Sûreté de fonctionnement & Retour d'Expériences

Dependability and Feedback Data Collection

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Automatic Control, Reliability and Health Management of Systems

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Contents

Reliability and Failure Rate function

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Failure Rate Basics

- ‘Lifetime’ of a system/component is mostly determined by its *time to failure*.

- Lifetime Or time to failure :
  - non-negative random variable
  - characterized by distribution functions
  - very significant for reliability analysis, Survival analysis, risk analysis,
  - leads to probability of failure in next in next finite interval of time (preferably infinitesimal).

Probabilistic analysis

Basic concepts of probability

In reliability engineering , failures can be described as **random events.**

For, Random event E:

- probability denoted: $0 < P(E) < 1$
- impossible event: $P(E) = 0$
- certain event $P(E) = 1$
- Collection of all possible outcomes for a random process : Sample Space $S = \{E_1, E_2 \ldots E_n\}, P(S) = 1$
- Complement event $\overline{E}, \quad P(\overline{E}) = 1 - P(\mathrm{E})$
- Pr (A given B) : $P(\mathrm{A} \mid \mathrm{B}) = \dfrac{P(\mathrm{A} \cap \mathrm{B})}{P(B)}$
- *A* and *B are* independent **if and only if**

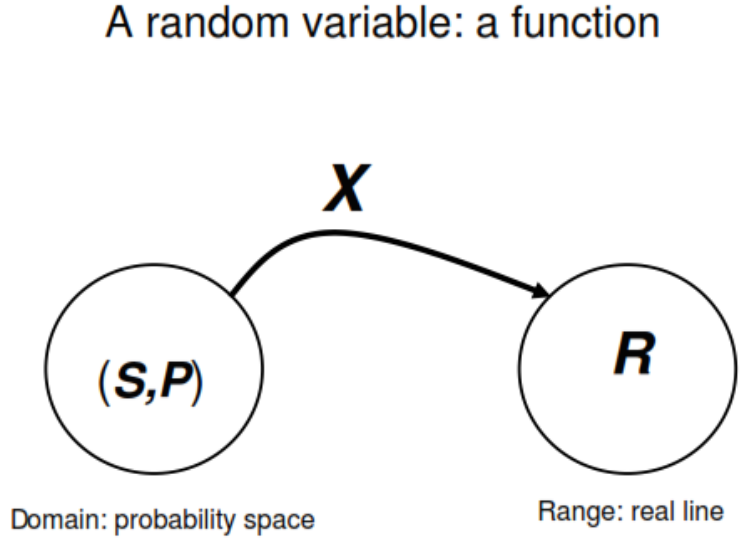
$$P(\mathrm{A} \cap \mathrm{B}) = P(\mathrm{A})P(\mathrm{B})$$

- Then, if *A* and *B* are independent:

$$P(\mathrm{A} \cap \mathrm{B}) = P(\mathrm{A} \mid \mathrm{B})P(B)$$

- Baye's formula: Conditional probability of two events

$$P(\mathrm{A}\,|\,\mathrm{B}) = \frac{P(A \cap B)}{P(\mathrm{B})} = \frac{P(\mathrm{B}\,|\,\mathrm{A})\,\mathrm{P}(\mathrm{A})}{P(\mathrm{B}\,|\,\mathrm{A})\,\mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}\,|\,\bar{\mathrm{A}})\,\mathrm{P}(\bar{\mathrm{A}})}$$



Random Variables (RV) :

The outcome "x" of a random experiment in sample space S, can be described by a random variable RV : $X(x) \in \mathbb{R}$

Example:

Modélisation de RV → Sorties de l'expérience « Lancer un dé avec la variable X »

Pour une valeur numérique donnée x ,

Définition de l'événement : tous les résultats possibles associés aux valeurs de la variable **aléatoire X inférieur à x .**

- Pour **x** = 4,72 → l'événement {**X** ≤ 4,72} correspond à l'union des résultats (1 U 2 U 3 U 4) ;

