000 | 0.08534 | 0.91466 |
| −2.36000 | 0.00914 | 0.99086 | −1.86000 | 0.03144 | 0.96856 | −1.36000 | 0.08691 | 0.91309 |
| −2.35000 | 0.00939 | 0.99061 | −1.85000 | 0.03216 | 0.96784 | −1.35000 | 0.08851 | 0.91149 |
| −2.34000 | 0.00964 | 0.99036 | −1.84000 | 0.03288 | 0.96712 | −1.34000 | 0.09012 | 0.90988 |
| −2.33000 | 0.00990 | 0.99010 | −1.83000 | 0.03362 | 0.96638 | −1.33000 | 0.09176 | 0.90824 |
| −2.32000 | 0.01017 | 0.98983 | −1.82000 | 0.03438 | 0.96562 | −1.32000 | 0.09342 | 0.90658 |
| −2.31000 | 0.01044 | 0.98956 | −1.81000 | 0.03515 | 0.96485 | −1.31000 | 0.09510 | 0.90490 |
| −2.30000 | 0.01072 | 0.98928 | −1.80000 | 0.03593 | 0.96407 | −1.30000 | 0.09680 | 0.90320 |
| −2.29000 | 0.01101 | 0.98899 | −1.79000 | 0.03673 | 0.96327 | −1.29000 | 0.09853 | 0.90147 |
| −2.28000 | 0.01130 | 0.98870 | −1.78000 | 0.03754 | 0.96246 | −1.28000 | 0.10027 | 0.89973 |
| −2.27000 | 0.01160 | 0.98840 | −1.77000 | 0.03836 | 0.96164 | −1.27000 | 0.10204 | 0.89796 |
| −2.26000 | 0.01191 | 0.98809 | −1.76000 | 0.03920 | 0.96080 | −1.26000 | 0.10383 | 0.89617 |
| −2.25000 | 0.01222 | 0.98778 | −1.75000 | 0.04006 | 0.95994 | −1.25000 | 0.10565 | 0.89435 |
| −2.24000 | 0.01255 | 0.98745 | −1.74000 | 0.04093 | 0.95907 | −1.24000 | 0.10749 | 0.89251 |
| −2.23000 | 0.01287 | 0.98713 | −1.73000 | 0.04182 | 0.95818 | −1.23000 | 0.10935 | 0.89065 |
| −2.22000 | 0.01321 | 0.98679 | −1.72000 | 0.04272 | 0.95728 | −1.22000 | 0.11123 | 0.88877 |
| −2.21000 | 0.01355 | 0.98645 | −1.71000 | 0.04363 | 0.95637 | −1.21000 | 0.11314 | 0.88686 |
| −2.20000 | 0.01390 | 0.98610 | −1.70000 | 0.04457 | 0.95543 | −1.20000 | 0.11507 | 0.88493 |
| −2.19000 | 0.01426 | 0.98574 | −1.69000 | 0.04551 | 0.95449 | −1.19000 | 0.11702 | 0.88298 |
| −2.18000 | 0.01463 | 0.98537 | −1.68000 | 0.04648 | 0.95352 | −1.18000 | 0.11900 | 0.88100 |
| −2.17000 | 0.01500 | 0.98500 | −1.67000 | 0.04746 | 0.95254 | −1.17000 | 0.12100 | 0.87900 |
| −2.16000 | 0.01539 | 0.98461 | −1.66000 | 0.04846 | 0.95154 | −1.16000 | 0.12302 | 0.87698 |
| −2.15000 | 0.01578 | 0.98422 | −1.65000 | 0.04947 | 0.95053 | −1.15000 | 0.12507 | 0.87493 |
| −2.14000 | 0.01618 | 0.98382 | −1.64000 | 0.05050 | 0.94950 | −1.14000 | 0.12714 | 0.87286 |
| −2.13000 | 0.01659 | 0.98341 | −1.63000 | 0.05155 | 0.94845 | −1.13000 | 0.12924 | 0.87076 |
| −2.12000 | 0.01700 | 0.98300 | −1.62000 | 0.05262 | 0.94738 | −1.12000 | 0.13136 | 0.86864 |
| −2.11000 | 0.01743 | 0.98257 | −1.61000 | 0.05370 | 0.94630 | −1.11000 | 0.13350 | 0.86650 |
| −2.10000 | 0.01786 | 0.98214 | −1.60000 | 0.05480 | 0.94520 | −1.10000 | 0.13567 | 0.86433 |
| −2.09000 | 0.01831 | 0.98169 | −1.59000 | 0.05592 | 0.94408 | −1.09000 | 0.13786 | 0.86214 |
| −2.08000 | 0.01876 | 0.98124 | −1.58000 | 0.05705 | 0.94295 | −1.08000 | 0.14007 | 0.85993 |
| −2.07000 | 0.01923 | 0.98077 | −1.57000 | 0.05821 | 0.94179 | −1.07000 | 0.14231 | 0.85769 |
| −2.06000 | 0.01970 | 0.98030 | −1.56000 | 0.05938 | 0.94062 | −1.06000 | 0.14457 | 0.85543 |
| −2.05000 | 0.02018 | 0.97982 | −1.55000 | 0.06057 | 0.93943 | −1.05000 | 0.14686 | 0.85314 |
| −2.04000 | 0.02067 | 0.97933 | −1.54000 | 0.06178 | 0.93822 | −1.04000 | 0.14917 | 0.85083 |

# Annex : Student *t* distribution Chart

**TABLE A.2**
**Critical *t* values with *ν* degrees of freedom**

| *ν* | α 0.100 | 0.050 | 0.025 | 0.010 | 0.005 |
|---|---|---|---|---|---|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 1.886 | 2.920 | 4.303 | 6.695 | 9.925 |
| 3 | 1.639 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.799 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| ∞ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |