Degradation tolerant optimal control design for linear discrete-time systems

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Abstract. This paper develops a degradation tolerant approach based on optimal control. Compared to classical control design, the aim of this work is to decelerate the rate of evolution of degradation by minimizing a quadratic cost function of control input, tracking error and rate of degradation. A linear quadratic regulator (LQR) and tracker (LQT) are expanded, in a finite and infinite-horizon, for a discrete-time linear system in presence of degradation of its components. The performance of the systems is affected by the degradation in an affine manner. First, a LQR is developed for finite and infinite horizon, then the structure of a LQT is designed for finite-horizon. By tuning the weighting matrices, the performance of the system in closed loop can be modified so that the system can achieve its mission before the occurrence of component failure and the remaining useful life can be extended. Electro-Mechanical Actuator (EMA) system is widely used in modern automobiles, transportation and industrial processes and is adopted to verify the effectiveness of the proposed control scheme.

Keywords: Linear Quadratic Tracker · Fault-Tolerant Control · Health Management.

1 Introduction

The evolution of complex and autonomous systems, such as modern aircraft, unmanned aerial vehicles, and automated industrial processes requires the development and the implementation of new control technologies that maintain system stability and that address incipient failure. Traditional control system design focuses only on performance and stability without taking into account the effects of aging, fatigue, and damage to the concerned components. For this purpose, the implementation of Prognostics and Health Management (PHM) systems [12] turns out to be an important part of maintenance activities and the prognostics process becomes one of the main action levers in the search for global performance.

Recent approaches have applied modern control techniques such as adaptive or robust control to address situations where the degree of failure may be unknown. In [2], an adaptive failure control is developed for incipient failure modes, which do not affect the stability of the system, but lead to a catastrophic failure. For various industrial and mission critical systems that operate in closed loop, Fault Tolerant control design has been developed [10], [1] in order to compensate under passive or active approaches fault occurrence but more recently new methods are sought such that useful life of critical systems can be extended. In this context, health-aware control has recently emerged as one of the domains where control synthesis is sought, based upon the state of health (SOH) and/or Remaining Useful Life (RUL) prognostics of critical components [8]. RUL prognostics are obtained using degradation models which estimate their parameters online and adapt the degradation dynamics based upon individual assets. Few notable works have proposed methods to design control laws that seek to extend the RUL of component/system as [13], [14] and [5]. Also, [4] proposed Reliability or RUL based constraints within MPC. In such a framework, a model predictive control (MPC) framework is adopted to produce a controller that adjusts its reference points and ensures robustness to particular failures, thus reducing its impact on the system. In the same framework, [3] presents a new methodology for failure-tolerant design of electromechanical actuator (EMA), it takes advantage of online estimates of the remaining useful life (RUL) of a defaulted component and reconfigures the existing MPC control authority. In [11], a comprehensive framework for monitoring the RUL of the system is developed by using the post-prognostic information. The RUL controller uses a cost function that adjusts the performance requirements and the desired RUL values.

This paper examines a degradation tolerant approach based on optimal control [15], [9] by building a quadratic cost function of control input, tracking error and rate of degradation. First, it is developed for a general discrete-time linear system affected by a linear degradation in an affine manner, then implemented for a specific application. The application example is an EMA, which is widely used in aerospace and industrial processes in place of hydraulic actuators, where a system failure would be potentially dangerous and extremely costly. The usage of EMA brings some specific problems associated to thermal balance, reflected inertia, parasitic motion due failure response [7]. Another important damage mechanism for EMA is the degradation of friction coefficient due to usury. Sliding and rolling friction cause material wear, which enhances the coefficient of friction [6].

This paper is arranged as follows. Section 2 introduces the problem statement and the issue addressed. Section 3 presents the proposed reconfiguration approach in finite and infinite horizon. Section 4 examines the feasibility of the proposed approach using an example of an EMA. Finally, the conclusion summarizes the significant advances and includes plans for future work.