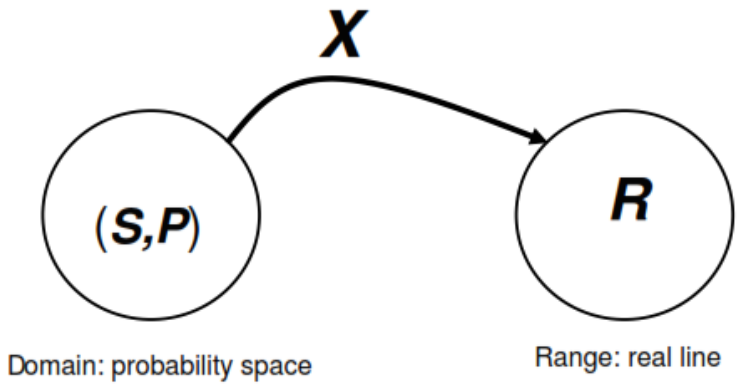
- l'événement {X ≤ 0} est l'ensemble nul → les résultats du jet des dés ne sont associés à aucune valeur négative de X.

- pour x=∞ l'événement (**X** ≤ ∞ } est l'espace échantillon complet R .

- Baye's formula: Conditional probability of two events

$$P(\mathrm{A}\,|\,\mathrm{B}) = \frac{P(A \cap B)}{P(\mathrm{B})} = \frac{P(\mathrm{B}\,|\,\mathrm{A})\,\mathrm{P}(\mathrm{A})}{P(\mathrm{B}\,|\,\mathrm{A})\,\mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}\,|\,\bar{\mathrm{A}})\,\mathrm{P}(\bar{\mathrm{A}})}$$

A random variable: a function



Random Variables (RV) :

- More useful : can describe cont. and discrete processes, phenomena,
- RV takes numerical values according to some probability distribution.
  - cont. (real numbers)
  - discrete (usually non-negative integers)
- **Probability distribution assigns → Probability to each value of:**

Discrete Variables

- Described by Probability Mass function (PMF) $p(x)$

Describes shape of probability distribution

Continuous Variables

- Probability Density function (PDF) $f(x)$

Describes shape of probability distribution

Cumulative Distribution Function CDF $F(x)$

CDF gives the cumulative probability $\Pr\{X \le x\} = F(x)$

Discrete Distributions :

- For any discrete distribution, PMF associates probability to each discrete RV.

$$p(x) = PMF\{X = x\}$$

- CDF :

$$F(x) = \Pr\{X \le x\} = \sum_{all\ \xi}^{x} p(\xi)$$

- CDF is monotonically increasing:

$$0 \le F(x) \le 1, \quad F(0) = 0, F(\infty) = 1$$

- For any discrete distribution:

$$0 \le p(x) \le 1$$

$$\sum_{all\ x} p(x) = 1$$

Mean

$$\mu = \sum_{all\ x} x\ p(x)$$

Variance

$$\sigma^2 = \sum_{all\ x} (x - \mu)^2\ p(x)$$

Continuous Distributions: $0 \le F(x) \le 1, \quad F(0) = 0, F(\infty) = 1$

- CDF : $\Pr\{X \le x\} = F(x) = \int_{-\infty}^{-\infty} p(\xi)$
- PDF: $f(x) = \frac{F(x)}{dx}$
- Probability of being between two RV values:

$$\Pr\{a \le X \le b\} = \int_a^b f(x) = F(b) - F(a)$$

$$\Pr\{x \le X \le x + dx\} = F(x + dx) - F(x) = f(x)dx$$

- Mean $E(X) = \mu = \int_{-\infty}^{-\infty} x f(x)\, dx$
- Variance $\mathrm{Var}(X) = \sigma^2 = \int_{-\infty}^{-\infty} (x - \mu)^2 f(x)\, dx$

Remark: PDF $f_X(x)$ is not a probability but probability per unit **x**.

A PDF when multiplied by **dx** gives the probability of **X** falling in interval $[x, x + dx]$

Reliability Function

Reliability: Probability that a system will function over some period time ***t*** .

- Continuous random variable T , $T \geq 0$ → *time to failure* of system component
- **Reliability** expressed as : $R(t) = \Pr\{T \geq t\}$

$$0 \leq R(t) \leq 1, \quad R(0) = 1, R(\infty) = 0$$

- Consider, $F(t) = 1 - R(t) = \Pr\{T < t\}$

$$0 \leq F(t) \leq 1, \quad F(0) = 0, F(\infty) = 1$$

Reliability Function

Probability that time to failure **T** is **greater than** or equal to ***t***. ➔ *Reliability*

PDF of Failure Distribution
(describes shape of failure distribution)

CDF of Failure Distribution ➔ failure pro.
(and, not ,failure function !)

