Sûreté de fonctionnement & Retour d'Expériences

Dependability and Feedback Data Collection

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Major reliability distributions

Time to failure is a random event →Probability distribution

Discrete or Continuous ??

Constant failure rate models

- Exponential Reliability function (cont.)
- Poisson process (**discrete**)

Time dependent failure rate models

- Weibull distribution (cont.)
- Normal distribution (cont.)
- Lognormal distribution (cont.)

Discrete

RVs can take discrete values, countable, …

Ex:

- number of demands of failure,
- Analyze one-shot systems
- Number of successful launches of a missile out of '*n*' launches.

Continuous

RVs can take continuous values, not constrained to distinct discrete values,

Ex:

- time to failure over an interval
- Successful launch of missile depends on
  - Age,
  - Time spent in storage…

Problem has to be treated as continuous one.

Constant rate : Exponential reliability Function

- Constant failure rate (CFR) → exponential probability distribution
- Most common failure model :
  - failure due to random events,
  - Prevalent during 'useful' life of component.

Assume : $\lambda(t) = \lambda,\ t \geq 0, \lambda > 0$

$$R(t) = \exp\left(-\int_0^t \lambda(u)du \right)$$

$$= \exp(-\lambda t),\ t \geq 0$$

$$F(t) = 1 - \exp(-\lambda t)$$

$$f(t) = -\frac{dR(t)}{dt} = \lambda \exp(-\lambda t)$$

$$\text{MTTF} = \int_0^\infty \exp(-\lambda t)\, dt = \left.\frac{\exp(-\lambda t)}{-\lambda}\right|_0^\infty = \frac{1}{\lambda}$$

$$Var, \boldsymbol{\sigma}^2 = \int_0^\infty \left(t - \frac{1}{\lambda} \right)^2 \lambda \exp(-\lambda t) dt = \frac{1}{\lambda^2}$$

$$Standard\ deviation, \boldsymbol{\sigma} = \frac{1}{\lambda} = \text{MTTF}$$

- Implies: variability of failure times increases as MTTF decreases
  - often , observed in practice.
- Also, $R(t = MTTF) = \exp - \left(\frac{\text{MTTF}}{\text{MTTF}} \right) = e^{-1} = 0.368$
  - implies: component having CFR has slightly more than 'one-third ' of chance to survive its MTTF.

Exponential reliability Function

Some properties:

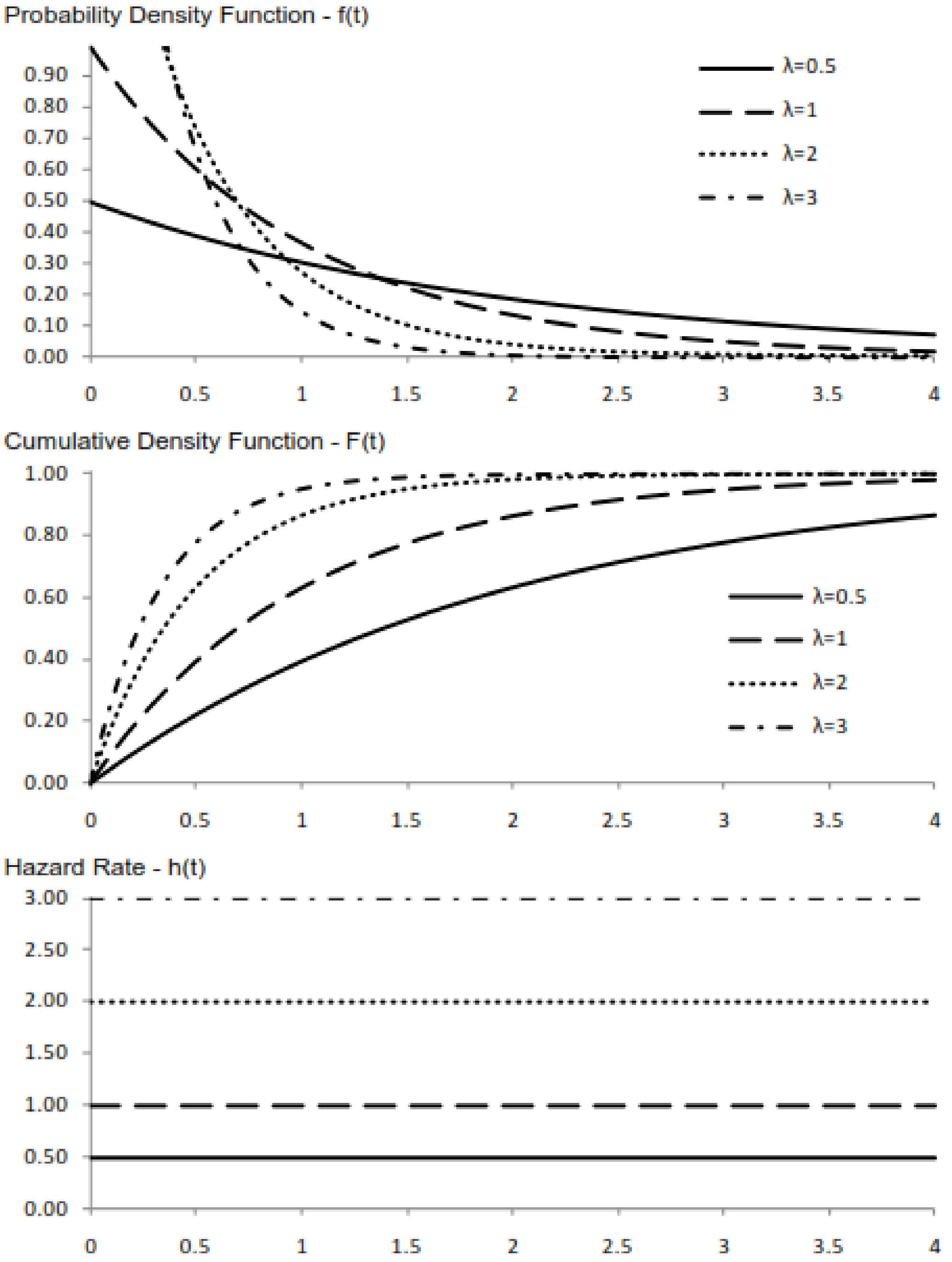
- Memory lessness: *T* (time to failure) is independent of how long the component has operated !!

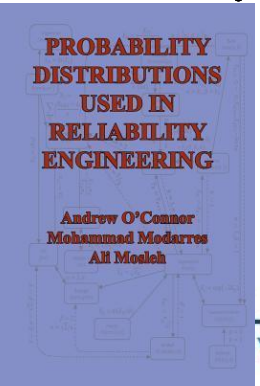
    - probability that component operates for next 1000 Hrs is same if component is new, aged, or already operated for 1000 Hrs !!

    - CFR does not take into account age, degradation, wear etc.
    - complete random failure nature.

$$R(t\,|\,x) = \frac{R(t+x)}{R(x)} = \frac{\exp\left(-\lambda(t+x)\right)}{\exp\left(-\lambda x\right)} = \exp\left(-\lambda t\right) = R(t)$$

- *T* depends on length of operating time not current age.