2 Problem Formulation

The degradation of components of an active system affects directly the remaining useful life of the system and consequently its usability and productivity. The state of degradation or deterioration considered as a health indicator are affected by the action of the controller. Hence the importance of development of an optimal approach for performing a control action that takes into account the stability and the performance requirements of the system and also the SOH.
This paper focuses on linear MIMO discrete-time systems represented by the state transition, control and observation matrices, $A_1 \in \mathbb{R}^{n\times n}$, $A_2 \in \mathbb{R}^{n\times l}$, $B_1 \in \mathbb{R}^{n\times m}$ and $C_1 \in \mathbb{R}^{p\times n}$.

$$x_{k+1} = A_1x_k + A_2d_k + B_1u_k \tag{1}$$

$$y_k = C_1x_k \tag{2}$$

where $u \in \mathbb{R}^m$, $x \in \mathbb{R}^n$, $d \in \mathbb{R}^l$ and $y \in \mathbb{R}^p$ correspond respectively to the input, state of the system, state of degradation and measurement vectors. The system is supposed to be affected by the degradation in an affine manner and the degradation evolution is described by the following state-space representation:

$$d_{k+1} = A_3x_k + A_4d_k \tag{3}$$

with $A_3 \in \mathbb{R}^{l\times n}$ and $A_4 \in \mathbb{R}^{l\times l}$. In most cases, the evolution of the degradation is monotonic and irreversible, moreover, it is unknown. In this work, the current state of degradation is assumed to be dependent on the previous state of degradation and also the previous state of the system. The evolution of the degradation is controlled implicitly by the input since it is sensitive to the state of the system.
In order to maintain the performance of the system while minimizing the energy and the speed of evolution of degradation, a quadratic utility function is defined by:

$$\mathscr{U}_k = x_k^TQx_k + u_k^TRu_k + \Delta d_k^TQ_1\Delta d_k \tag{4}$$

where Δd_k is the rate of evolution of degradation described by(5).

$$\Delta d_k = d_{k+1} - d_k = (A_4 - I)d_k + A_3x_k \tag{5}$$

The utility function (4) is used to develop the performance index of a linear quadratic regulator problem, which gives the following quadratic cost function :

$$J_0 = \frac{1}{2}(x_N^T\bar{S}_Nx_N + \Delta d_N^T\bar{P}_N\Delta d_N) + \frac{1}{2}\sum_{k=0}^{N-1} x_k^TQx_k + u_k^TRu_k + \Delta d_k^TQ_1\Delta d_k \tag{6}$$

Q, Q_1, R, $\bar{S}_N$ and $\bar{P}_N$ are symmetric positive definite cost-weighting matrices and $|R|\neq 0$. The initial plant and degradation state are given as x_0 and d_0 respectively.

3 Optimal Reconfiguration Control

In this bellowing section, an optimal control based approach is developed that allows the synthesis of a state feedback control law using the minimization of a quadratic criterion involving the state, the control and the rate of evolution of degradation. The problem posed is to bring the state of equilibrium (zero) starting from a non-zero initial condition. This problem is equivalent to bringing the state to any reference track. Hence, in section 3.1, the solution of the problem is developed for a LQR and then in section 3.2 the solution is extended to a LQT problem.

3.1 Linear Quadratic Regulator

The solution for the LQR is reached by determining the control sequence u_0, u_1, ...,u_{N-1} that minimizes J_0 in (6). Using (5) to eliminate Δd in (6) gives:

$$\begin{aligned} J_0 = \frac{1}{2}[x_N^T(\bar{S}_N + A_3^T\bar{P}_N A_3)x_N + d_N^T(A_4 - I)^T\bar{P}_N(A_4 - I)d_N \\ + d_N^T(A_4 - I)^T\bar{P}_N A_3 x_N + x_N^T A_3^T{}_N(A_4 - I)d_N] \\ + \frac{1}{2}\sum_{k=0}^{N-1}[x_k^T(Q + A_3^T Q_1 A_3)x_k + u_k^T R u_k + x_k^T A_3^T Q_1(A_4 - I)d_k \\ + d_k^T(A_4 - I)^T Q_1 A_3 x_k + d_k^T(A_4 - I)^T Q_1(A_4 - I)d_k] \end{aligned} \tag{7}$$