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

Failure Rate or Hazard Rate Function

- **Instantaneous** Failure rate is very significant for reliability analysis , WHY??

- Consider time interval $[t, t+\Delta t]$

What is probability of failure in a given time interval (t, t+dt) given item functions well till time *t*???

- Prob of failure in this interval given that, it did not occur before is :

$$\Pr\{t \le T \le t+\Delta t \mid T \ge t\}$$

$$= \frac{\Pr\{t \le T \le t+\Delta t\}}{\Pr\{T \ge t\}}$$

$$= \frac{F(t+\Delta t) - F(t)}{R(t)}$$

Reliability function : Probability that item survives till time *t*

Failure Rate or Hazard Rate Function

Consider the quotient:

$$\lambda_{\Delta t}(t) = \frac{F(t+\Delta t) - F(t)}{R(t)\Delta t}$$

Define ***failure rate*** $\lambda(t)$ as $\Delta t \to 0$:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{\Pr\{t \le T \le t+\Delta t \mid T \ge t\}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{R(t)\Delta t}$$

$\lambda(t)$:

- is the instantaneous ***failure rate or hazard rate function***.
- provides alternative way of describing failure distribution.
- denotes, **conditional probability** of failure in $[t, t+\Delta t]$
- uniquely defines the continuous CDF $F(t)$

$$\lambda(t) = \frac{f(t)}{R(t)}$$ ← Instantaneous failure rate!

so, ***Failure rate function*** characterizes *failure distribution.*

and, **reliability function**!

$$R(t) = \exp\left(-\int_0^t \lambda(u)du\right)$$

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{F'(t)}{1-F(t)}$$

$$F(t) = 1 - \exp\left(-\int_0^t \lambda(u)du\right)$$

First order Differential equation with known initial condition $F(0)=0$

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Mean time to Failure (MTTF)

We saw *time to failure T.*

- *W*hat is MTTF? expectation of lifetime T !

(T is stochastic random variable)

$$MTTF = m = \int_0^\infty R(t)dt$$

Area under reliability function defines the MTTF.

integration by parts

$$mean, \mathrm{E[T]} = m = \int_0^\infty t\ f(t)dt$$

$$= \int_0^\infty -\frac{dR(t)}{dt} t\ dt$$

$$= -tR(t)\Big|_0^\infty + \int_0^\infty R(t)dt$$

$$MTTF = m = \int_0^\infty R(t)dt$$

- Median time to failure: $R(t_{med}) = 0.5 = \Pr[T \geq t_{med}]$

  divides distribution into two halves (50% before t_{med} *and 50%* after t_{med}).

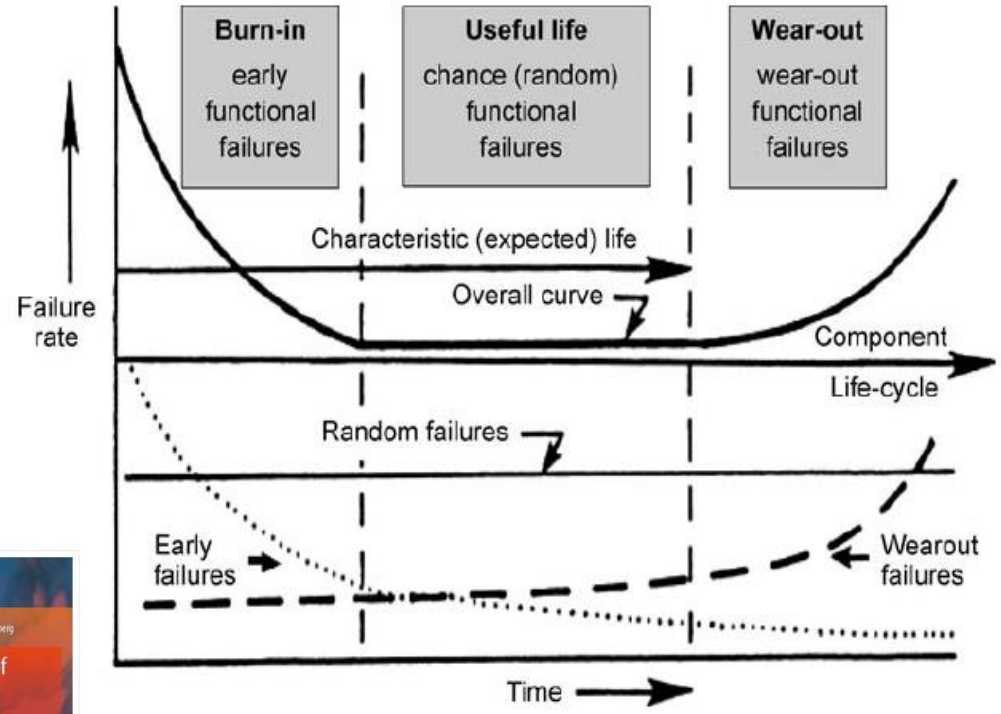
Failure Hazard Rate Curve : Bathtub Curve

