POLYTECH NANCY

TD 1

Reliability basics and Survival Analysis

Course : *Sûreté de fonctionnement & Retour d'Expériences*

(Dependability and Feedback Data Collection)

Mayank Jha

1 The following PDF for a random variable T, the time (in operating hours) to failure of an electric generator.

a. What is the reliability for a 100 Hr operating life?

$$f(t) = \begin{cases} \dfrac{0.001}{(0.001t+1)^2}; t \geq 0 \\ 0; \end{cases}$$

b. What is the design life if a reliability of 0.95 is desired?

2 Consider the probability density function

$$f(t) = \begin{cases} 0.002 \times e^{-0.002t}; & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

with 't' in hours.

a. what is reliability function.
b. What is MTTF?
c. What is median time to failure?
d. Consider another reliability function

$$R_2(t) = \frac{1000 - t}{1000} \qquad ; 0 \leq t \leq 1000$$

What is $MTTF_2$?

e. Compute the reliabilities for operating time of 400 Hr for **a** and **d**. What do you observe? Is MTTF enough to characterize the two reliability functions?

UNIVERSITÉ DE LORRAINE

Mayank Shekhar JHA, Associate Professor (Maitre de Conférences), Polytech Nancy, University of Lorraine. Email : mayank-shekhar.jha{at}univ-lorraine.fr

f. The variance f a failure distribution represents average square distance of a failure time from M T T F. It is given as:

$$\sigma^2 = \int_0^\infty \left[(t - MTTF)^2 f(t)\right] dt$$

Show that

$$\sigma^2 = \int_0^\infty (t^2 f(t) dt) - (MTTF)^2$$

g. Find variances for each failure distribution in **a** and **d**.

h. Give conclusions.

3. Given a linear hazard rate function a failure rate) as:

$$\lambda(t) = 5 \times 10^{-6} t$$

where t is measured in hours. What is design life if a reliability of 0.98 is desired?

4- Given

$$\lambda(t) = \frac{0.5}{1000} \times \left(\frac{t}{1000}\right)^{-0.5}$$

with 't' in years.

a- Comment on the nature of failure rate. Is it increasing, decreasing or a constant?

b. Given a desired reliability of 090, find the design life.

c- Given that the considered item has 'burn-in' period (already used) of 6 months, then, find the conditional reliability and the new design life.

d. Justify the difference in design life. what happens when the failure rate function is an increasing function? Comment.

5- Consider reliability function

$$R(t) = \frac{a^2}{(a+t)^2} \qquad ; t \geq 0, \; a > 0$$

The hazard rate function is

$$\lambda(t) = \frac{2}{a+t}$$

Find the residual MTTF given a burn-in period of To. what is MTTF when T_0 = O.

6. The reliability of a rotating blade (turbine) is given as:

$$R(t) = \left(1 - \frac{t}{t_0}\right)^2 ; 0 \leq t \leq t_0$$

with T_0 as maximum life of blade.

a. Are the blades undergoing degradation?
b. Compute MTTF as function of maximum life.
c. If maximum life is 2000 operating hours, what is the design life for reliability of 0. 9?

7. A uniform failure distribution has the property that equal intervals of time has equal failure probability. The density function is given as'

$$f(t) = \frac{1}{b}; 0 \leq t \leq b$$

Analyze this general failure distribution by finding $F(t); R(t), \lambda(t), MTTF, t_{med}, \; \sigma$.

8. A washing machine is advertised to have more than 10-year life. Given its PDF below, determine its reliability for next 10 years if it has survived a 1- year warranty period:

$$f(t) = 0.1 \times (1 + 0.05t)^{-3}; t \geq 0$$

What is the MTT F before the warranty period and what is the MTTF after the warranty period assuming it has still survived?