To solve the problem of LQR, we have first to begin with the Hamiltonian in order to derive the necessary conditions. The Hamiltonian function is defined by the following equation:

$$\begin{aligned} H_k = \frac{1}{2}[x_k^T(Q + A_3^T Q_1 A_3)x_k + u_k^T R u_k + x_k^T A_3^T Q_1(A_4 - I)d_k + d_k^T(A_4 - I)^T Q_1 A_3 x_k \\ + d_k^T(A_4 - I)^T Q_1(A_4 - I)d_k] + \lambda_{k+1}[A_1 x_k + A_2 d_k + B_1 u_k] \end{aligned} \tag{8}$$

where the costate of the system $\lambda_k \in \mathbb{R}^n$ and it's given by:

$$\lambda_k = \frac{\partial H_k}{\partial x_k} = (Q + A_3^T Q_1 A_3)x_k + A_1^T\lambda_{k+1} + A_3^T Q_1(A_4 - I)d_k \tag{9}$$

Solving the stationarity condition $\frac{\partial H_k}{\partial u_k} = 0$, yields to:

$$u_k = -R^{-1}B_1^T\lambda_{k+1} \tag{10}$$

If the optimal λ_k can be found, (10) can therefore be used to find the optimal control. Moreover, the boundary condition is given by:

$$\lambda_N = \frac{\partial \Phi_N}{\partial x_N} = (\bar{S}_N + A_3^T\bar{P}_N A_3)x_N + A_3^T\bar{P}_N(A_4 - I)d_N \tag{11}$$

with

$$\Phi_N = \frac{1}{2}[x_N^T(\bar{S}_N + A_3^T\bar{P}_N A_3)x_N + d_N^T(A_4 - I)^T\bar{P}_N(A_4 - I)d_N \\ + d_N^T(A_4 - I)^T\bar{P}_N A_3 x_N + x_N^T A_3^T\bar{P}_N(A_4 - I)d_N] \tag{12}$$

Thus, assuming that a linear relation like (11) holds for all times $k \leq N$:

$$\lambda_k = S_k x_k + P_k d_k \tag{13}$$

for a sequence of $n \times n$ matrices S_k and $n \times l$ vectors P_k. If a consistent formula for these postulated S_k and P_k can be found, then (13) is a valid assumption. To do this, use (10) and (13) in (1) to get:

$$x_{k+1} = (I + B_1R^{-1}B1^T S_{k+1})^{-1}[(A_1 - B_1R^{-1}B_1^T P_{k+1}A_3)x_k \\ + (A_2 - B_1R^{-1}B_1^T P_{k+1}A_4)d_k] \tag{14}$$

Using (13) and (14) in the costate equation (9) gives:

$$\begin{aligned} S_k x_k + P_k d_k = (Q + A_3^T Q_1 A_3)x_k \\ + A_1^T S_{k+1}(I + B_1R^{-1}B_1^T S_{k+1})^{-1}(A_1 - B_1R^{-1}B_1^T P_{k+1}A_3)x_k \\ + A_1^T S_{k+1}(I + B_1R^{-1}B_1^T S_{k+1})^{-1}(A_2 - B_1R^{-1}B_1^T P_{k+1}A_4)d_k \\ + A_1^T P_{k+1}A_4 d_k + A_1^T P_{k+1}A_3 x_k + A_3^T Q_1(A_4 - I)d_k \end{aligned} \tag{15}$$

