Solution TD 3, Exo 4, 5.

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TD 4

Exercise 12: Grouped complete data

We consider all the failures since 1985, according to the table given. The data are considered as grouped complete data.

i	Year	Number of failures	Number of surviving	Reliability	$\hat{R}$	Density function $(PDF = \hat{f}(t))$	Hazard rate function $\hat{\lambda}(t)$
0	1984	0	300	1	0	0,05	0,05
1	1985	15	285	0,95	0,1	0,06666667	0,070175439
2	1986	20	265	0,8833333	0,2	0,06	0,067924528
3	1987	18	247	0,8233333	0,3	0,09	0,109311741
4	1988	27	220	0,7333333	0,4	0,11666667	0,159090909
5	1989	35	185	0,6166667	0,5	0,10333333	0,167567568
6	1990	31	154	0,5133333	0,6	0,15	0,292207792
7	1991	45	109	0,3633333	0,7	0,14333333	0,394495413
8	1992	43	66	0,22	0,8	0,22	1
9	1993	66	0	0	0,9		

The formulas we used are:

$$\mathrm{R}(t) = 1 - \frac{\sum_{num\ of\ failure=0}^{num\ of\ failure\ (i)} \text{number of failure}}{number\ of\ units}$$

$$\hat{R} = \frac{n_i}{n}$$

$$\hat{f}(t) = -\frac{n_{i+1} - n_{i+1}}{(t_{i+1} - t_i) * n}$$

$$\hat{\lambda}(t) = \frac{\hat{f}(t)}{\hat{R}(t)}$$

We want to determine the MTTF and the standard deviation for those data. The formulas are:

$$\widehat{MTTF} = \sum_{i=0}^{k-1} \bar{t_i} * \frac{n_i - n_{i+1}}{\mathrm{n}} \quad \text{and} \quad s^2 = \sum_{i=0}^{k-1} \bar{t_i}^2 * \frac{n_i - n_{i+1}}{\mathrm{n}} - (\widehat{MTTF})^2$$

<u>The calculation details are present below:</u>

t bar	t bar²	(ni-ni+1)/n	t bar * (ni-ni+1)/n	t bar² * (ni-ni+1)/n
0,5	0,25	0,05	0,025	0,0125
1,5	2,25	0,066666667	0,1	0,15
2,5	6,25	0,06	0,15	0,375
3,5	12,25	0,09	0,315	1,1025
4,5	20,25	0,116666667	0,525	2,3625
5,5	30,25	0,103333333	0,568333333	3,125833333
6,5	42,25	0,15	0,975	6,3375
7,5	56,25	0,143333333	1,075	8,0625
8,5	72,25	0,22	1,87	15,895

We can then compute:

$$\widehat{MTTF} = 5{,}6033333\ years$$

$$s^2 = 6{,}025988889 \text{ years}$$

$$\text{Standard deviation} = \sqrt{s^2} = 2{,}45478897 \text{ years}$$

TD 5

We want to estimate the reliability function with the product limit method and rank adjustment method.

a- If we have 12 units at risk

- *With the rank adjustment method*

n	Failure time	R(t)	F(t)	rank increment	adjusted rank	Adjusted Reliability
0	0	1	0		0	1
1	5	0,92307692	0,07692308	1	1	0,94354839
2	12	0,84615385	0,15384615	1	2	0,86290323
3	15+	0,84615385	0,15384615			
4	22	0,76153846	0,23846154	1,1	3,1	0,77419355
5	27	0,67692308	0,32307692	1,1	4,2	0,68548387
6	35+	0,67692308	0,32307692			
7	49	0,58021978	0,41978022	1,25714286	5,45714286	0,58410138
8	71+	0,58021978	0,41978022			
9	73	0,46417582	0,53582418	1,50857143	6,96571429	0,4624424
10	81	0,34813187	0,65186813	1,50857143	8,47428571	0,34078341
11	112+	0,34813187	0,65186813			
12	117	0,17406593	0,82593407	2,26285714	10,7371429	0,15829493

First, we can calculate the reliability R(t) using the formula

$$R(t) = \frac{(n_{i+1} - n)}{(n_{i+2} - n)^a} * R(t_{i-1})$$

With a being 1 if the value is uncensored and 0 if the value is censored.

Then we can compute:

$$F(t) = 1 - R(t)$$

To get the adjusted reliability we calculate the rank increment for each value after a censored data we use:

$$Rank\ increment = \frac{(n + 1 - n_{i-2})}{(1 + n - n_{i-1})}$$

Once we have the rank increment, we can compute the adjusted rank taking the given rank for the uncensored data and adding the rank increment for each rank after a censored data.

Finally, we have the adjusted reliability being

$$\hat{R}(t) = 1 - \frac{(adjusted\ rank - 0{,}3)}{(n + 0{,}4)}$$

We can see that the adjusted reliability is slightly different from the first reliability because it takes more in account the censored data.