Two parameter exponential distribution

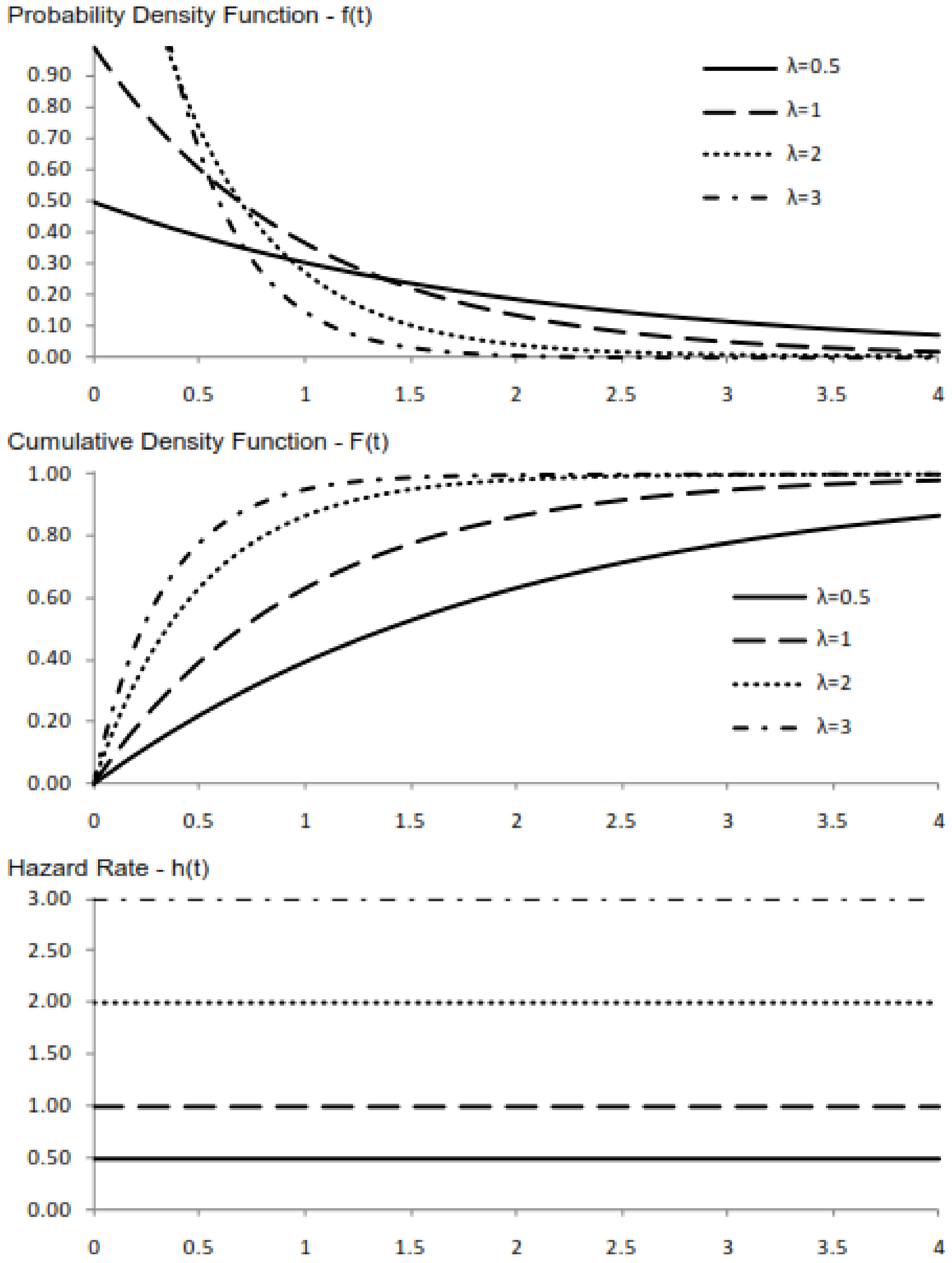
- If failure never occurs before time t_0, then t_0 is minimum time = guaranteed lifetime.

- *then* , t_0 shifts the distribution on right of x axis.

$$R(t) = \exp(-\lambda(t - t_0)),\ 0 < t_0 \le t < \infty$$

$$f(t) = -\frac{dR(t)}{dt} = \lambda \exp(-\lambda(t - t_0))$$

$$\mathrm{MTTF} = \frac{1}{\lambda} + t_0$$



Constant rate: Poisson Process

If,

- component having constant failure rate λ is immediately repaired,
- replaced upon failing,

then, **number of failures observed** over time interval $(0,t)$ -→ ***Poison distribution***

$$p_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, n = 0,1,2$$

- *Posisson distribution* is discrete distribution (not continuous like before!)

- Consider Z_i as random variable , time between failure *i-1* and *i* , with exponential distribution with failure rate λ .
- Then, time of kth failure, Y_k is sum of *k* exponential random variables:
- Y_k → Gamma Distribution (see Probability course) ,

then CDF: (k^{th} failure will occur by time *t*)

$$Y_k = \sum_{i=1}^{k} T_i$$

$$\text{mean } Y_k = \frac{k}{\lambda}; \text{var } Y_k = \frac{k}{\lambda^2}$$

$$\Pr\{Y_k \le t\} = F_{Y_k}(t) = 1 - e^{-\lambda t}\sum_{i=0}^{k-1}\frac{(\lambda t)^i}{i!}$$

$$P_n(t) = \Pr\{Y_n \le t\} - \Pr\{Y_{n+1} \le t\} = F_{Y_n}(t) - F_{Y_{n+1}}(t)$$

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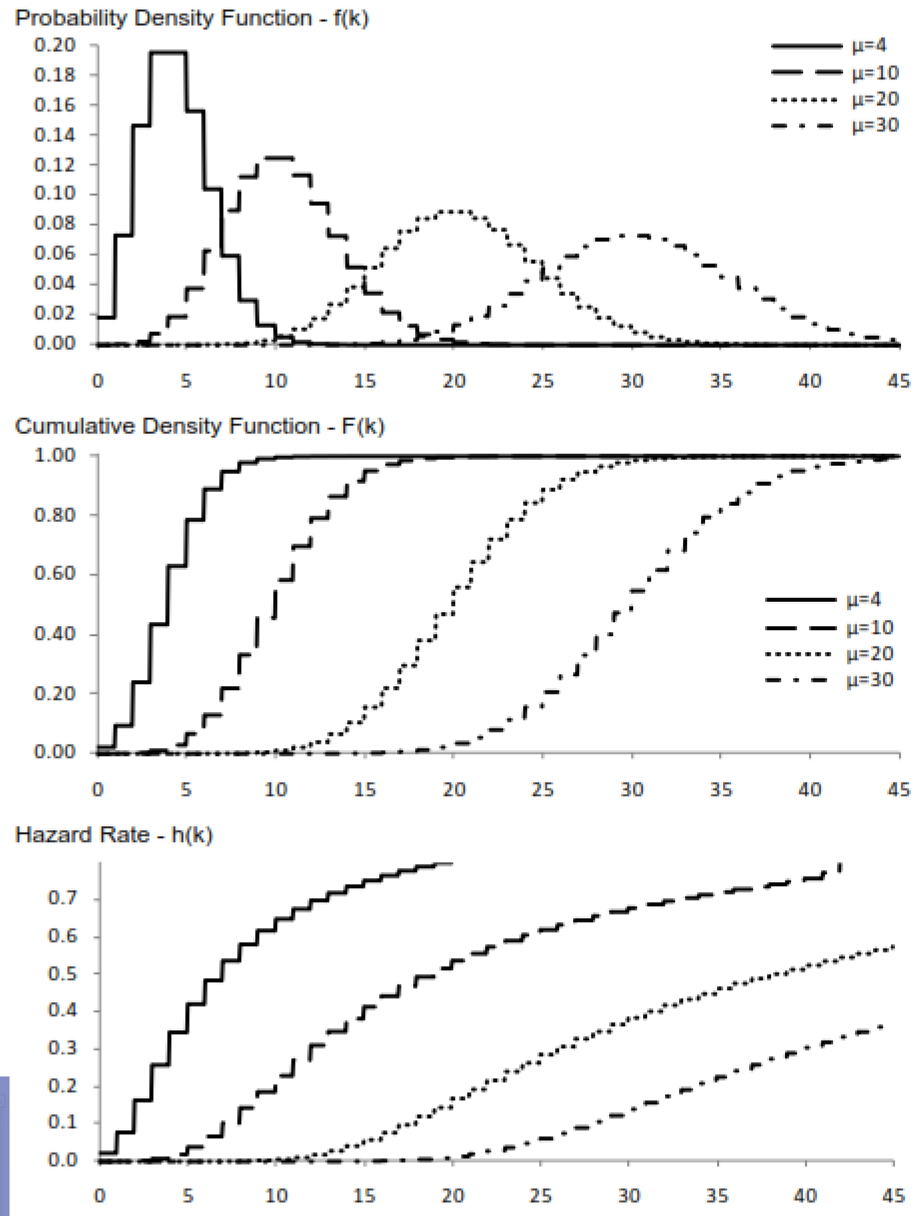
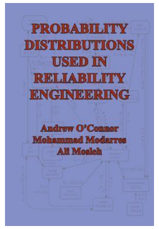
Remarks:

- What is the probability of having no failure in time t ?? $\Pr\{Z \geq t\}$

$$p_0(t) = \frac{e^{-\lambda t}(\lambda t)^0}{0!} = e^{-\lambda t} = R(t)$$

- Poisson process used in **inventory analysis** to determine number of spare components when time between failures is exponential.