This equation must hold for all state sequences x_k and d_k given any x_0 and d_0, therefore we can write :

$$S_k = Q + A_3^T Q_1 A_3 + A_1^T S_{k+1}(I + B_1R^{-1}B_1^T S_{k+1})^{-1}(A_1 - B_1R^{-1}B_1^T P_{k+1}A_3) \\ + A_1^T P_{k+1}A_3 \tag{16}$$

and

$$P_k = A_1^T S_{k+1}(I + B_1R^{-1}B_1^T S_{k+1})^{-1}(A_2 - B_1R^{-1}B_1^T P_{k+1}A_4) \\ + A_1^T P_{k+1}A_4 + A_3^T Q_1(A_4 - I) \tag{17}$$

By comparing (11) and (13), the boundary conditions for these recursions are $S_N = \bar{S}_N + A_3^T\bar{P}_N A_3$ and $P_N = A_3^T\bar{P}_N(A_4 - I)$. Since the auxiliary sequences S_k and P_k can now be computed, assumption (13) was a valid one, and the optimal control is:

$$u_k = -R^{-1}B_1^T(S_{k+1}x_{k+1} + P_{k+1}d_{k+1}) \tag{18}$$

Substituting (1) and (3) in (18) yields to:

$$u_k = -(R + B_1^T S_{k+1}B_1)^{-1}B^T(S_{k+1}A_1 + P_{k+1}A_3)x_k \\ - (R + B_1^T S_{k+1}B_1)^{-1}B^T(S_{k+1}A_2 + P_{k+1}A_4)d_k \tag{19}$$

Defining the gains sequences :

$$K_k^x = (R + B_1^T S_{k+1} B_1)^{-1} B^T (S_{k+1} A_1 + P_{k+1} A_3) \tag{20}$$

$$K_k^d = (R + B_1^T S_{k+1} B_1)^{-1} B^T (S_{k+1} A_2 + P_{k+1} A_4) \tag{21}$$

The control takes the form:

$$u_k = -K_k^x x_k - K_k^d d_k \tag{22}$$

Equations (16) and (17) are solved backwards in time starting from time N. As $k \to -\infty$, the sequences S_k and P_k converge to a steady-state matrices S_∞ and P_∞, then the corresponding steady-state gains are:

$$K_\infty^x = (R + B_1^T S_\infty B_1)^{-1} B^T (S_\infty A_1 + P_\infty A_3) \tag{23}$$

$$K_\infty^d = (R + B_1^T S_\infty B_1)^{-1} B^T (S_\infty A_2 + P_\infty A_4) \tag{24}$$

Note: The augmented system is formed by the state of the system and the degradation, and $A \in \mathbb{R}^{(n+l)\times(n+l)}$, $B \in \mathbb{R}^{(n+l)\times m}$ and $C \in \mathbb{R}^{p\times(n+l)}$ are respectively the state transition, the control and the observation matrices of the augmented system. If (A, B) is stabilizable and (A, C) is detectable, S_k and P_k converge to unique steady-state matrices S_∞ and P_∞.

3.2 Linear Quadratic Tracker

This section synthetizes an optimal control law that forces the system to track a desired reference trajectory r_k over a specified time interval $[0, N]$. The cost function must involve the tracking error, the input, and Δd to force the state to reach the reference and to decelerate the speed of evolution of degradation:

$$\begin{aligned} J_0 = \frac{1}{2}[(C_1 x_N - r_N)^T \bar{S}_N (C_1 x_N - r_N) + \Delta d_N^T \bar{P}_N \Delta d_N] \\ + \frac{1}{2}\sum_{k=0}^{N-1} (C_1 x_k - r_k)^T Q (C_1 x_k - r_k) + u_k^T R u_k + \Delta d_k^T Q_1 \Delta d_k \end{aligned} \tag{25}$$

In this case, the Hamiltonian function becomes:

$$\begin{aligned} H_k = \frac{1}{2}[x_k^T (C_1^T Q C_1 + A_3^T Q_1 A_3) x_k + u_k^T R u_k + d_k^T (A4 - I)^T Q_1 (A_4 - I) d_k + r_k^T Q r_k \\ - x_k^T C_1^T Q r_k - r_k^T Q C_1 x_k + d_k^T (A_4 - I)^T Q_1 A_3 x_k + x_k^T A_3^T Q_1 (A_4 - I) d_k] \\ + \lambda_{k+1} [A_1 x_k + A_2 d_k + B_1 u_k] \end{aligned} \tag{26}$$