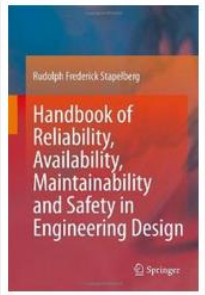
This curve is used to represent the failure rate pattern.

Hazard Curve rate can be divided in three regions:

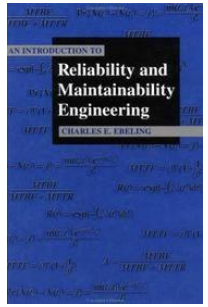
- Decreasing (burn in period , or infant mortality period → early failures)
  - Ex: design errors, manufacturing defects, welding erros, poor quality control, etc.
- Constant (Useful life period → random failures)
  - Ex: Electronic components, random failures, Environmental anomalies , Human errors, “Acts of God”, …
- Increasing (wear out phase)
  - Ex: Failure due to degradation, ageing, fatigue, Friction, Corrosion …

Failure rate → most important aspect for reliability analysis!!!



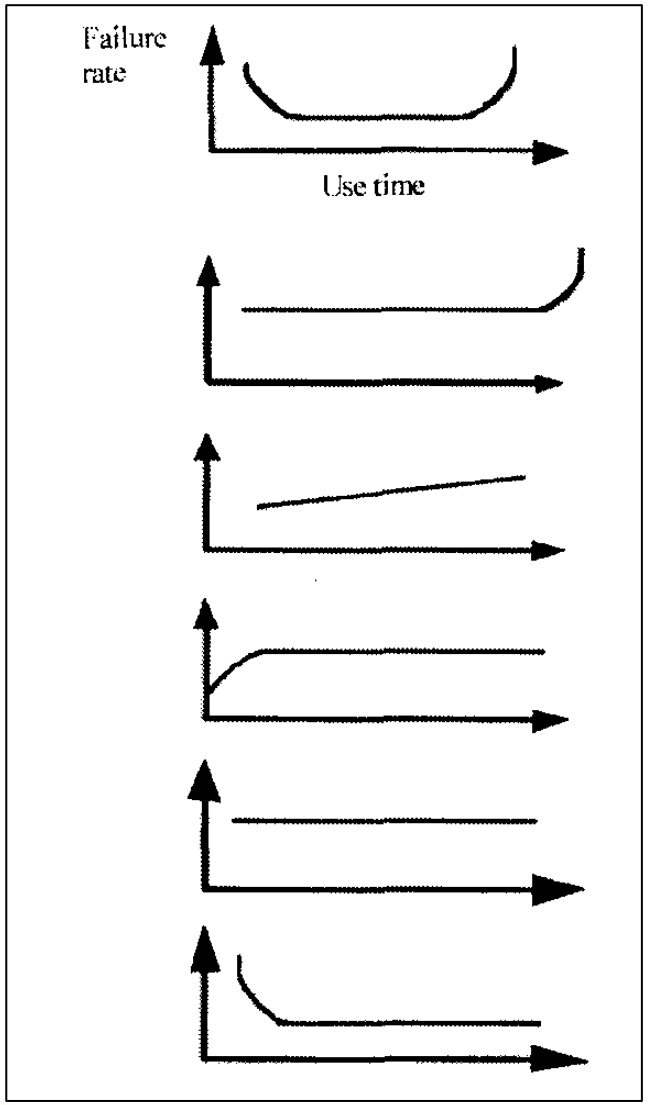


$$\lambda(t) = \begin{cases} c_0 - c_1 t + \lambda & 0 \le t \le \dfrac{c_0}{c_1} \\ \lambda & \dfrac{c_0}{c_1} < t \le t_0 \\ c_2(t - t_0) + \lambda & t_0 < t \end{cases}$$



$$R(t) = \begin{cases} \exp -\{(c_0 + \lambda)t - c_1(t^2/2)\} & 0 \le t \le \dfrac{c_0}{c_1} \\ \exp -\left(\lambda t + \dfrac{c_0^2}{2c_1}\right) & \dfrac{c_0}{c_1} < t \le t_0 \\ \exp -\left\{\left(\dfrac{c_2}{2}\right)(t - t_0)^2 + \lambda t + \left(\dfrac{c_0^2}{2c_1}\right)\right\} & t_0 < t \end{cases}$$

Failure Hazard Rate Curves



Bathtub curve: infant mortality followed by stable and wear out periods.

Constant failure rate followed by pronounced wear out period

Gradually increasing failure rate, no wear out age

Low failure rate when component is new, followed increase to constant level

Constant failure rate over useful life.

Infant mortality followed by constant or slowly increasing failure rate

Zeng, S. W., Reliability Engineering and System Safety, Vol. 55, pp. 151-162, 1997.

Conditional Reliability and Mean Remaining Lifetime.

- How much **longer** will an item of age ***x*** live??
- Very important for predictive maintenance!!!