Time dependent Failure models : Weibull distribution

- used to model : increasing failure rate as well as decreasing failure rates.
- Characterized by hazard (failure) rate function as: $\lambda(t) = at^b$

increasing $a > 0, b > 0$

decreasing $a > 0, b < 0$

- Mathematical convenience:

$$\lambda(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} ;\beta,\alpha > 0, t \geq 0$$

$$R(t) = \exp\left[-\int_0^t \frac{\beta}{\alpha}\left(\frac{u}{\alpha}\right)^{\beta-1} du\right] = e^{-(t/\alpha)^\beta}$$

$$f(t) = -\frac{dR(t)}{dt} = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-(t/\alpha)^\beta}$$

$$MTTF : \alpha\,\Gamma\left(1+\frac{1}{\beta}\right)$$

$$\sigma^2 = \alpha^2\left\{\Gamma\left(1+\frac{2}{\beta}\right)-\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right\}$$

$$with\ \Gamma(x) = \int_0^\infty y^{x-1}e^{-y}dy$$

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Weibull distribution

β → shape parameter.

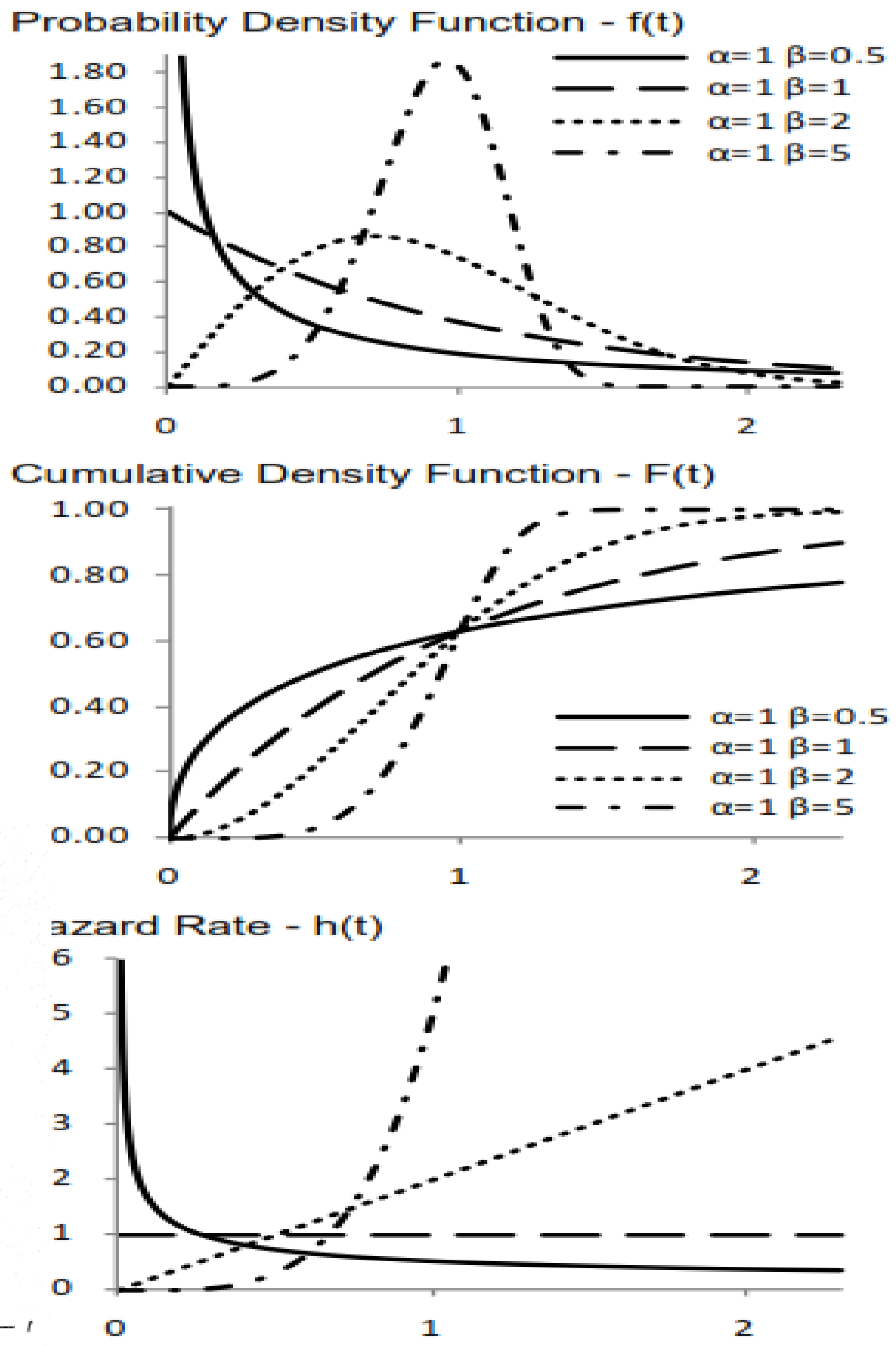
For :

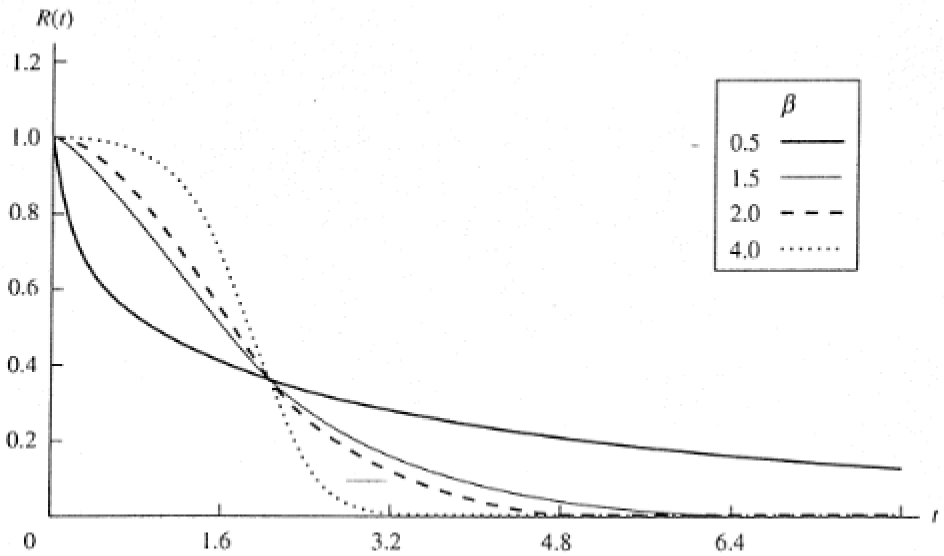
- for $\beta < 1$ PDF shape is similar to Exponential
- for larger values , Ex: $\beta > 3$ symmetrical shape like

Normal Distribution.

- For $1 < \beta < 3$ PDF is skewed.

- For $\beta = 1$, $\lambda(t) = \dfrac{1}{\alpha}$, a constant , distribution →exponential





Weibull distribution

β → shape parameter.