The costate equation (9) and the boundary condition (11) are replaced by:

$$\lambda_k = \frac{\partial H_k}{\partial x_k} = (C^T Q C + A_3^T Q_1 A_3) x_k + A_1^T \lambda_{k+1} + A_3^T Q_1 (A_4 - I) d_k - C_1^T Q r_k \tag{27}$$

$$\lambda_N = \frac{\partial \Phi_N}{\partial x_N} = (C_1^T \bar{S}_N C_1 + A_3^T \bar{P}_N A_3) x_N + A_3^T \bar{P}_N (A_4 - I) d_N - C_1^T \bar{S}_N r_N \quad (28)$$

with

$$\begin{aligned}\Phi_N = \frac{1}{2}[x_N^T (C_1^T \bar{S}_N C_1 + A_3^T \bar{P}_N A_3) x_N + d_N^T (A4 - I)^T \bar{P}_N (A_4 - I) d_N + r_N^T \bar{S}_N r_N \\ - x_N^T C_1^T \bar{S}_N r_N - r_N^T \bar{S}_N C_1 x_N + d_N^T (A_4 - I)^T \bar{P}_N A_3 x_N \\ + x_N^T A_3^T \bar{P}_N (A_4 - I) d_N]\end{aligned} \quad (29)$$

From the looks of (28), it seems reasonable to assume that for all $k \leq N$, the costate equation can be written as:

$$\lambda_k = S_k x_k + P_k d_k - v_k \quad (30)$$

Using (30) in the state equation (1) to get:

$$\begin{aligned}x_{k+1} = (I + B_1 R^{-1} B1^T S_{k+1})^{-1} [(A_1 - B_1 R^{-1} B_1^T P_{k+1} A_3) x_k \\ + (A_2 - B_1 R^{-1} B_1^T P_{k+1} A_4) d_k + B_1 R^{-1} B_1^T v_{k+1}]\end{aligned} \quad (31)$$

Using (31) and (30) in the costate equation (27) gives

$$\begin{aligned}S_k x_k + P_k d_k - v_k = {} & [C_1^T Q C_1 + A_3^T Q_1 A_3] x_k \\ & + A_1^T S_{k+1} [I + B_1 R^{-1} B_1^T S_{k+1}]^{-1} [A_1 - B_1 R^{-1} B_1^T P_{k+1} A_3] x_k \\ & + A_1^T S_{k+1} [I + B_1 R^{-1} B_1^T S_{k+1}]^{-1} [A_2 - B_1 R^{-1} B_1^T P_{k+1} A_4] d_k \\ & + A_1^T S_{k+1} [I + B_1 R^{-1} B_1^T S_{k+1}]^{-1} B_1 R^{-1} B_1^T v_{k+1} \\ & + A_1^T P_{k+1} A_4 d_k + A_1^T P_{k+1} A_3 x_k \\ & + A_3^T Q_1 (A_4 - I) d_k - A_1 v_{k+1} - C_1^T Q r_k\end{aligned} \quad (32)$$

This equation must hold for all state sequences x_k and d_k given any x_0 and d_0, therefore allows to write:

$$\begin{aligned}S_k = {} & C^T Q C + A_3^T Q_1 A_3 \\ & + A_1^T S_{k+1} (I + B_1 R^{-1} B_1^T S_{k+1})^{-1} (A_1 - B_1 R^{-1} B_1^T P_{k+1} A_3) + A_1^T P_{k+1} A_3\end{aligned} \quad (33)$$

$$\begin{aligned}P_k = {} & A_1^T S_{k+1} (I + B_1 R^{-1} B_1^T S_{k+1})^{-1} (A_2 - B_1 R^{-1} B_1^T P_{k+1} A_4) \\ & + A_1^T P_{k+1} A_4 + A_3^T Q_1 (A_4 - I)\end{aligned} \quad (34)$$

$$v_k = A_1^T v_{k+1} - A_1^T S_{k+1} (I + B_1 R^{-1} B_1^T S_{k+1})^{-1} B_1 R^{-1} B_1^T v_{k+1} + C^T Q r_k \quad (35)$$