- Conditional reliability describes reliability of the item, given that item has operated for time *x* (functional) at *t*=*x*.

- ***Mean Remaining (residual) Lifetime (MRL)*** is obtained from $R\ (t \mid x)$

$$MRL(x) = \int_0^\infty R\ (t \mid x)dt = \int_x^\infty \frac{R\ (u)}{R(x)}du$$

$$= \frac{1}{R(x)}\int_x^\infty R\ (u)\,du$$

where $u = t + x$

$$R\ (t \mid x) = \Pr\{T \geq x+t \mid T \geq x\}$$

$$= \frac{\Pr\{T \geq x+t \mid T \geq x\}}{\Pr\{T \geq x\}}$$

$$= \frac{R(x+t)}{R(x)}$$

$$= \frac{\exp\left(-\int_0^{x+t} \lambda(u)du\right)}{\exp\left(-\int_0^{x} \lambda(u)du\right)}$$

$$R\ (t \mid x) = \exp\left(-\int_x^{x+t} \lambda(u)du\right)$$

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Remarks:

- MRL defines the mean lifetime left for an item of age *x*.
- MRL function at time *x* , considers information about whole remaining interval *(t*, infinity).

- when *x* =0, implies a new item , (age=0), then MRL(0) =?

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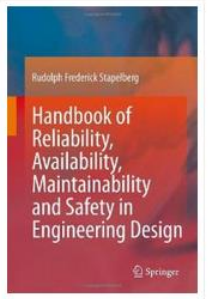
Remarks:

- MRL defines the mean lifetime left for an item of age *x*.
- MRL function at time *x* , considers information about whole remaining interval *(t*, infinity).

- when *x* =0, implies a new item , (age=0), then MRL(0) = MTTF !!

Consider: $$h(x) = \frac{MRL(x)}{MTTF}$$

- When time to failure *T,* has an exponential distribution, then *h(x)=1* for all *x*.
- When *T* has a Weibull distribution:
  - $\beta < 1$, decreasing failure rate, *h(x)* is an increasing function.
  - $\beta > 1$, *increasing failure rate, h(x) is a decreasing function.*



Guide : **Fides (reliability)**

- reliability calculation for ***electronic components and systems***.
- Fides is a DGA (French armament industry supervision agency) study conducted by a European consortium :

Airbus France – Eurocopter – GIAT Industries – MBDA Missile systems – THALES Airborne