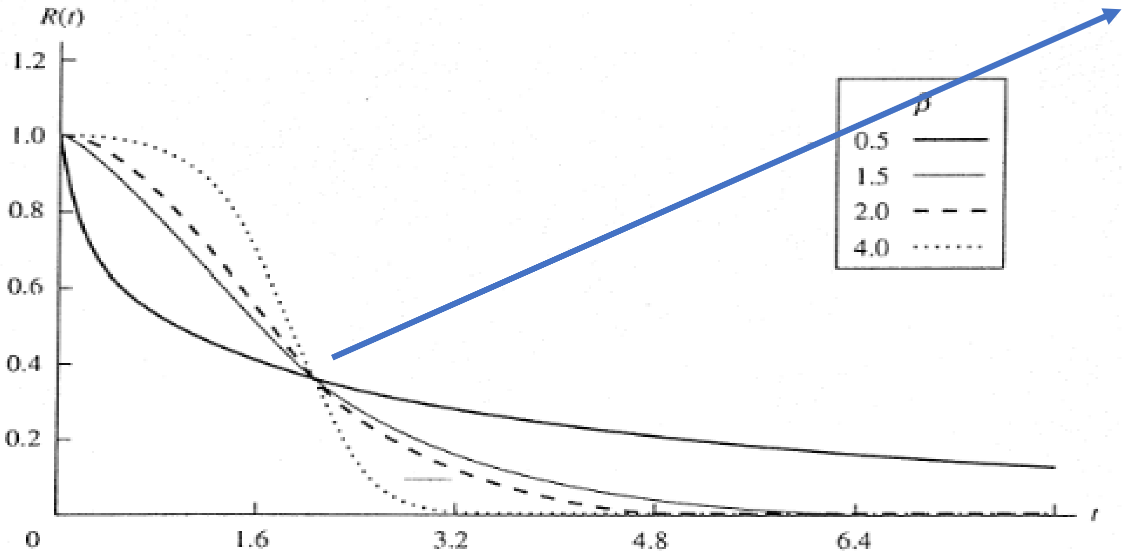
When $t = \alpha$:

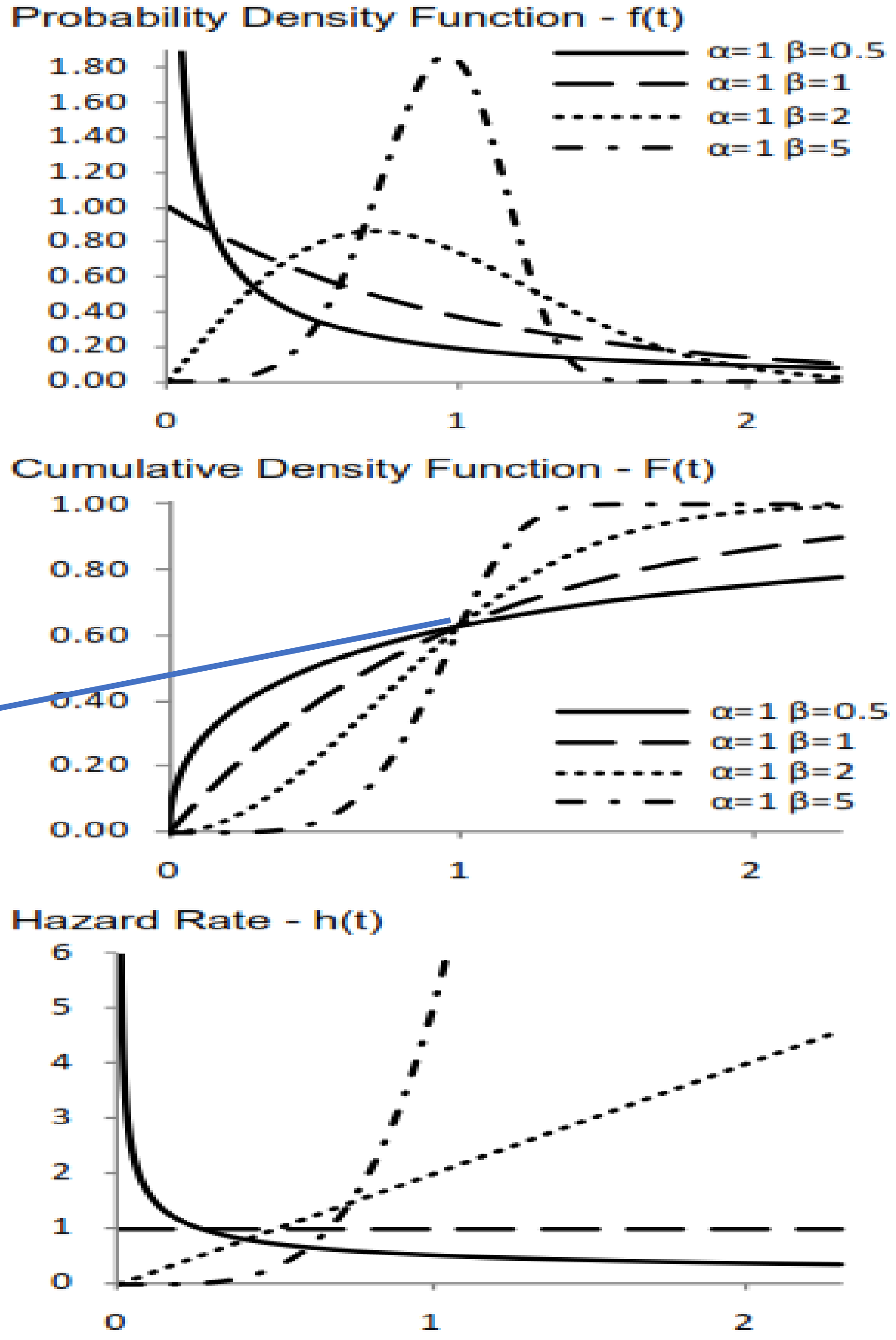
$$R(t=\alpha) = e^{-(t/\alpha)^{\beta}} = e^{-(\alpha/\alpha)^{\beta}} = e^{-1} = 0.368$$

- 63.2% of Weibull failures occur by time $t = \alpha$

regardless of the value of shape parameter β

CDF and reliability curves pass through the same point where $t = \alpha$





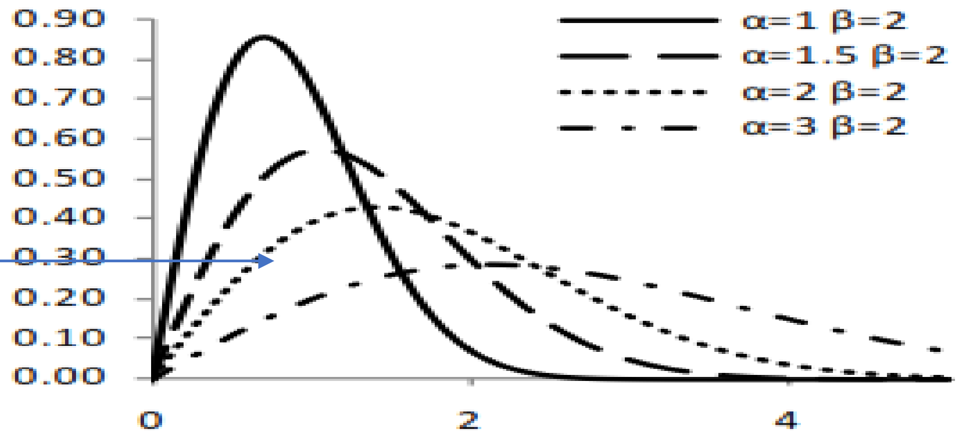
Weibull distribution

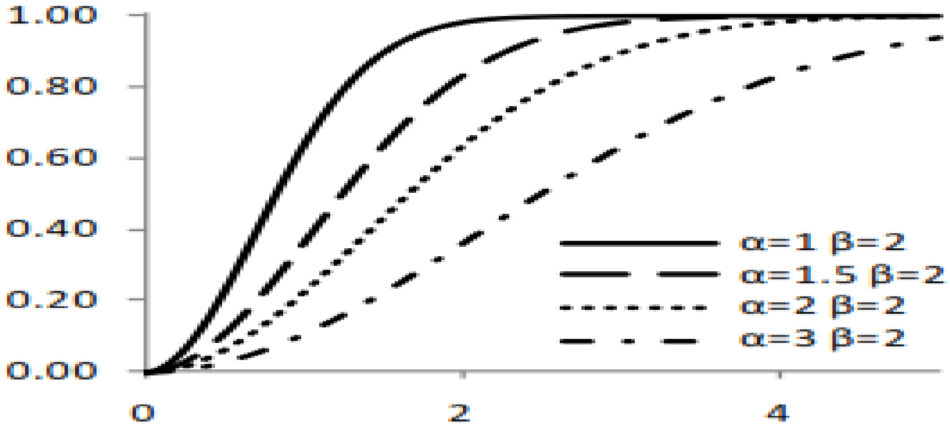
α → scale parameter : influences both mean and spread of distribution.