By comparing (28) and (30), the boundary conditions for these recursions are $S_N = C_1^T \bar{S}_N C_1 + A_3^T \bar{P}_N A_3$, $P_N = A_3^T \bar{P}_N (A_4 - I)$ and $v_N = -C^T \bar{S}_N r_N$. Since the sequences S_k, P_k and v_k can be computed, assumption (30) is valid, and the optimal control is:

$$u_k = -K_k^x x_k - K_k^d d_k + K_k^v v_{k+1} \tag{36}$$

with

$$K_k^x = (R + B_1^T S_{k+1} B_1)^{-1} B^T (S_{k+1} A_1 + P_{k+1} A_3) \tag{37}$$

$$K_k^d = (R + B_1^T S_{k+1} B_1)^{-1} B^T (S_{k+1} A_2 + P_{k+1} A_4) \tag{38}$$

$$K_k^v = (R + B_1^T S_{k+1} B_1)^{-1} B^T \tag{39}$$

If (A, B) is stabilizable and (A, C) is detectable, the tracker gains K_k^x, K_k^d and K_k^v reach steady-state values K_∞^x, K_∞^d and K_∞^v as $N \to \infty$.

A disadvantage of this formulation is the need of computing v_k using backward recursion (35) (Fig. 1). To solve this problem, an infinite-horizon LQT in a causal manner was proposed in [6]. In the following, to verify the effectiveness of the developed control schemes, a finite horizon LQT is implemented on an EMA linear model.

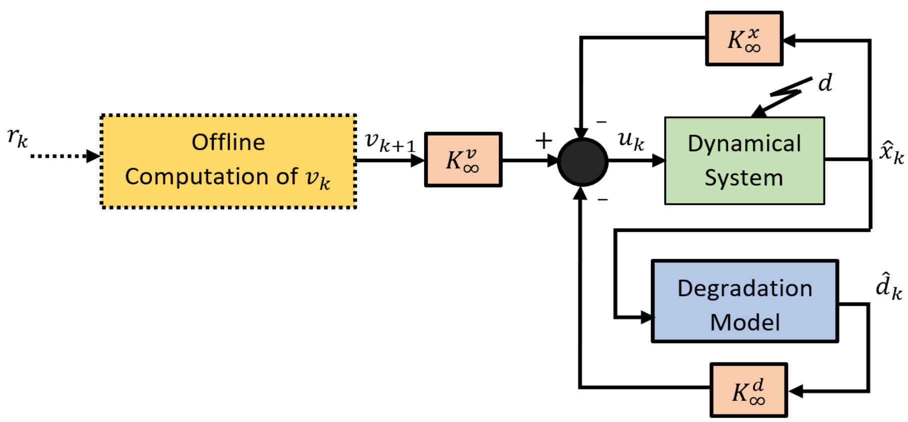


Fig. 1: LQT design

4 EMA Application Example

4.1 Actuator Model

A 5th order state-space model [4] was developed by using (1) and (2) to represent the actuator dynamics. The state vector $x = [i_m\ \theta_m\ \omega_m\ \theta_l\ \omega_l\ ]^T$ is described by the motor current, motor position, motor speed, load position and load speed, the control input $u = [\theta_{ref}]$ is constitute of the reference position (the external load disturbance is supposed to be negligible), and the output vector $y = [i_m\ \theta_l]^T$ is defined by the motor current and the load position. The coefficients of the EMA model are given in Table 1. The transition and the control matrices are provided respectively in (40), (41) .