Systems – THALES Avionics – THALES Research & Technology – THALES Underwater Systems

TABLE A.1

Standardized normal probabilities: $\Phi(z) = \int_{-\infty}^{z} (1/\sqrt{2\pi}) e^{-y^2/2}\, dy$

z	$\Phi(z)$	$1-\Phi(z)$	z	$\Phi(z)$	$1-\Phi(z)$	z	$\Phi(z)$	$1-\Phi(z)$
−4.00000	0.00003	0.99997	−3.51000	0.00022	0.99978	−3.02000	0.00126	0.99874
−3.99000	0.00003	0.99997	−3.50000	0.00023	0.99977	−3.01000	0.00131	0.99869
−3.98000	0.00003	0.99997	−3.49000	0.00024	0.99976	−3.00000	0.00135	0.99865
−3.97000	0.00004	0.99996	−3.48000	0.00025	0.99975	−2.99000	0.00139	0.99861
−3.96000	0.00004	0.99996	−3.47000	0.00026	0.99974	−2.98000	0.00144	0.99856
−3.95000	0.00004	0.99996	−3.46000	0.00027	0.99973	−2.97000	0.00149	0.99851
−3.94000	0.00004	0.99996	−3.45000	0.00028	0.99972	−2.96000	0.00154	0.99846
−3.93000	0.00004	0.99996	−3.44000	0.00029	0.99971	−2.95000	0.00159	0.99841
−3.92000	0.00004	0.99996	−3.43000	0.00030	0.99970	−2.94000	0.00164	0.99836
−3.91000	0.00005	0.99995	−3.42000	0.00031	0.99969	−2.93000	0.00169	0.99831
−3.90000	0.00005	0.99995	−3.41000	0.00032	0.99968	−2.92000	0.00175	0.99825
−3.89000	0.00005	0.99995	−3.40000	0.00034	0.99966	−2.91000	0.00181	0.99819
−3.88000	0.00005	0.99995	−3.39000	0.00035	0.99965	−2.90000	0.00187	0.99813
−3.87000	0.00005	0.99995	−3.38000	0.00036	0.99964	−2.89000	0.00193	0.99807
−3.86000	0.00006	0.99994	−3.37000	0.00038	0.99962	−2.88000	0.00199	0.99801
−3.85000	0.00006	0.99994	−3.36000	0.00039	0.99961	−2.87000	0.00205	0.99795
−3.84000	0.00006	0.99994	−3.35000	0.00040	0.99960	−2.86000	0.00212	0.99788
−3.83000	0.00006	0.99994	−3.34000	0.00042	0.99958	−2.85000	0.00219	0.99781
−3.82000	0.00007	0.99993	−3.33000	0.00043	0.99957	−2.84000	0.00226	0.99774
−3.81000	0.00007	0.99993	−3.32000	0.00045	0.99955	−2.83000	0.00233	0.99767
−3.80000	0.00007	0.99993	−3.31000	0.00047	0.99953	−2.82000	0.00240	0.99760
−3.79000	0.00008	0.99992	−3.30000	0.00048	0.99952	−2.81000	0.00248	0.99752
−3.78000	0.00008	0.99992	−3.29000	0.00050	0.99950	−2.80000	0.00255	0.99745
−3.77000	0.00008	0.99992	−3.28000	0.00052	0.99948	−2.79000	0.00264	0.99736
−3.76000	0.00008	0.99992	−3.27000	0.00054	0.99946	−2.78000	0.00272	0.99728
−3.75000	0.00009	0.99991	−3.26000	0.00056	0.99944	−2.77000	0.00280	0.99720
−3.74000	0.00009	0.99991	−3.25000	0.00058	0.99942	−2.76000	0.00289	0.99711
−3.73000	0.00009	0.99991	−3.24000	0.00060	0.99940	−2.75000	0.00298	0.99702
−3.72000	0.00010	0.99990	−3.23000	0.00062	0.99938	−2.74000	0.00307	0.99693
−3.71000	0.00010	0.99990	−3.22000	0.00064	0.99936	−2.73000	0.00317	0.99683
−3.70000	0.00011	0.99989	−3.21000	0.00066	0.99934	−2.72000	0.00326	0.99674
−3.69000	0.00011	0.99989	−3.20000	0.00069	0.99931	−2.71000	0.00336	0.99664