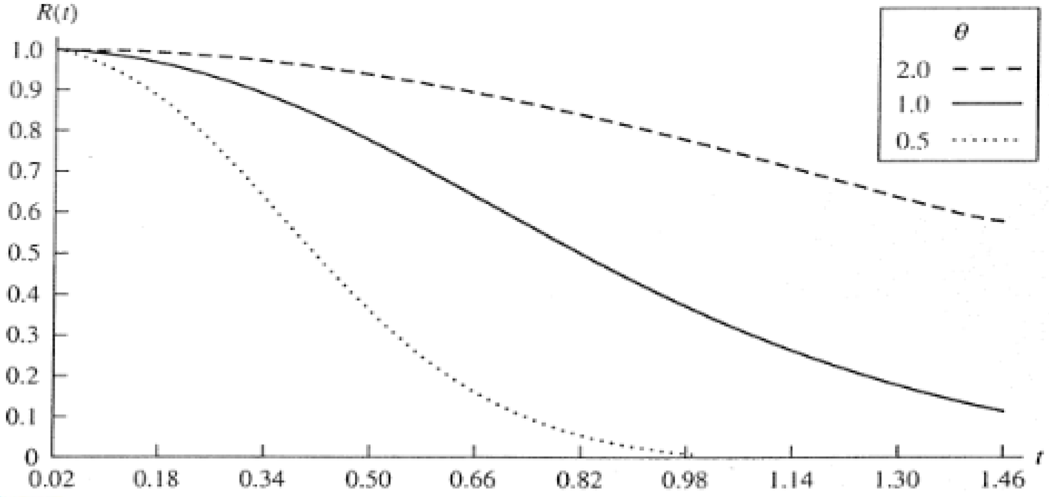
As α increases, reliability increases at a given point in time.

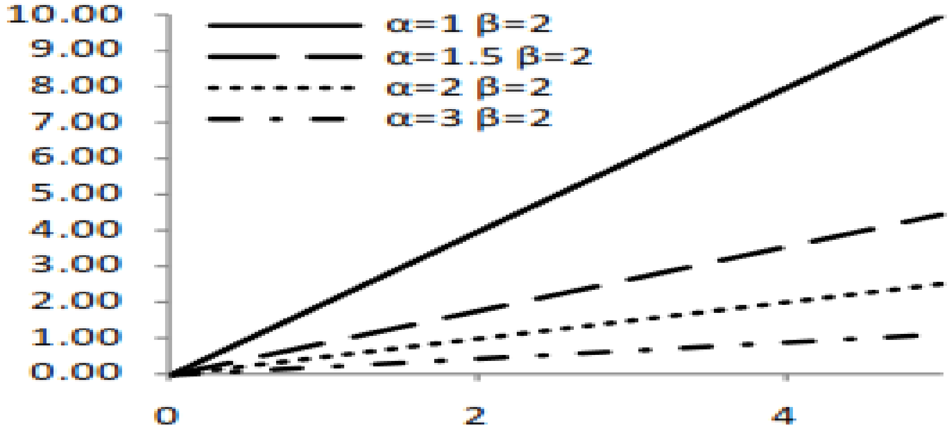
Slope of failure rate decreases as α increases.

α : characteristic life.









Weibull : Design life and Median

- Given a desired reliability R, $R(t) = e^{-(t/\alpha)^{\beta}} = R$

design life is found from: $t_R = \alpha(-\ln R)^{1/\beta}$

Remarks: When R=0.99,

- $t_{0.99}$ is referred **B1 life** → time at which 1% of population has failed!

- $t_{0.999}$ is referred **B.1 life** → time at which 0.1% of population has failed!

Median gives good central measure of skewed distribution.
For small $\beta < 3$: median is better than mean for central tendency.

Median time to failure: $t_{0.5} = t_{med} = \alpha(-\ln 0.5)^{1/\beta} = \alpha(0.69)^{1/\beta}$

Weibull distribution: Conditional Reliability

- Consider a burn-in period T_0 , then conditional probability :

$$R\ (t\,|\,T_0) = \frac{R(t+T_0)}{R(T_0)}$$

$$R(t\,|\,T_0) = \frac{e^{-(t+T_0/\alpha)^{\beta}}}{e^{-(T_0/\alpha)^{\beta}}} = \exp\left[-\left(\frac{t+T_0}{\alpha}\right)^{\beta} + \left(\frac{T_0}{\alpha}\right)^{\beta}\right]$$

Weibull distribution: Three parameter Weibull

- Consider a minimum life t_0 , $T > t_0$ → three parameter Weibull assumes no failure takes place before t_0.

t_0 : *location parmeter*

$$R(t) = \exp\left[-\left(\frac{t-t_0}{\alpha}\right)^{\beta}\right];\ t \geq t_0$$

$$\lambda(t) = \frac{\beta}{\alpha}\left(\frac{t-t_0}{\alpha}\right)^{\beta-1}\ ;\ t \geq t_0$$

$$MTTF: t_0 + \alpha\,\Gamma\left(1+\frac{1}{\beta}\right)$$

$$t_{med} = t_0 + \alpha(0.69315)^{1/\beta}$$

$$t_R = t_0 + \alpha\,(-\ln R)^{1/\beta}$$

Normal distribution

- Used mostly to model : fatigue, wear-out phenomena.
- Bell shaped curve.

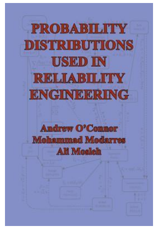
$$f(t)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{1}{2}\frac{(t-\mu)^2}{\sigma^2}\right];-\infty<t<\infty$$

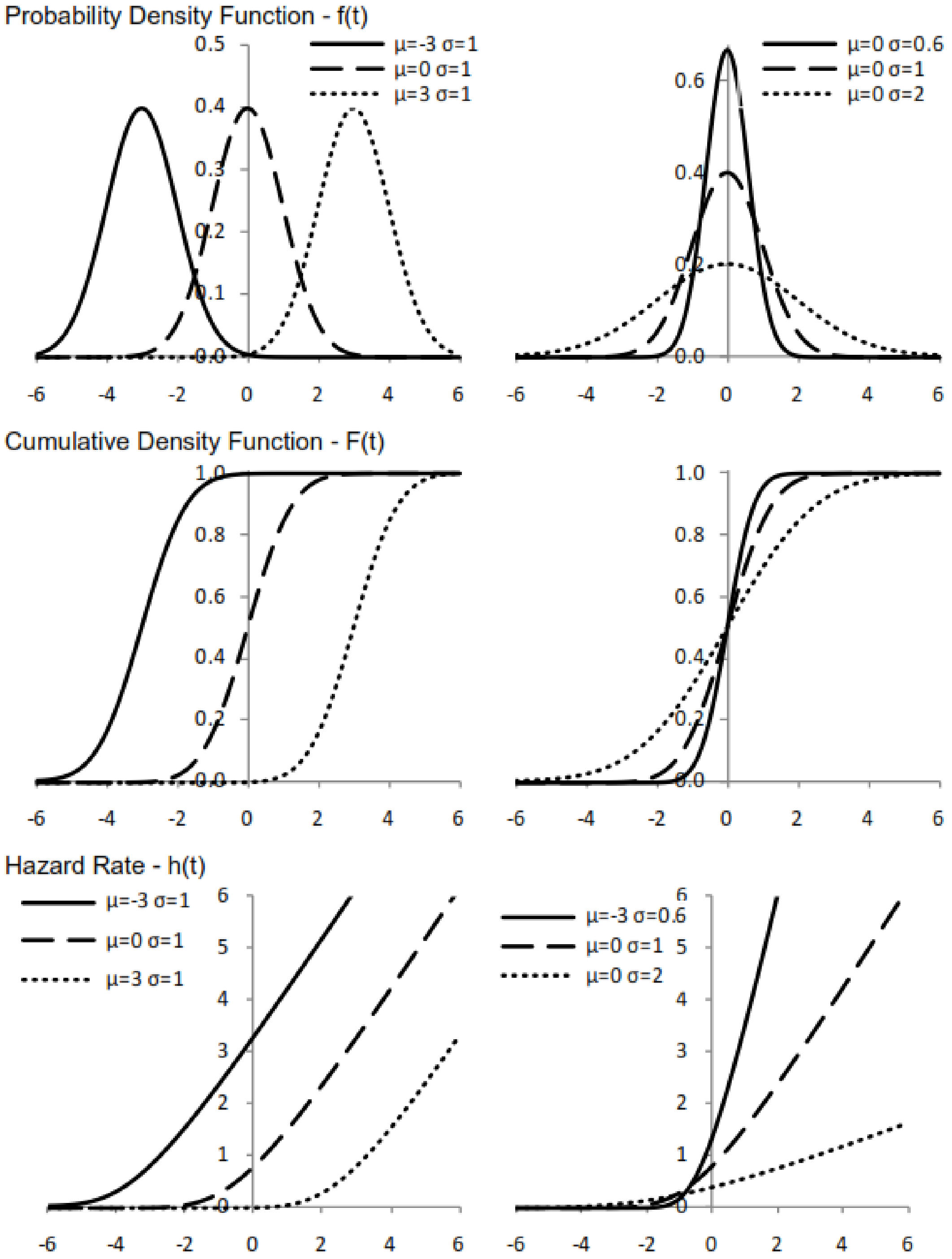
$$R(t)=\int_t^{\infty}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{1}{2}\frac{(t'-\mu)^2}{\sigma^2}\right]dt'$$

- No closed form solution to above. Must be solved numerically.
- Normal → not a true representation of failure distribution.