$$A_1 = \begin{bmatrix} -\frac{R_{tt}}{L_{tt}} & -\frac{K_{p1}K_{p2}}{L_{tt}} & -\frac{K_{p1}+K_e}{L_{tt}} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{K_t}{J_m} & -\frac{K_{cs}}{J_m N_{cm}^2} & -\frac{b_m}{J_m} & \frac{K_{cs}N_{cl}}{J_m N_{cm}} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{K_{cs}N_{cl}}{J_l N_{cm}} & 0 & -\frac{K_l+K_{cs}N_{cl}^2}{J_l} & -\frac{b_l}{J_l} \end{bmatrix} \tag{40}$$

$$B_1 = \begin{bmatrix} \frac{K_{p1}K_{p2}N_{cm}}{L_{tt}N_{cl}} & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{41}$$

All the state of the system and the degradation are supposed to be measurable, but we only want to track the load position, thus :

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{42}$$

4.2 Model of degradation

The winding temperature is proportional to the power loss in the copper windings [2], [3], [4], thus proportional to the absolute value of motor current. The relation between the motor current and the winding-to-ambient temperature can be described by a first order thermo-electrical model with $R_0 = 2.4 \times 10^{-1} A.K/W.s$ and $T_0 = 10^{-3} s$:

$$\dot{d}(t) = R_0 |i_m(t)| + T_0 d(t) \tag{43}$$

Temperature and time affect the electrical endurance qualities of insulation. Moreover, an increase of temperature leads to winding insulation breakdown, which is a primary failure mechanism for the EMA's motors. In the following, only the rate of evolution of temperature is intended to be decelerated.

Remark: The augmented system formed by the EMA system and the state of the winding temperature is stabilizable and detectable. The continuous augmented system is discretized using the Zero-Order Hold method (ZOH) for $T_s = 0.01s$.

4.3 Results and Simulation

The weight matrices Q and R are chosen in such a way to get a null steady state error. When increasing the value of matrix R, a larger penalty is applied to the aggressiveness of the control action, and the control gains are decreasing. Choosing a large value of R means trying to stabilize the system with less energy and by choosing a large value of Q, the error between the load position and the reference has vanished. The main purpose of this work is to decelerate the rate of evolution of winding temperature, this can be achieved by increasing the value of Q_1. In the following, Q and R are fixed and have the values :

Fig. 2: Evolution of load position and winding temperature for different Q_1

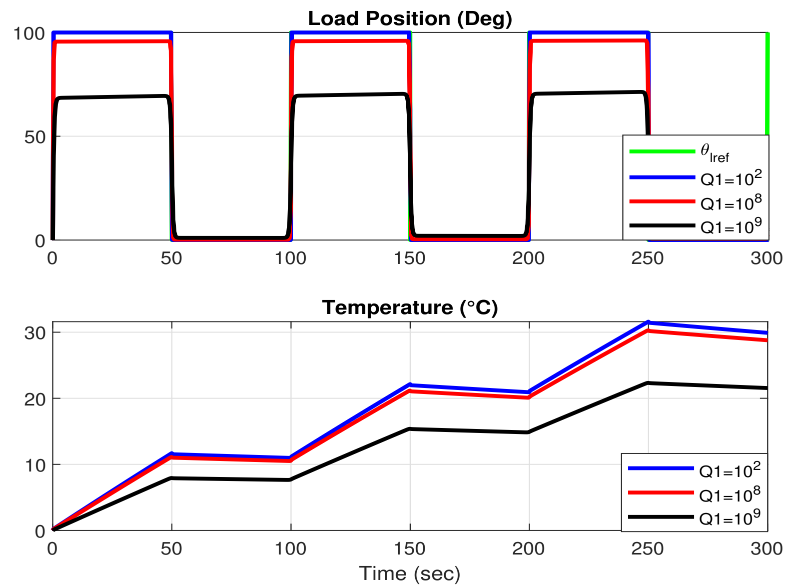
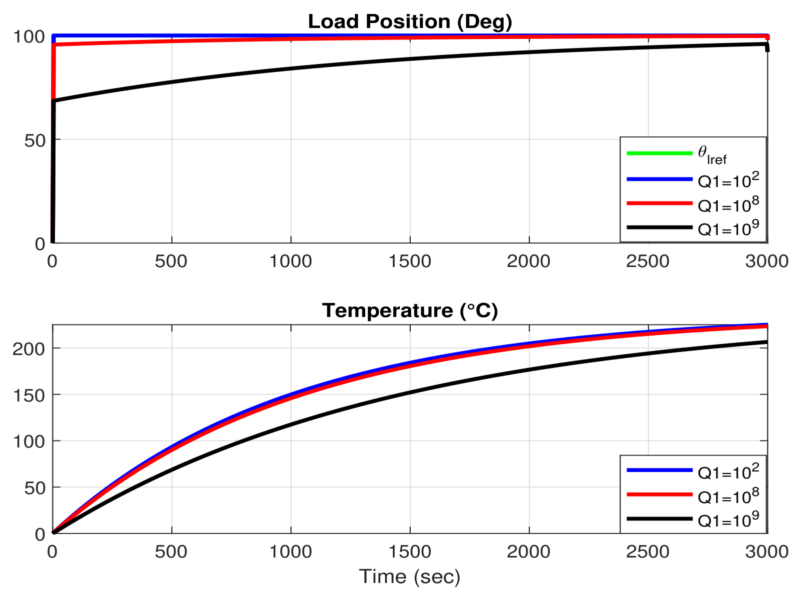