−3.68000	0.00012	0.99988	−3.19000	0.00071	0.99929	−2.70000	0.00347	0.99653
−3.67000	0.00012	0.99988	−3.18000	0.00074	0.99926	−2.69000	0.00357	0.99643
−3.66000	0.00013	0.99987	−3.17000	0.00076	0.99924	−2.68000	0.00368	0.99632
−3.65000	0.00013	0.99987	−3.16000	0.00079	0.99921	−2.67000	0.00379	0.99621
−3.64000	0.00014	0.99986	−3.15000	0.00082	0.99918	−2.66000	0.00391	0.99609
−3.63000	0.00014	0.99986	−3.14000	0.00084	0.99916	−2.65000	0.00402	0.99598
−3.62000	0.00015	0.99985	−3.13000	0.00087	0.99913	−2.64000	0.00415	0.99585
−3.61000	0.00015	0.99985	−3.12000	0.00090	0.99910	−2.63000	0.00427	0.99573
−3.60000	0.00016	0.99984	−3.11000	0.00094	0.99906	−2.62000	0.00440	0.99560
−3.59000	0.00016	0.99984	−3.10000	0.00097	0.99903	−2.61000	0.00453	0.99547
−3.58000	0.00017	0.99983	−3.09000	0.00100	0.99900	−2.60000	0.00466	0.99534
−3.57000	0.00018	0.99982	−3.08000	0.00103	0.99897	−2.59000	0.00480	0.99520
−3.56000	0.00019	0.99981	−3.07000	0.00107	0.99893	−2.58000	0.00494	0.99506
−3.55000	0.00019	0.99981	−3.06000	0.00111	0.99889	−2.57000	0.00508	0.99492
−3.54000	0.00020	0.99980	−3.05000	0.00114	0.99886	−2.56000	0.00523	0.99477
−3.53000	0.00021	0.99979	−3.04000	0.00118	0.99882	−2.55000	0.00539	0.99461
−3.52000	0.00022	0.99978	−3.03000	0.00122	0.99878	−2.54000	0.00554	0.99446
−2.53000	0.00570	0.99430	−2.03000	0.02118	0.97882	−1.53000	0.06301	0.93699
−2.52000	0.00587	0.99413	−2.02000	0.02169	0.97831	−1.52000	0.06426	0.93574
−2.51000	0.00604	0.99396	−2.01000	0.02222	0.97778	−1.51000	0.06552	0.93448
−2.50000	0.00621	0.99379	−2.00000	0.02275	0.97725	−1.50000	0.06681	0.93319
−2.49000	0.00639	0.99361	−1.99000	0.02330	0.97670	−1.49000	0.06811	0.93189
−2.48000	0.00657	0.99343	−1.98000	0.02385	0.97615	−1.48000	0.06944	0.93056
−2.47000	0.00676	0.99324	−1.97000	0.02442	0.97558	−1.47000	0.07078	0.92922
−2.46000	0.00695	0.99305	−1.96000	0.02500	0.97500	−1.46000	0.07214	0.92786
−2.45000	0.00714	0.99286	−1.95000	0.02559	0.97441	−1.45000	0.07353	0.92647
−2.44000	0.00734	0.99266	−1.94000	0.02619	0.97381	−1.44000	0.07493	0.92507
−2.43000	0.00755	0.99245	−1.93000	0.02680	0.97320	−1.43000	0.07636	0.92364
−2.42000	0.00776	0.99224	−1.92000	0.02743	0.97257	−1.42000	0.07780	0.92220
−2.41000	0.00798	0.99202	−1.91000	0.02807	0.97193	−1.41000	0.07927	0.92073
−2.40000	0.00820	0.99180	−1.90000	0.02872	0.97128	−1.40000	0.08076	0.91924
−2.39000	0.00842	0.99158	−1.89000	0.02938	0.97062	−1.39000	0.08226	0.91774
−2.38000	0.00866	0.99134	−1.88000	0.03005	0.96995	−1.38000	0.08379	0.91621
−2.37000	0.00889	0.99111	−1.87000	0.03074	0.96926	−1.37000	0.08534	0.91466
−2.36000	0.00914	0.99086	−1.86000	0.03144	0.96856	−1.36000	0.08691	0.91309
−2.35000	0.00939	0.99061	−1.85000	0.03216	0.96784	−1.35000	0.08851	0.91149