Why? RV ranges as $-\infty<t<\infty$

- reasonable approximation of failure process.





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Normal distribution

Consider transformation: $z = \dfrac{T-\mu}{\sigma}$ mean =0, variance=1

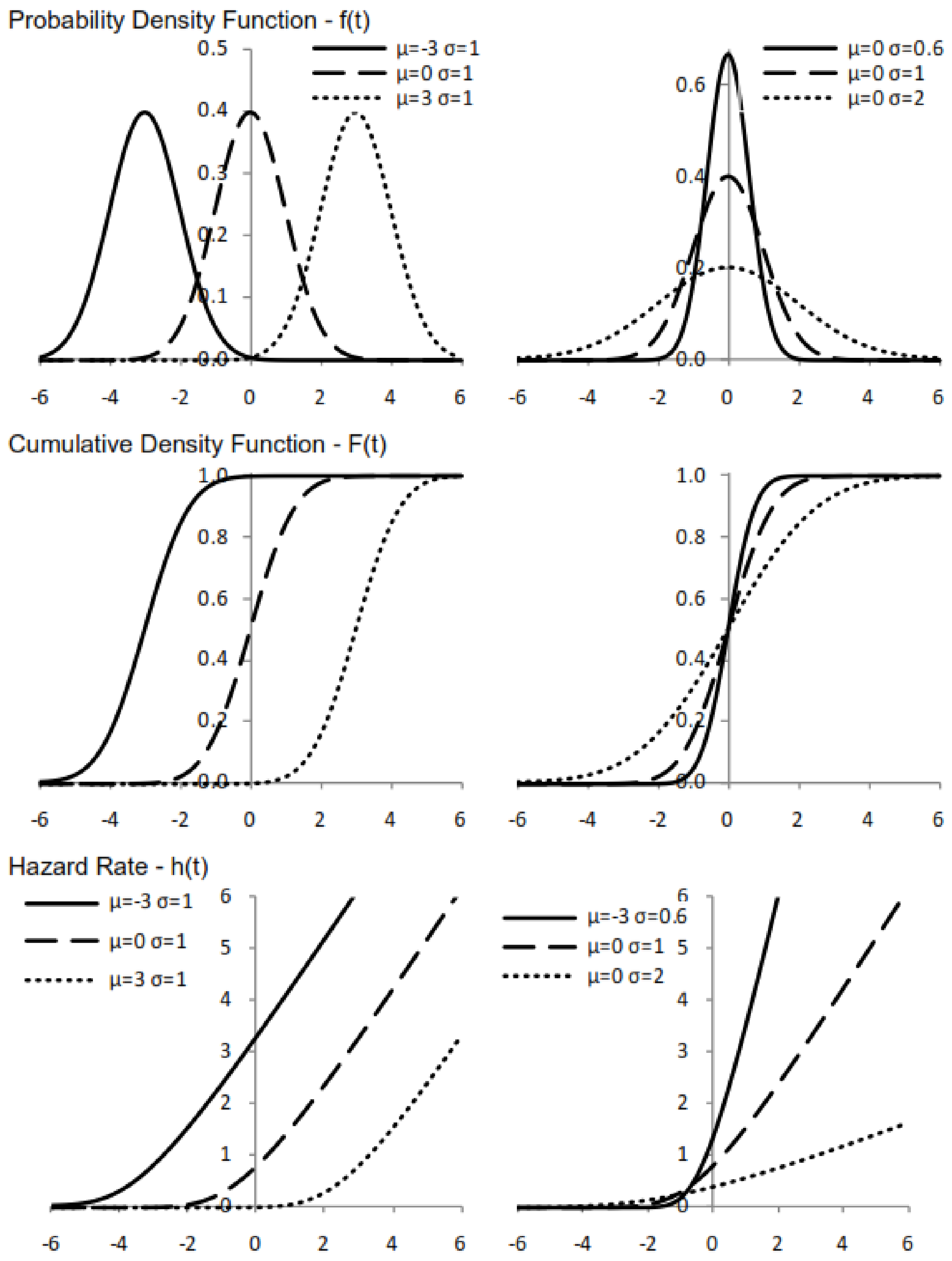
$$\text{PDF: } \phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

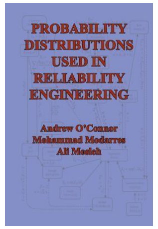
$$\text{CDF: } \Phi(z) = \int_{-\infty}^{z} \phi(z')dz'$$

$$F(t) = \Pr\{T \le t\} = \Pr\left\{\frac{T-\mu}{\sigma} \le \frac{t-\mu}{\sigma}\right\}$$

$$= \Pr\left\{z \le \frac{t-\mu}{\sigma}\right\} = \Phi\left(\frac{t-\mu}{\sigma}\right)$$

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma}\right)$$





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Log normal distribution

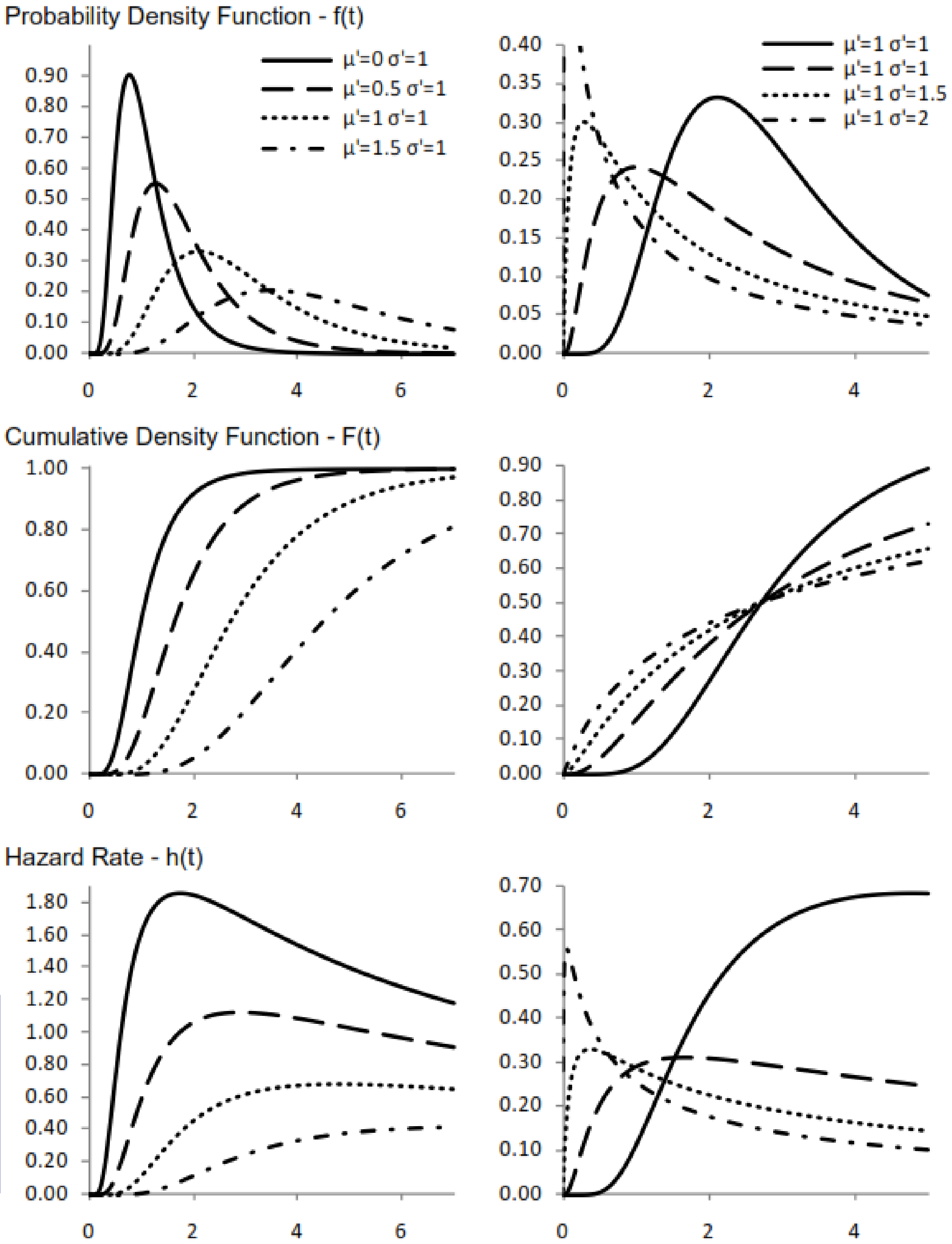
- Normal distribution has RV range as: $-\infty < t < \infty$
- Modification to constrain RV in non-negative real domain.
- Like Weibull, it takes many shapes.
- Usually, data that fits Weibull , it fits Log normal.