Fig. 3: Evolution of load position and winding temperature for a constant reference track



$$R = I[1] \qquad ; \qquad Q = 40.10^2.I[5]$$

and

$$\bar{S}_N = Q \qquad ; \qquad \bar{P}_N = Q_1$$

For different value of Q_1 and for a finite horizon $N = 30000$, the results are represented in Fig. 2. It can be seen that by increasing the value of Q_1 the steady state error increases and the maximal temperature value decreases.

Moreover, for a constant reference track and for $N = 300000$, Fig. 3 shows that for $Q_1 = 10^9$ the settling time is greater than for $Q_1 = 10^2$ and $Q_1 = 10^8$, and the steady state error is bigger, than it starts to decrease with time. To reduce more the rate of evolution of degradation, the value of Q_1 can be increased but this will increase the rising and the response time.

5 Conclusion and Future Work

Reconfigurable control strategies have gained attention in the control community in recent years to improve the survivability of critical systems under failure conditions. This paper proposed a degradation tolerant control design, a finite and an infinite horizon optimization approach was developed. This approach was examined for a linear electromechanical actuator system and simulation results show its feasibility. By tuning the matrix Q_1, the rate of evolution of winding temperature can be decreased thus prevents winding insulation breakdown.
Future work will focus on how to implement LQG control to estimate non-measurable states. Furthermore, the integration of remaining useful life in the cost function to extend its value.

Table 1: List of used symbols and constants [4].

Sym	Description	Units	Value
b_l	Load damping	in·lbf/rad/s	2.5×10^{-1}
b_m	Motor damping	in·lbf/rad/s	1×10^{-4}
K_{cs}	Coupling stiffness	rad/rad	1×10^{5}
K_e	Back-emf coef.	V/rad/s	1.1×10^{-1}
K_l	Load stiffness	in·lbf/rad/s	2×10^{-3}
K_{p1}	Motor speed gain	V/rad/s	1
K_{p2}	Motor position gain	s^{-1}	1
K_t	Motor torque coef.	in·lbf/A	1.01
J_l	Load inertia	in·lbf·s^2	2×10^{-3}
J_m	Motor inertia	in·lbf·s^2	2.1×10^{-3}
L_{tt}	Turn-to-turn induct.	H	3×10^{-4}
R_{tt}	Turn-to-turn resist	Ω	1.6×10^{-1}
N_{cl}	Load coupling	–	1
N_{cm}	Motor coupling	–	1

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