−2.34000	0.00964	0.99036	−1.84000	0.03288	0.96712	−1.34000	0.09012	0.90988
−2.33000	0.00990	0.99010	−1.83000	0.03362	0.96638	−1.33000	0.09176	0.90824
−2.32000	0.01017	0.98983	−1.82000	0.03438	0.96562	−1.32000	0.09342	0.90658
−2.31000	0.01044	0.98956	−1.81000	0.03515	0.96485	−1.31000	0.09510	0.90490
−2.30000	0.01072	0.98928	−1.80000	0.03593	0.96407	−1.30000	0.09680	0.90320
−2.29000	0.01101	0.98899	−1.79000	0.03673	0.96327	−1.29000	0.09853	0.90147
−2.28000	0.01130	0.98870	−1.78000	0.03754	0.96246	−1.28000	0.10027	0.89973
−2.27000	0.01160	0.98840	−1.77000	0.03836	0.96164	−1.27000	0.10204	0.89796
−2.26000	0.01191	0.98809	−1.76000	0.03920	0.96080	−1.26000	0.10383	0.89617
−2.25000	0.01222	0.98778	−1.75000	0.04006	0.95994	−1.25000	0.10565	0.89435
−2.24000	0.01255	0.98745	−1.74000	0.04093	0.95907	−1.24000	0.10749	0.89251
−2.23000	0.01287	0.98713	−1.73000	0.04182	0.95818	−1.23000	0.10935	0.89065
−2.22000	0.01321	0.98679	−1.72000	0.04272	0.95728	−1.22000	0.11123	0.88877
−2.21000	0.01355	0.98645	−1.71000	0.04363	0.95637	−1.21000	0.11314	0.88686
−2.20000	0.01390	0.98610	−1.70000	0.04457	0.95543	−1.20000	0.11507	0.88493
−2.19000	0.01426	0.98574	−1.69000	0.04551	0.95449	−1.19000	0.11702	0.88298
−2.18000	0.01463	0.98537	−1.68000	0.04648	0.95352	−1.18000	0.11900	0.88100
−2.17000	0.01500	0.98500	−1.67000	0.04746	0.95254	−1.17000	0.12100	0.87900
−2.16000	0.01539	0.98461	−1.66000	0.04846	0.95154	−1.16000	0.12302	0.87698
−2.15000	0.01578	0.98422	−1.65000	0.04947	0.95053	−1.15000	0.12507	0.87493
−2.14000	0.01618	0.98382	−1.64000	0.05050	0.94950	−1.14000	0.12714	0.87286
−2.13000	0.01659	0.98341	−1.63000	0.05155	0.94845	−1.13000	0.12924	0.87076
−2.12000	0.01700	0.98300	−1.62000	0.05262	0.94738	−1.12000	0.13136	0.86864
−2.11000	0.01743	0.98257	−1.61000	0.05370	0.94630	−1.11000	0.13350	0.86650
−2.10000	0.01786	0.98214	−1.60000	0.05480	0.94520	−1.10000	0.13567	0.86433
−2.09000	0.01831	0.98169	−1.59000	0.05592	0.94408	−1.09000	0.13786	0.86214
−2.08000	0.01876	0.98124	−1.58000	0.05705	0.94295	−1.08000	0.14007	0.85993
−2.07000	0.01923	0.98077	−1.57000	0.05821	0.94179	−1.07000	0.14231	0.85769
−2.06000	0.01970	0.98030	−1.56000	0.05938	0.94062	−1.06000	0.14457	0.85543
−2.05000	0.02018	0.97982	−1.55000	0.06057	0.93943	−1.05000	0.14686	0.85314
−2.04000	0.02067	0.97933	−1.54000	0.06178	0.93822	−1.04000	0.14917	0.85083

Annex : Student *t* distribution Chart

TABLE A.2

Critical t values with ν degrees of freedom

	α				
ν	0.100	0.050	0.025	0.010	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.695	9.925
3	1.639	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.799
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
∞	1.282	1.645	1.960	2.326	2.576