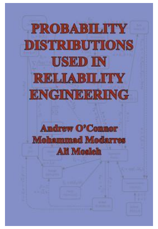
$$f(t) = \frac{1}{\sqrt{2\pi\sigma_n^2}\, t} \exp\left[-\frac{1}{2} \frac{(\ln t - \mu_n)^2}{\sigma_n^2} \right]; 0 \le t$$

$$F(t) = \Phi\left\{ \frac{t-\mu}{\sigma} \right\}, t \ge 0$$

$$R(t) = 1 - \Phi\left\{ \frac{t-\mu}{\sigma} \right\}$$

$$MTTF = \exp\left\{ \mu_n + \frac{\sigma^2}{2} \right\}$$





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Sources of Failure Data

Organisations:

- **Reliability Analysis Center (RAC) :** Nonelectronic Parts Reliability Data (NPRD) reports by US Airforce.
- **Defense Technical Information Center :** Reliability data for defense equipment.
- **Parts Reliability Information Center (PRINCE):** Reliability of systems related to space
- **Institute of Electrical and Electronics Engineers (IEEE) :** failure data concerning various electrical related items.

Data Banks:

- **Nuclear Plant Reliability Data System (NPRDS):**Failure data on equipment used in nuclear power plants.
- **Equipment Reliability Information System (ERIS):** failure data on equipment used in electric power generation.
- **SYREL: Reliability Data Bank:** failure data on equipment used in power generation (UK).
- **OREDA (Offshore Reliability Data) - version 4 (2002)** : recueil européen concernant les matériels des compagnies pétrolières.
- **IEEE Standard 500 - 1984 (États-Unis) - Guide to the Collection and Presentation of Electrical, Electronic, Sensing Component, and Mechanical Equipment Reliability Data for Nuclear Power Generating Stations**

Guide : **Fides (reliability)**

- reliability calculation for ***electronic components and systems*.**
- Fides is a DGA (French armament industry supervision agency) study conducted by a European consortium :

Airbus France – Eurocopter – GIAT Industries – MBDA Missile systems – THALES Airborne

Systems – THALES Avionics – THALES Research & Technology – THALES Underwater Systems

TABLE A.1

Standardized normal probabilities: $\Phi(z) = \int_{-\infty}^{z} (1/\sqrt{2\pi}) e^{-y^2/2}\, dy$

z	$\Phi(z)$	$1-\Phi(z)$	z	$\Phi(z)$	$1-\Phi(z)$	z	$\Phi(z)$	$1-\Phi(z)$
−4.00000	0.00003	0.99997	−3.51000	0.00022	0.99978	−3.02000	0.00126	0.99874
−3.99000	0.00003	0.99997	−3.50000	0.00023	0.99977	−3.01000	0.00131	0.99869
−3.98000	0.00003	0.99997	−3.49000	0.00024	0.99976	−3.00000	0.00135	0.99865
−3.97000	0.00004	0.99996	−3.48000	0.00025	0.99975	−2.99000	0.00139	0.99861
−3.96000	0.00004	0.99996	−3.47000	0.00026	0.99974	−2.98000	0.00144	0.99856
−3.95000	0.00004	0.99996	−3.46000	0.00027	0.99973	−2.97000	0.00149	0.99851
−3.94000	0.00004	0.99996	−3.45000	0.00028	0.99972	−2.96000	0.00154	0.99846
−3.93000	0.00004	0.99996	−3.44000	0.00029	0.99971	−2.95000	0.00159	0.99841
−3.92000	0.00004	0.99996	−3.43000	0.00030	0.99970	−2.94000	0.00164	0.99836
−3.91000	0.00005	0.99995	−3.42000	0.00031	0.99969	−2.93000	0.00169	0.99831
−3.90000	0.00005	0.99995	−3.41000	0.00032	0.99968	−2.92000	0.00175	0.99825
−3.89000	0.00005	0.99995	−3.40000	0.00034	0.99966	−2.91000	0.00181	0.99819
−3.88000	0.00005	0.99995	−3.39000	0.00035	0.99965	−2.90000	0.00187	0.99813
−3.87000	0.00005	0.99995	−3.38000	0.00036	0.99964	−2.89000	0.00193	0.99807
−3.86000	0.00006	0.99994	−3.37000	0.00038	0.99962	−2.88000	0.00199	0.99801
−3.85000	0.00006	0.99994	−3.36000	0.00039	0.99961	−2.87000	0.00205	0.99795
−3.84000	0.00006	0.99994	−3.35000	0.00040	0.99960	−2.86000	0.00212	0.99788
−3.83000	0.00006	0.99994	−3.34000	0.00042	0.99958	−2.85000	0.00219	0.99781
−3.82000	0.00007	0.99993	−3.33000	0.00043	0.99957	−2.84000	0.00226	0.99774
−3.81000	0.00007	0.99993	−3.32000	0.00045	0.99955	−2.83000	0.00233	0.99767
−3.80000	0.00007	0.99993	−3.31000	0.00047	0.99953	−2.82000	0.00240	0.99760
−3.79000	0.00008	0.99992	−3.30000	0.00048	0.99952	−2.81000	0.00248	0.99752
−3.78000	0.00008	0.99992	−3.29000	0.00050	0.99950	−2.80000	0.00255	0.99745
−3.77000	0.00008	0.99992	−3.28000	0.00052	0.99948	−2.79000	0.00264	0.99736
−3.76000	0.00008	0.99992	−3.27000	0.00054	0.99946	−2.78000	0.00272	0.99728
−3.75000	0.00009	0.99991	−3.26000	0.00056	0.99944	−2.77000	0.00280	0.99720
−3.74000	0.00009	0.99991	−3.25000	0.00058	0.99942	−2.76000	0.00289	0.99711
−3.73000	0.00009	0.99991	−3.24000	0.00060	0.99940	−2.75000	0.00298	0.99702
−3.72000	0.00010	0.99990	−3.23000	0.00062	0.99938	−2.74000	0.00307	0.99693
−3.71000	0.00010	0.99990	−3.22000	0.00064	0.99936	−2.73000	0.00317	0.99683
−3.70000	0.00011	0.99989	−3.21000	0.00066	0.99934	−2.72000	0.00326	0.99674
−3.69000	0.00011	0.99989	−3.20000	0.00069	0.99931	−2.71000	0.00336	0.99664
−3.68000	0.00012	0.99988	−3.19000	0.00071	0.99929	−2.70000	0.00347	0.99653
−3.67000	0.00012	0.99988	−3.18000	0.00074	0.99926	−2.69000	0.00357	0.99643
−3.66000	0.00013	0.99987	−3.17000	0.00076	0.99924	−2.68000	0.00368	0.99632
−3.65000	0.00013	0.99987	−3.16000	0.00079	0.99921	−2.67000	0.00379	0.99621
−3.64000	0.00014	0.99986	−3.15000	0.00082	0.99918	−2.66000	0.00391	0.99609
−3.63000	0.00014	0.99986	−3.14000	0.00084	0.99916	−2.65000	0.00402	0.99598
−3.62000	0.00015	0.99985	−3.13000	0.00087	0.99913	−2.64000	0.00415	0.99585
−3.61000	0.00015	0.99985	−3.12000	0.00090	0.99910	−2.63000	0.00427	0.99573
−3.60000	0.00016	0.99984	−3.11000	0.00094	0.99906	−2.62000	0.00440	0.99560
−3.59000	0.00016	0.99984	−3.10000	0.00097	0.99903	−2.61000	0.00453	0.99547
−3.58000	0.00017	0.99983	−3.09000	0.00100	0.99900	−2.60000	0.00466	0.99534
−3.57000	0.00018	0.99982	−3.08000	0.00103	0.99897	−2.59000	0.00480	0.99520
−3.56000	0.00019	0.99981	−3.07000	0.00107	0.99893	−2.58000	0.00494	0.99506
−3.55000	0.00019	0.99981	−3.06000	0.00111	0.99889	−2.57000	0.00508	0.99492
−3.54000	0.00020	0.99980	−3.05000	0.00114	0.99886	−2.56000	0.00523	0.99477
−3.53000	0.00021	0.99979	−3.04000	0.00118	0.99882	−2.55000	0.00539	0.99461
−3.52000	0.00022	0.99978	−3.03000	0.00122	0.99878	−2.54000	0.00554	0.99446

z	$\Phi(z)$	$1-\Phi(z)$	z	$\Phi(z)$	$1-\Phi(z)$	z	$\Phi(z)$	$1-\Phi(z)$
−2.53000	0.00570	0.99430	−2.03000	0.02118	0.97882	−1.53000	0.06301	0.93699
−2.52000	0.00587	0.99413	−2.02000	0.02169	0.97831	−1.52000	0.06426	0.93574
−2.51000	0.00604	0.99396	−2.01000	0.02222	0.97778	−1.51000	0.06552	0.93448
−2.50000	0.00621	0.99379	−2.00000	0.02275	0.97725	−1.50000	0.06681	0.93319
−2.49000	0.00639	0.99361	−1.99000	0.02330	0.97670	−1.49000	0.06811	0.93189
−2.48000	0.00657	0.99343	−1.98000	0.02385	0.97615	−1.48000	0.06944	0.93056
−2.47000	0.00676	0.99324	−1.97000	0.02442	0.97558	−1.47000	0.07078	0.92922
−2.46000	0.00695	0.99305	−1.96000	0.02500	0.97500	−1.46000	0.07214	0.92786
−2.45000	0.00714	0.99286	−1.95000	0.02559	0.97441	−1.45000	0.07353	0.92647
−2.44000	0.00734	0.99266	−1.94000	0.02619	0.97381	−1.44000	0.07493	0.92507
−2.43000	0.00755	0.99245	−1.93000	0.02680	0.97320	−1.43000	0.07636	0.92364
−2.42000	0.00776	0.99224	−1.92000	0.02743	0.97257	−1.42000	0.07780	0.92220
−2.41000	0.00798	0.99202	−1.91000	0.02807	0.97193	−1.41000	0.07927	0.92073
−2.40000	0.00820	0.99180	−1.90000	0.02872	0.97128	−1.40000	0.08076	0.91924
−2.39000	0.00842	0.99158	−1.89000	0.02938	0.97062	−1.39000	0.08226	0.91774
−2.38000	0.00866	0.99134	−1.88000	0.03005	0.96995	−1.38000	0.08379	0.91621
−2.37000	0.00889	0.99111	−1.87000	0.03074	0.96926	−1.37000	0.08534	0.91466
−2.36000	0.00914	0.99086	−1.86000	0.03144	0.96856	−1.36000	0.08691	0.91309
−2.35000	0.00939	0.99061	−1.85000	0.03216	0.96784	−1.35000	0.08851	0.91149
−2.34000	0.00964	0.99036	−1.84000	0.03288	0.96712	−1.34000	0.09012	0.90988
−2.33000	0.00990	0.99010	−1.83000	0.03362	0.96638	−1.33000	0.09176	0.90824
−2.32000	0.01017	0.98983	−1.82000	0.03438	0.96562	−1.32000	0.09342	0.90658
−2.31000	0.01044	0.98956	−1.81000	0.03515	0.96485	−1.31000	0.09510	0.90490
−2.30000	0.01072	0.98928	−1.80000	0.03593	0.96407	−1.30000	0.09680	0.90320
−2.29000	0.01101	0.98899	−1.79000	0.03673	0.96327	−1.29000	0.09853	0.90147
−2.28000	0.01130	0.98870	−1.78000	0.03754	0.96246	−1.28000	0.10027	0.89973
−2.27000	0.01160	0.98840	−1.77000	0.03836	0.96164	−1.27000	0.10204	0.89796
−2.26000	0.01191	0.98809	−1.76000	0.03920	0.96080	−1.26000	0.10383	0.89617
−2.25000	0.01222	0.98778	−1.75000	0.04006	0.95994	−1.25000	0.10565	0.89435
−2.24000	0.01255	0.98745	−1.74000	0.04093	0.95907	−1.24000	0.10749	0.89251
−2.23000	0.01287	0.98713	−1.73000	0.04182	0.95818	−1.23000	0.10935	0.89065
−2.22000	0.01321	0.98679	−1.72000	0.04272	0.95728	−1.22000	0.11123	0.88877
−2.21000	0.01355	0.98645	−1.71000	0.04363	0.95637	−1.21000	0.11314	0.88686
−2.20000	0.01390	0.98610	−1.70000	0.04457	0.95543	−1.20000	0.11507	0.88493
−2.19000	0.01426	0.98574	−1.69000	0.04551	0.95449	−1.19000	0.11702	0.88298
−2.18000	0.01463	0.98537	−1.68000	0.04648	0.95352	−1.18000	0.11900	0.88100
−2.17000	0.01500	0.98500	−1.67000	0.04746	0.95254	−1.17000	0.12100	0.87900
−2.16000	0.01539	0.98461	−1.66000	0.04846	0.95154	−1.16000	0.12302	0.87698
−2.15000	0.01578	0.98422	−1.65000	0.04947	0.95053	−1.15000	0.12507	0.87493
−2.14000	0.01618	0.98382	−1.64000	0.05050	0.94950	−1.14000	0.12714	0.87286
−2.13000	0.01659	0.98341	−1.63000	0.05155	0.94845	−1.13000	0.12924	0.87076
−2.12000	0.01700	0.98300	−1.62000	0.05262	0.94738	−1.12000	0.13136	0.86864
−2.11000	0.01743	0.98257	−1.61000	0.05370	0.94630	−1.11000	0.13350	0.86650
−2.10000	0.01786	0.98214	−1.60000	0.05480	0.94520	−1.10000	0.13567	0.86433
−2.09000	0.01831	0.98169	−1.59000	0.05592	0.94408	−1.09000	0.13786	0.86214
−2.08000	0.01876	0.98124	−1.58000	0.05705	0.94295	−1.08000	0.14007	0.85993
−2.07000	0.01923	0.98077	−1.57000	0.05821	0.94179	−1.07000	0.14231	0.85769
−2.06000	0.01970	0.98030	−1.56000	0.05938	0.94062	−1.06000	0.14457	0.85543
−2.05000	0.02018	0.97982	−1.55000	0.06057	0.93943	−1.05000	0.14686	0.85314
−2.04000	0.02067	0.97933	−1.54000	0.06178	0.93822	−1.04000	0.14917	0.85083

Annex : Student *t* distribution Chart

TABLE A.2
Critical *t* values with ν degrees of freedom

	α				
ν	0.100	0.050	0.025	0.010	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.695	9.925
3	1.639	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.799
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
∞	1.282	1.645	1.960	2.326	2.576